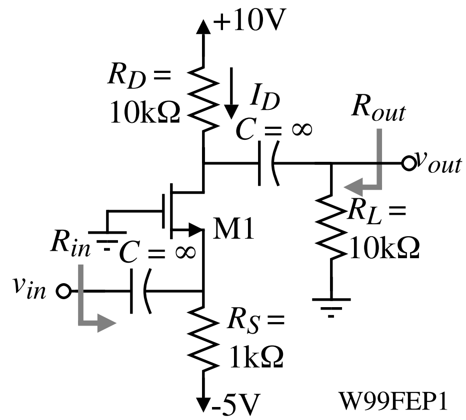
Homework Assignment No. 7 - Solutions

1.) If $K = 0.1\text{mA/V}^2$ and $V_t = 1\text{V}$ for the n-channel MOSFET shown, find the dc value of I_D, V_{GS}, and V_{DS}. If the dc value of $I_D = 1\text{mA}$ (not necessarily the answer above) find the value of the small-signal input resistance, R_{in}, voltage gain, v_{out}/v_{in}, and output resistance, R_{out}.



Solution

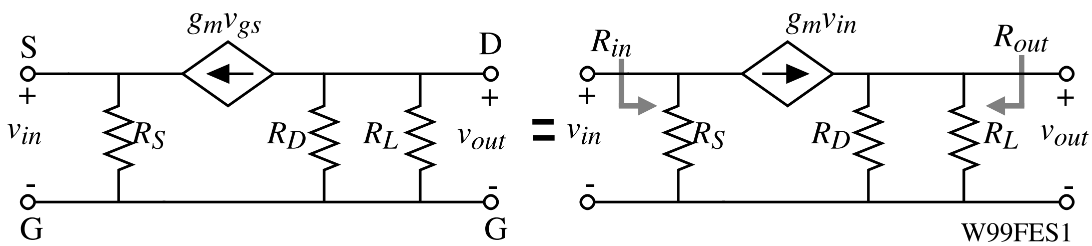
(a.) The key equations are $I_D = K(V_{GS}-V_t)^2$ and $5 = V_{GS}+I_DR_S$. Substituting the first into the second gives, $5 = V_{GS} + 0.1(V_{GS}-1)^2 \rightarrow 5 = V_{GS} + 0.1V_{GS}^2 - 0.2V_{GS} + 0.1$

or $V_{GS}^2 + 8V_{GS} - 49 = 0 \rightarrow V_{GS} = -4 \pm \frac{1}{2}\sqrt{64+4\cdot 49} = -4 \pm 8.0623 = 4.0623\text{V}$

$I_D = 0.1\text{mA}(4.0623-1)^2 = 0.9377\text{mA}$ and $V_{DS} = 15\text{V} - I_D(11\text{k}\Omega) = 4.68\text{V}$

$\therefore$ $\boxed{V_{GS} = 4.0623\text{V},\ I_D = 0.9377\text{mA and } V_{DS} = 4.68\text{V}}$

(b.) The small signal model for this amplifier is shown below. r_{ds} has been neglected since λ was not given. The small signal parameter g_m is: $g_m = 2\sqrt{KI_D} = 0.632\text{mA/V}$



The voltage gain is found as,

$$v_{out} = +g_m(R_D \| R_L)v_{in} \rightarrow \frac{v_{out}}{v_{in}} = g_m(R_D \| R_L) = 0.632\cdot 5 = 3.16\text{V/V}$$

$$\therefore \boxed{\frac{v_{out}}{v_{in}} = 3.16\text{V/V}}$$

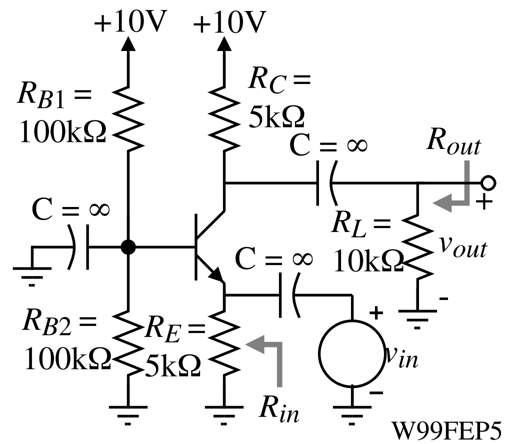
The output resistance is simply $R_D \| R_S$ and is $\boxed{R_{out} = 5\text{k}\Omega}$

