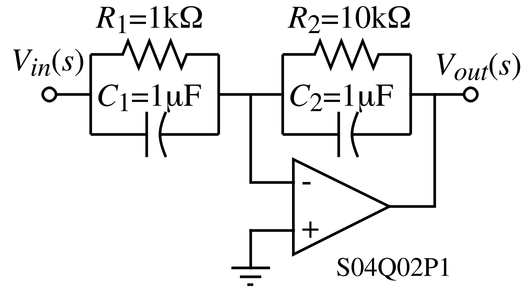
Homework Assignment No. 3 - Solution

1.) A circuit using an ideal op amp is shown.
(a.) Find the s-domain transfer function, $V_{out}(s)/V_{in}(s)$ and solve for the numerical values of all roots.
(b.) Assuming the answer to part (a.) is given below, sketch an asymptotic magnitude and phase frequency response of this transfer function.



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$$\frac{V_{out}(s)}{V_{in}(s)} = -10\left(\frac{s+100}{s+10}\right)$$

Solution

(a.) The easiest way to get the transfer function is to remember the generalized form,

$$\frac{V_{out}(s)}{V_{in}(s)} = -\frac{Z_2(s)}{Z_1(s)}$$

where $Z_2(s) = \dfrac{R_2(1/sC_2)}{R_2 + (1/sC_2)} = \dfrac{R_2}{sC_2R_2 + 1}$ and $Z_1(s) = \dfrac{R_1(1/sC_1)}{R_1 + (1/sC_1)} = \dfrac{R_1}{sC_1R_1 + 1}$

Thus,

$$\frac{V_{out}(s)}{V_{in}(s)} = -\left(\frac{R_2}{R_1}\right)\frac{sC_1R_1 + 1}{sC_2R_2 + 1} = -10\left(\frac{\frac{s}{1000} + 1}{\frac{s}{100} + 1}\right) \rightarrow$$

Pole at -100 rads/s

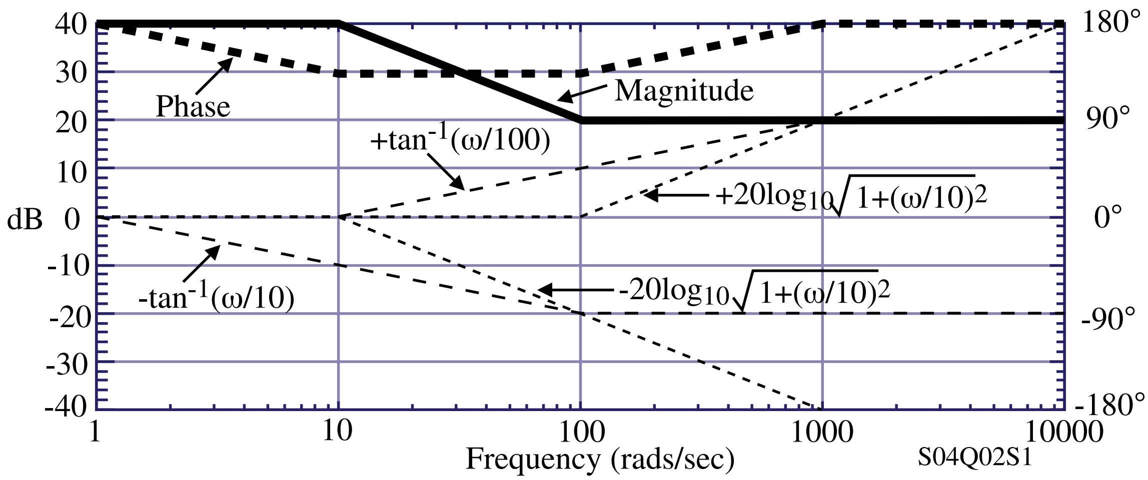
Zero at -1000 rads/s

(b.) If $\dfrac{V_{out}(s)}{V_{in}(s)} = A_v(s) = -10\left(\dfrac{s+100}{s+10}\right) = -100\left(\dfrac{\frac{s}{100}+1}{\frac{s}{10}+1}\right)$, the magnitude and phase are

$$|A_v(j\omega)| = 20\log_{10}(100) + 20\log_{10}\sqrt{1+\left(\frac{\omega}{100}\right)^2} - 20\log_{10}\sqrt{1+\left(\frac{\omega}{10}\right)^2}$$

$$\text{Arg}[A_v(j\omega)] = \pm 180° + \tan^{-1}\left(\frac{\omega}{100}\right) - \tan^{-1}\left(\frac{\omega}{10}\right)$$

Plotting gives,



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2.) The differential amplifier below uses an ideal op amp. Find the values of R_1, R_2, R_3 and R_4 if the single-ended input resistances, R_{in1} and R_{in2} are to be 100kΩ and the output voltage is to be $v_{out} = 10(v_1 - v_2)$.

