Homework Assignment No. 5 - Solution

1.) Problem 13.102 (13.91) of the text.

$$g_m = \sqrt{2\left(500\frac{\mu A}{V^2}\right)(100\mu A)(1+0.02(5))} = 332\mu S \quad | \quad r_o = \frac{50+5V}{100\mu A} = 550k\Omega$$

$$A_v = -\left(\frac{6.8M\Omega}{6.8M\Omega+0.1M\Omega}\right)(332\mu S)\left(550k\Omega \| 50k\Omega \| 120k\Omega\right) = -10.9$$

2.) Problem 13.111 (13.100) of the text. [A_v = -4.60 V/V]

$$g_m = \frac{2}{3}\sqrt{1mA(1mA)[1+0.015(9)]} = 710\mu S \quad | \quad r_o = \frac{\frac{1}{0.015}+9V}{1mA} = 75.7k\Omega$$

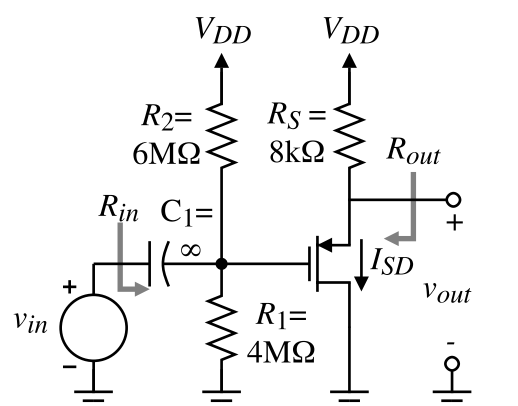
$$A_v = -\left(\frac{1M\Omega}{1M\Omega+10k\Omega}\right)(710\mu S)\left(75.7k\Omega \| 7.5k\Omega \| 160k\Omega\right) = -4.60V/V$$

3.) Problem 13.118 (13.108) of the text.

$$R_{in} = R_G = 6.8M\Omega \quad | \quad R_{out} = 50k\Omega \| r_o$$

$$r_o = \frac{(50+5)V}{0.1mA} = 550k\Omega \quad | \quad R_{out} = 50k\Omega \| 550k\Omega = 45.8k\Omega$$

4.) A PMOS common-drain amplifier is shown. Assume the parameters of the transistor are k_F = 0.5mA/V^2, V_{TP} = -1V, and λ = 0. (a.) If I_{SD} = 0.5mA, find the small signal model parameter values for g_m and r_o. (b.) Find an algebraic expression for the small signal input resistance, R_{in}, the output resistance, R_{out}, and the voltage gain, v_{out}/v_{in}. (c.) Numerically evaluate the small signal input resistance, R_{in}, the output resistance, R_{out}, and the voltage gain, v_{out}/v_{in}.



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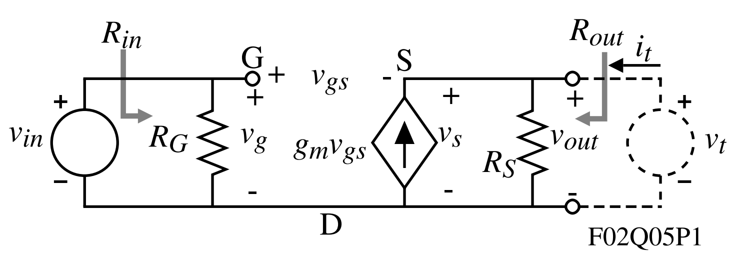
Solution

(a.) $g_m = \sqrt{2I_{SD}k_P} = \sqrt{2 \cdot 0.5 \cdot 0.5}$ mS = $\underline{0.707\text{mS}}$ and $r_o = \underline{\underline{\infty}}$

(b.) First we need a small signal model.



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Obviously, $\boxed{R_{in} = R_G = R_1 \| R_2}$. For R_{out} we apply the voltage source, v_t, and set v_{in} = 0 and solve for v_t/i_t which equivalent to R_{out}.

Problem 4 - Continued

$\therefore\ i_t = G_S v_t - g_m v_{gs} = G_S v_t - g_m(v_g - v_s) = G_S v_t - g_m(0 - v_s)$

$= G_S v_t + g_m v_s = G_S v_t + g_m v_t = (G_S + g_m) v_t$

$$\therefore \quad R_{out} = \frac{v_t}{i_t} = \frac{1}{G_S + g_m} = \frac{R_S}{1 + g_m R_S} \quad \rightarrow \quad \boxed{R_{out} = \frac{R_S}{1 + g_m R_S}}$$