The input resistance is found as,

$$i_{in} = \frac{v_{in}}{R_S} + g_m v_{in} \qquad \rightarrow \qquad R_{in} = \frac{v_{in}}{i_{in}} = \frac{1}{R_S + \frac{1}{g_m}} = \frac{1}{g_m} \| R_S = \frac{1\text{k}\Omega}{1 + 0.632} = 613\Omega$$

$\therefore$ $\boxed{R_{in} = 613\Omega}$

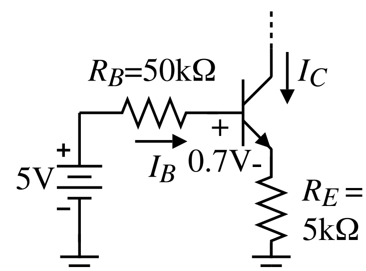
2.) (a.) If ß = 100 and V_T = 25mV of the NPN transistor shown, solve for the dc value of collector current. (b.) If this collector current is 1mA (so if you found I_C incorrectly, you will not be penalized on the rest of the problem) find the small signal values of the input resistance, R_{in}, output resistance, R_{out}, and voltage gain, v_{out}/v_{in}.



W99FEP5

<u>*Solution*</u>

(a.) The circuit used to calculate the collector current is shown. The base current is

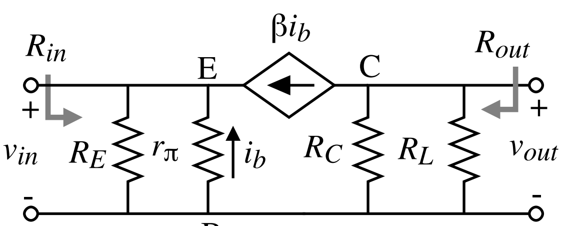


W99FES5A

$$I_B = \frac{5 - 0.7}{50k\Omega + 101\cdot 5k\Omega} = 7.75\mu A$$

$$\therefore \quad \boxed{I_C = 100\cdot I_B = 0.75mA}$$

(b.) The small signal parameters are $g_m = \frac{1mA}{25mV} = 0.04mS$ and $r_\pi = 101\cdot 25\Omega = 2.525k\Omega$. The small-signal model for the above circuit is given as



W99FES5B

$$i_{in} = \frac{v_{in}}{R_E} - (1+\beta)i_b = \frac{v_{in}}{R_E} + \frac{(1+\beta)v_{in}}{r_\pi} \quad \rightarrow \quad \boxed{R_{in} = \frac{v_{in}}{i_{in}} = R_E || \frac{1}{g_m} = 24.87\Omega}$$

$$v_{out} = -\beta i_b(R_C||R_L) = -\beta(R_C||R_L)\frac{-v_{in}}{r_\pi} \quad \rightarrow \quad \boxed{\frac{v_{out}}{v_{in}} = \frac{100\cdot 3.33}{2.525} = 132V/V}$$

Finally, $\boxed{R_{out} = R_C||R_L = 3.33k\Omega}$

14.26

$$I_B = \frac{(5-0.7)V}{1M\Omega + (100+1)430k\Omega} = 96.8nA \;\mid\; I_C = 9.68\mu A \;\mid\; V_{CE} = 10 - 430000 I_E = 5.80V$$

Active region is correct. | $r_\pi = \dfrac{100(0.025V)}{9.68\mu A} = 258k\Omega$ | $r_o = \dfrac{(60+5.80)V}{9.68\mu A} = 6.80M\Omega$ - neglected

In the ac model, R_1 appears in parallel with r_π. The circuit appears to be using a transistor with $r_\pi^{'} = 500k\Omega \| r_\pi = 170k\Omega$ and $\beta_o^{'} = g_m r_\pi^{'} = 40(9.68\mu A)170k\Omega = 65.8$

$R_L = 500k\Omega \| 430k\Omega \| 500k\Omega = 158k\Omega$ | $R_{in} = r_\pi^{'} + (\beta_o^{'}+1)R_L = 170k\Omega + 66.8(158k\Omega) = 10.7\ M\Omega$

$$A_v = \frac{(\beta_o^{'}+1)R_L}{r_\pi^{'} + (\beta_o^{'}+1)R_L}\left(\frac{R_{in}}{R_I + R_{in}}\right) = \frac{66.8(158k\Omega)}{170k\Omega + 66.8(158k\Omega)}\left(\frac{10.7M\Omega}{500\Omega + 10.7M\Omega}\right) = +0.984$$