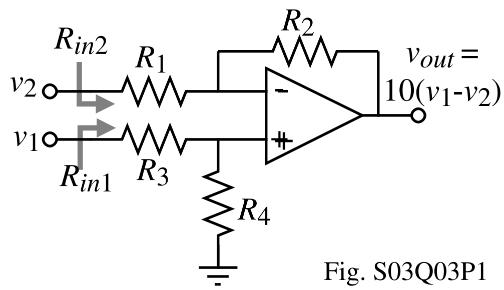


Fig. S03Q03P1

Solution

The first step is to find v_{out} as a function of v_1 and v_2 and to find R_{in1} and R_{in2}.

The output voltage can be found by using superposition applied to the inputs v_1 and v_2. The result is,

$$v_{out} = \left(\frac{v_{out}}{v_1}\right)\Big|_{v_2=0} + \left(\frac{v_{out}}{v_2}\right)\Big|_{v_1=0} = \left(\frac{R_1+R_2}{R_1}\right)\left(\frac{R_4}{R_3+R_4}\right) v_1 - \left(\frac{R_2}{R_1}\right) v_2$$

$R_{in1} = R_3 + R_4$ (remember to set v_2 to zero in this calculation – only one excitation at a time)

$R_{in2} = R_1$ (remember to set v_1 to zero in this calculation – only one excitation at a time)

From the input resistance results, we can write that,

$R_3 + R_4 = 100\text{k}\Omega$ and $\underline{\underline{R_1 = 100\text{k}\Omega}}$

Substituting these values in the voltage gain expression gives,

$$v_{out} = \left(\frac{R_1+R_2}{100\text{k}\Omega}\right)\left(\frac{R_4}{100\text{k}\Omega}\right) v_1 - \left(\frac{R_2}{100\text{k}\Omega}\right) v_2 = 10(v_1 - v_2)$$

This gives us $\underline{\underline{R_2 = 1\text{M}\Omega.}}$ Substituting this back into the voltage gain expression gives,

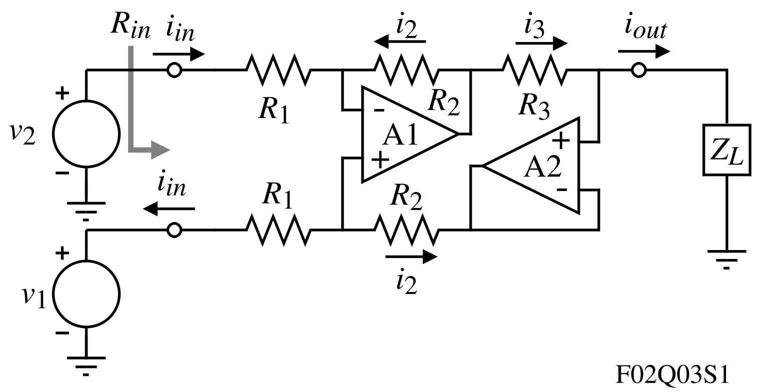
$$v_{out} = \left(\frac{1100\text{k}\Omega}{100\text{k}\Omega}\right)\left(\frac{R_4}{100\text{k}\Omega}\right) v_1 - 10\, v_2 = 10(v_1 - v_2) \rightarrow R_4 = \frac{1000\text{k}\Omega}{11} = \underline{\underline{90.9\text{k}\Omega}}$$

Since the sum of R_3 and R_4 must equal 100kΩ, we get

$R_3 = 100\text{k}\Omega - 90.9\text{k}\Omega = \underline{\underline{9.1\text{k}\Omega}}$

Substituting these values back into the top three equations satisfies the requirements.

3.) Assume that the op amps are ideal and find i_{out} as a function of the inputs, v_1 and v_2. Find the input resistance defined as $R_{in} = (v_2 - v_1)/i_{in}$.



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<u>*Solution*</u>

From the circuit we can write the following equations based on an ideal op amp:

$$i_{out} = i_3, \qquad v_2 - v_1 = 2R_1 i_{in}, \qquad i_2R_2 + i_2R_2 = i_3R_3, \qquad i_{in} = -i_2$$

$$\therefore \qquad i_{out} = i_3 = \frac{2R_2 i_2}{R_3} = \frac{2R_2}{R_3}(-\, i_{in}) = \frac{2R_2}{R_3}\left(-\frac{v_2 - v_1}{2R_1}\right) = \frac{R_2}{R_1R_3}(v_1 - v_2)$$

$$\boxed{i_{out} = \frac{R_2}{R_1R_3}(v_1 - v_2)}$$

The input resistance, R_{in} is seen to be equal to $2R_1$. $\boxed{R_{in} = 2R_1}$

