Oscillator Demonstration

The condition for oscillation to occur in a single loop feedback system is simply that the loop gain must equal $1/0^\circ$. Both the magnitude and phase are important and allow us to solve for (1) the frequency at which oscillation occurs and (2) the value of loop gain at this frequency necessary for oscillation. The loop gain is defined as $-AB$ where $A$ and $B$ are shown in Fig. 1. The

\[
\begin{align*}
V_f & \rightarrow B \\
X_i & = 0 \\
\text{No input for oscillator) \rightarrow A} \\
X_i & \rightarrow X_o \text{ output}
\end{align*}
\]

Fig. 1

minus is due to the inversion that $X_f$ receives as it passes through the summer. $A$ is the amplifier's forward transfer gain and $B$ is the reverse transmission factor. Since we want the condition that

\[
\text{Loop gain} = -AB = 1/0^\circ
\]

(1)

then either the amplifier $A$ or the feedback network $B$ (or both) must contribute a phase shift of $180^\circ$ so that condition of (1) is satisfied.

Let us consider an example of an oscillator shown in Fig. 2. This oscillator is called a Wien bridge oscillator.

\[
\begin{align*}
V_f' & \rightarrow V_f \\
V_f' & \rightarrow Ks^{\omega}_{RC} \\
V_f & \rightarrow RC \\
V_o & \rightarrow \text{Ideal voltage amplifier of gain k}
\end{align*}
\]

Fig. 2

The loop gain can be calculated as $V_f'/V_f$ where the loop has been opened at the amplifier's input. (In opening loops, one must be careful not to change the performance of the network. Since the input impedance of the amplifier is infinite, we have met this requirement. One could also open the loop at the output of the amplifier.) The loop gain is found as

\[
\frac{V_f'}{V_f} = -AB = \frac{Ks^{\omega}_{RC}}{s^2 + 3\omega_{RC}s + \omega_{RC}^2}
\]

where

\[
\omega_{RC} = \frac{1}{RC}
\]

Equating (2) to $1/0^\circ$ gives two equations. They are:

\[
\frac{|V_f'|}{V_f} = \frac{\omega K \omega_{RC}}{\sqrt{(\omega_{RC}^2 - \omega^2)^2 + (3\omega_{RC}\omega)^2}} = 1
\]

(4)
and

\[
\text{Arg}[V_f/V_f] = \frac{\pi}{2} - \tan^{-1}\left[\frac{3\omega_{RC}^2}{\omega_{RC}^2 - \omega^2}\right] = 0^\circ
\]  

(5)

From the phase shift we get the frequency of oscillation. Thus, from (5) we see that when \(\omega = \omega_{RC}\) that \(\text{Arg}[V_f/V_f] = 0^\circ\). Therefore, the frequency of oscillation, \(\omega_0\), is given as

\[
\omega_0 = \frac{1}{RC}
\]  

(6)

The necessary value of \(K\) for oscillation is found from the magnitude expression substituting the value of \(\omega_0\). Thus from (4) and (6) we get

\[
K = 3
\]  

(7)

Consequently, we have now found the operating condition of the oscillator.

Unfortunately, if Fig. 2 was connected with the above values it would probably not oscillate. The reason is that to start the oscillator going requires \(K\) greater than 3. However, once the oscillator is oscillating then \(K\) must return to the value of 3. In addition, there is nothing in the problem that allows us to predict the amplitude of oscillation. Therefore, we must consider some further aspects of practical oscillators.

The normal transfer function of the amplifier \(K\) in Fig. 2 could be represented as shown in Fig. 3.

However if we modify this transfer function through nonlinear methods we can obtain a transfer function such as that shown in Fig. 4.

This transfer function is suitable for determining the amplitude of oscillation and functions in the following manner. At low signal levels the value of \(K\) is greater than 3, thus the oscillator can build up in oscillation. However, as the amplitude grows above \(V_A\), the effective gain of the amplifier begins to decrease because of the lesser value of \(K\) (i.e. \(K_2\)) at higher levels. At some value of magnitude, the effective (or average) value of \(K\) will be exactly 3 and the oscillator will produce a constant level sinusoid. It is difficult to predict this value because of the nonlinearity of the transfer functions. However, it is easy to change the amplitude by simply changing \(K_1, K_2\), or both.

The distortion of the sinusoid will be greatly influenced by the shape of Fig. 4. For low distortion one prefers a very soft (gradual) limiting effect. Thermistors, light bulbs, and other devices which change resistance as the power increases have been used to generate very low distortion sinusoids.
Demonstration Oscillator

B network  Amplifier  Limiter

Secondary Oscillator