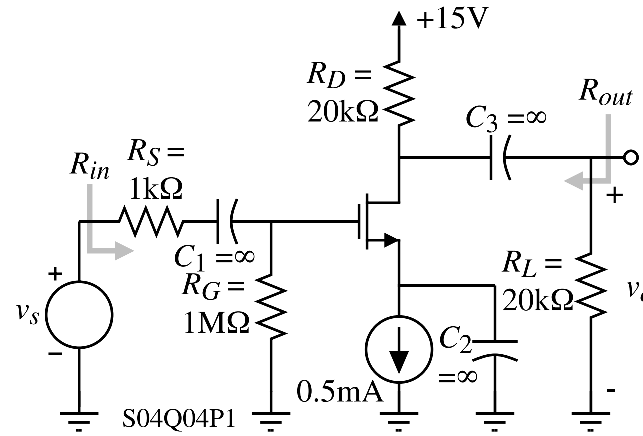
The output voltage can be expressed as,

$$v_{out} = g_m R_S v_{gs} = g_m R_S(v_g - v_s) = g_m R_S(v_{in} - v_{out})$$

$$\therefore \quad v_{out}(1 + g_m R_S) = g_m R_S v_{in} \quad \rightarrow \quad \boxed{\frac{v_{out}}{v_{in}} = \frac{g_m R_S}{1 + g_m R_S}}$$

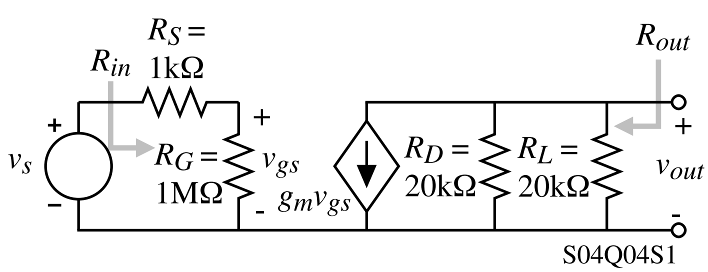
(c.) $R_{in} = R_G = R_1 || R_2 = \underline{2.4\text{M}\Omega}$, $R_{out} = \frac{8\text{k}\Omega}{1+0.707\cdot 8} = \underline{1.2\text{k}\Omega}$ and $\frac{v_{out}}{v_{in}} = \frac{0.707\cdot 8}{1+0.707\cdot 8} = \underline{0.85\text{V/V}}$

5.) A NMOS common-source inverting amplifier is shown. Assume the parameters of the transistor are $K_N = 1\text{mA/V}^2$, $V_{TN} = 1\text{V}$, and $\lambda = 0$. (a.) Find the small signal model parameter values for g_m and r_{ds}. (b.) Find an algebraic expression for the small signal input resistance, R_{in}, the output resistance, R_{out}, and the voltage gain, v_{out}/v_s. (c.) Numerically evaluate the small signal input resistance, R_{in}, the output resistance, R_{out}, and the voltage gain, v_{out}/v_{in}.



Solution

(a.) Small-signal model (Note the dc current source is replaced by an infinite):



Find g_m and r_{ds}: $\quad g_m = \sqrt{2K_N I_D} = \sqrt{2\cdot 1\cdot 0.5} = \underline{1\text{mS}}$ and $r_{ds} = \underline{\underline{\infty}}$ (ignore V_{DS})

(b.) Find R_{in}, R_{out}, and v_{out}/v_s.

$\boxed{R_{in} = R_S + R_G}$ $\quad\quad$ $\boxed{R_{out} = R_D || R_L}$

Problem 5 - Continued

$$\frac{v_{out}}{v_s} = \frac{v_{out}}{v_{gs}}\frac{v_{gs}}{v_s} = (-g_m R_{out})\left(\frac{R_G}{R_G + R_S}\right) = -g_m\left(\frac{R_D R_L}{R_D + R_L}\right)\left(\frac{R_G}{R_G + R_S}\right)$$

$$\boxed{\frac{v_{out}}{v_s} = -g_m \left(\frac{R_D R_L}{R_D + R_L}\right)\left(\frac{R_G}{R_G + R_S}\right)}$$

(c.) Evaluate R_{in}, R_{out}, and v_{out}/v_s..

$R_{in} = 1\text{k}\Omega + 1\text{M}\Omega \approx \underline{\underline{1\text{M}\Omega}}$ $\qquad$ $R_{out} = R_D || R_L = \underline{\underline{10\text{k}\Omega}}$

$$\frac{v_{out}}{v_s} = \frac{v_{out}}{v_{gs}}\frac{v_{gs}}{v_s} = -g_m\left(\frac{R_D R_L}{R_D + R_L}\right)\left(\frac{R_G}{R_G + R_S}\right) = (-1\text{mS})(10\text{k}\Omega)(1) = \underline{\underline{-10\text{V/V}}}$$