4.) Problem 11.38 (12.24) of the text

Applying op-amp assumption 1 to the circuit on the next page, the voltage at the top of R_2 is v_{O2}, and applying op-amp assumption 2,

$$\frac{\mathbf{v_S}}{R_1} = -\frac{\mathbf{v_{O2}}}{R_2} \quad \text{or} \quad \mathbf{v_{O2}} = -\mathbf{v_S}\frac{R_2}{R_1}$$

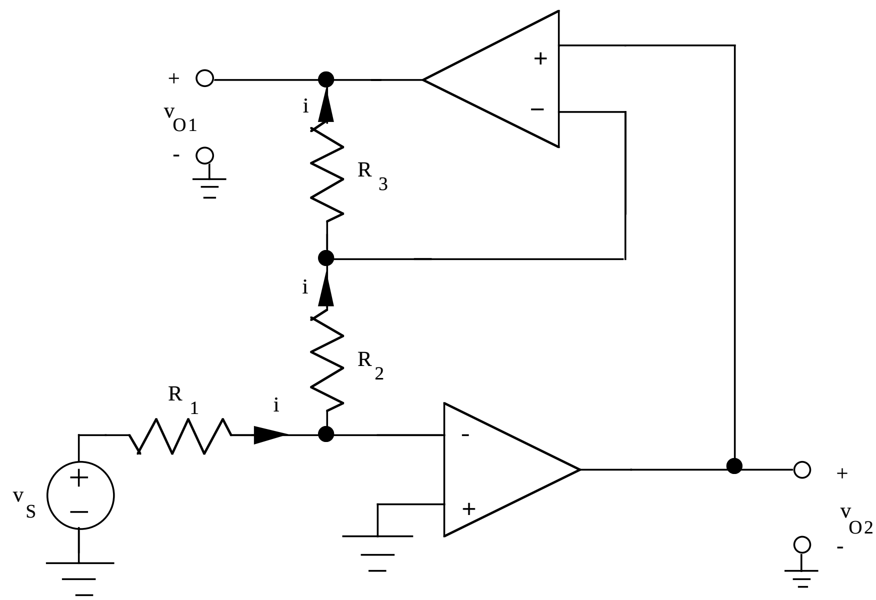
Since the op-amp input currents are zero, and

$$\mathbf{i} = \frac{\mathbf{v_S}}{R_1}, \qquad \mathbf{v_{O1}} = -\mathbf{i}R_2 - \mathbf{i}R_3 = -\left(\frac{R_2}{R_1} + \frac{R_3}{R_1}\right)\mathbf{v_S}$$

Alternatively, the voltage at the bottom of R_2 is zero, so

$$\mathbf{v_{O1}} = \left(1 + \frac{R_3}{R_2}\right)\mathbf{v_{O2}} = \left(1 + \frac{R_3}{R_2}\right)\left(-\frac{R_2}{R_1}\right)\mathbf{v_S} = -\left(\frac{R_2}{R_1} + \frac{R_3}{R_1}\right)\mathbf{v_S}$$

See next page for figure



5.) Problem 11.39 of the text.

$$V_O = -V_{REF}\left(\frac{R}{4R}+\frac{R}{8R}\right) = -3.2\left(\frac{1}{4}+\frac{1}{8}\right) = -1.2\ V$$

$$V_O = -V_{REF}\left(\frac{R}{2R}+\frac{R}{16R}\right) = -3.2\left(\frac{1}{2}+\frac{1}{16}\right) = -1.8\ V$$

0000	0.000 V	1000	-1.60 V
0001	-0.200 V	1001	-1.80 V
0010	-0.400 V	1010	-2.00 V
0011	-0.600 V	1011	-2.20 V
0100	-0.800 V	1100	-2.40 V
0101	-1.00 V	1101	-2.60 V
0110	-1.20 V	1110	-2.80 V
0111	-1.40 V	1111	-3.00 V

6.) Problem 11.98 (12.74) of the text.

$$\beta = \frac{2k\Omega}{2k\Omega + 40k\Omega} = \frac{1}{21} \quad | \quad A\beta = \frac{10^5}{21} = 4760 >> 1$$

$$\text{(a)}\ A_V = -\frac{R_2}{R_1} = -\frac{40k\Omega}{2k\Omega} = -20 \quad | \quad f_H = \beta f_T = \frac{3x10^6\,Hz}{21} = 143kHz$$

$$\text{(b)}\ A_V = (-20)^3 = -8000\ (78dB) \quad | \quad f_{H3} = 0.51f_H = 72.9kHz$$