$$R_{out} = R_E \| R_2 \| \frac{R_I + r_\pi^{'}}{\beta_o^{'}+1} = 430k\Omega \| 500k\Omega \| \frac{500\Omega + 170k\Omega}{66.8} = 2.52\ k\Omega$$

$$v_{be} = v_i \frac{r_\pi^{'}}{R_I + r_\pi^{'} + (\beta_o^{'}+1)R_L} = v_i \frac{170k\Omega}{500\Omega + 170k\Omega + 66.8(158k\Omega)} = 1.58x10^{-2} v_i$$

$$v_i \le \frac{0.005V}{1.58x10^{-2}} = 0.315\ V$$

14.38

$$V_{GS} = -12 + (33k\Omega)I_D \;\mid\; V_{GS} = -12 + \frac{(3.3x10^4)(2x10^{-4})}{2}(V_{GS}+1)^2$$

$$V_{GS} = -2.68V \;\&\; \mathrm{I_D} = \frac{(2x10^{-4})}{2}(V_{GS}+1)^2 = 282\mu A$$

$V_{DS} = -[24 - I_D(33k\Omega + 24k\Omega)] = -7.93\ V$ - Active region operation is correct.

$$g_m = \sqrt{2(2x10^{-4})(2.82x10^{-4})} = 3.36x10^{-4}\,S \;\mid\; \mathrm{R_I} \| R_S = 0.5k\Omega \| 33k\Omega = 493\Omega$$

Assume $\lambda = 0$, $r_o = \infty$ | $R_L = R_D \| R_3 = 24k\Omega \| 100k\Omega = 19.4k\Omega$

$$A_v = \frac{g_m R_L}{1 + g_m(R_I \| R_S)}\left(\frac{R_S}{R_I + R_S}\right) = \frac{0.336mS(19.4k\Omega)}{1 + 0.336mS(493\Omega)}\left(\frac{33k\Omega}{500\Omega + 33k\Omega}\right) = 5.51$$

$$A_i = 1\frac{R_S}{R_S + \frac{1}{g_m}}\left(\frac{R_D}{R_D + R_3}\right) = \frac{33k\Omega}{33k\Omega + 2.98k\Omega}\left(\frac{24k\Omega}{24k\Omega + 100k\Omega}\right) = 0.178$$

$$R_{in} = R_S \left\| \frac{1}{g_m} \right. = 2.73\ k\Omega \;\mid\; R_{out} = R_D = 24k\Omega$$

$$|\mathbf{v}_{gs}| = \mathbf{v}_i \frac{R_{IN}}{R_I + R_{IN}} \le 0.2|V_{GS}+1| \;\mid\; \mathbf{v}_i \frac{2.73k\Omega}{0.5k\Omega + 2.73k\Omega} \le 0.2(1.68) \to \mathbf{v_s} \le 0.398\ V$$

14.39

$$I_B = \frac{(9-0.7)V}{100k\Omega + (50+1)82k\Omega} = 1.94\mu A \;\;|\;\; I_C = 96.9\ \mu A$$

$V_{CE} = 18 - 82000 I_E - 39000 I_C = 6.12\ V$ | Active region operation is correct.

$$g_m = 40 I_C = 3.88mS \;\;|\;\; r_\pi = \frac{\beta_o}{g_m} = 12.9k\Omega \;\;|\;\; r_o = \frac{(50+6.12)V}{96.9\mu A} = 579k\Omega$$ - neglected

$$R_I \| R_E = 0.5k\Omega \| 82k\Omega = 497\Omega \;\;|\;\; R_L = R_C \| R_3 = 39k\Omega \| 100k\Omega = 28.1k\Omega$$

$$A_v = \frac{g_m R_L}{1 + g_m (R_I \| R_E)}\left(\frac{R_E}{R_I + R_E}\right) = \frac{3.88mS(28.1k\Omega)}{1 + 3.88mS(497\Omega)}\left(\frac{82k\Omega}{500\Omega + 82k\Omega}\right) = 37.0$$

$$R_{in} = 82k\Omega \left\| \frac{r_\pi}{\beta_o + 1} \right. = 252\Omega \;\;|\;\; A_i = A_v \frac{R_I + R_{in}}{R_3} = 37.0 \frac{500\Omega + 252\Omega}{100k\Omega} = 0.278$$

$$R_{out} = R_C = 39.0\ k\Omega \;\;|\;\; v_{eb} = v_i \frac{R_{in}}{R_I + R_{in}} \le 5.00mV \;\;|\;\; 0.335 v_i \le 5.00mV \;\;|\;\; v_i \le 14.9\ mV$$