QUIZ NO. 6 - SOLUTION

(Average score = 5.6/10 with 5 students not taking the quiz)

A PNP common-collector amplifier is shown. Assume the parameters of the transistor are β_F = 100, V_t = 25mV, and $V_A = \infty$. (a.) If I_C = 1 mA, find the small signal model parameter values for g_m and r_π. (b.) Find an algebraic expression for the small signal input resistance, R_{in}, the output resistance, R_{out}, the voltage gain, v_{out}/v_{in}, and the current gain, i_{out}/i_{in}. (c.) Numerically evaluate the small signal input resistance, R_{in}, the output resistance, R_{out}, the voltage gain, v_{out}/v_{in}, and the current gain, i_{out}/i_{in}.

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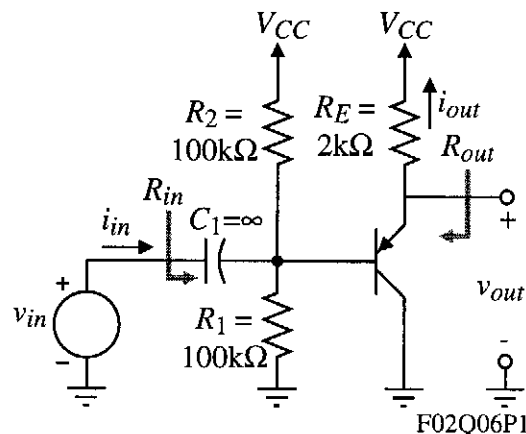
Solution

(a.) $g_m = \dfrac{I_C}{V_t} = \dfrac{1\text{mA}}{25\text{mV}} = \underline{40\text{ mS}}$

and $r_\pi = \dfrac{\beta_F}{g_m} = \dfrac{100}{40\text{mS}} = \underline{2.5\text{k}\Omega}$

(b.) The small-signal model is given below where $R_B = R_1 \| R_2$,

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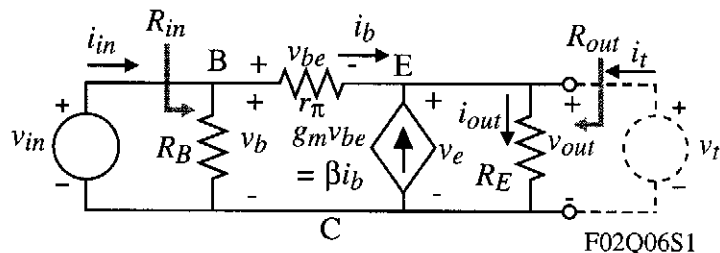
$$\boxed{R_{in} = R_B \| [r_\pi + (1+g_m r_\pi)R_E] = R_B \| [r_\pi + (1+\beta_F)R_E]}$$

For R_{out} we apply the voltage source, v_t, and set $v_{in} = 0$ ($v_{be} = -v_t$) and solve for v_t/i_t which equivalent to R_{out}.

$$i_t = G_E v_t - i_b(1+\beta_F) = G_E v_t + g_\pi(1+\beta_F)v_t = v_t(G_E + g_\pi + g_\pi\beta_F) = v_t(G_E + g_\pi + g_m)$$

$$\therefore R_{out} = \frac{v_t}{i_t} = \frac{1}{G_E + g_m + g_\pi} \rightarrow \boxed{R_{out} = \frac{R_E}{1 + g_m R_E + g_\pi R_E}}$$

The output voltage can be expressed as,

$$\frac{v_{out}}{v_{in}} = \left(\frac{v_{out}}{i_b}\right)\left(\frac{i_b}{i_{in}}\right)\left(\frac{i_{in}}{v_{in}}\right) = [(1+\beta_F)R_E]\left(\frac{R_B}{R_B + r_\pi + (1+\beta_F)R_E}\right)\left(\frac{1}{R_{in}}\right)$$

$$\therefore \boxed{\frac{v_{out}}{v_{in}} = \frac{(1+\beta_F)R_E R_B}{R_{in}[R_B + r_\pi + (1+\beta_F)R_E]}} \quad \text{and} \quad \boxed{\frac{i_{out}}{i_{in}} = \frac{v_{out}/R_E}{v_{in}/R_{in}} = \frac{R_{in}}{R_E}\left(\frac{v_{out}}{v_{in}}\right)}$$

(c.) Numerical evaluation:

$R_{in} = 50\text{k} \| [2.5\text{k}\Omega + (101)2\text{k}\Omega] = \underline{40.17\text{k}\Omega}$ $R_{out} = [0.5\text{mS} + 0.4\text{mS} + 40\text{mS}]^{-1} = \underline{24.45\Omega}$

$$\frac{v_{out}}{v_{in}} = \frac{101 \cdot 2\text{k}\Omega \cdot 50\text{k}\Omega}{40.17\text{k}\Omega[50\text{k}\Omega + 2.5\text{k}\Omega + (101)2\text{k}\Omega]} = \underline{0.988\text{V/V}} \quad \frac{i_{out}}{i_{in}} = \frac{40.17}{2}(0.988) = \underline{19.84\text{ A/A}}$$