Homework Assignment No. 13 - Solutions

Problem 1
Use the method of feedback analysis to find $v_2/v_1$, $R_{in} = v_1/i_1$, and $R_{out} = v_2/i_2$. Assume that all transistors are matched and that $g_m = 1\text{mS}$. Neglect $r_{ds}$.

Solution

The topology is series-series.

The circuit is redrawn below in order to help identify the various terms for the analysis:

![Circuit Diagram]

Calculation of the $A$ circuit:

$$A = \frac{i_o'}{v_s'} = \left( \frac{i_o'}{v_{gs2'}} \right) \left( \frac{v_{gs2'}}{v_{gs1'}} \right) = \left( -g_m \right) \left( \frac{-g_m R_2}{1 + g_m R_3} \right) \left( \frac{1}{1 + g_m R_3} \right) = (10^{-3})(-5)(-0.5) = 2.5 \text{ mS}$$

$$i_o' = A \frac{v_s'}{g_{mgs1'}} = 2.5 \text{ mS} \frac{1}{1 + 2.5} = 0.714 \text{ mS}$$

Since, $z_{11A} = \infty$, then $R_{in} = 50\text{k}\Omega || \infty = 50\text{k}\Omega$

$$v_2 = \frac{-i_o R_3}{v_s} = -i_o \frac{R_3}{v_s} = -0.714 \text{ mS}(1\text{k}\Omega) = -0.714 \text{ V/V}$$

$$R_o = (z_{22F} + R_3)(1 + A\beta) = \left( \frac{1}{g_m} + R_3 \right)(1 + A\beta) = 2\text{k}\Omega(3.5) = 7\text{k}\Omega$$

However, the output resistance specified by the problem, $R_{out}$, is found as

$$R_{out} = (R_o - R_3) || R_3 = 6\text{k}\Omega || 1\text{k}\Omega = 857\Omega$$
Problem 2
A feedback amplifier is shown. Use the methods of feedback analysis to find the numerical values of \( v_2/v_1 \), \( v_1/i_1 \), and \( v_2/i_2 \). Assume that all transistors are matched and that \( g_m = 1 \text{mS} \), and \( r_{ds} = \infty \).

Solution
The feedback circuit is shunt-series. You should know by now not to include \( R_1 \) in the feedback circuit. The simplified transistor circuit and the open-loop equivalent is shown below. It is easy to see that \( \beta = i_1F/i_2F (v_1F=0) = -1 \).

The small-signal, open-loop circuit is,

\[
A = \frac{i_o'}{i_1'} = \left( \frac{g_{m1}}{v_{gs1}} \right) \left( \frac{v_{gs2}}{v_{gs2}'} \right) \left( \frac{v_{g2}'}{v_{g2}} \right) \left( \frac{v_{gs1}'}{v_{gs1}} \right) \left( \frac{1}{i_1'} \right)
\]

Also, \( i_1' + \frac{v_{gs1}'}{R_4} + g_m v_{gs1}' = 0 \) \( \Rightarrow \)

\[
A = \frac{i_o'}{i_1'} = \left( \frac{1}{1+g_m R_3} \right) \left( -g_m R_2 \right) \left( \frac{-1}{R_4 + g_m} \right) = (-1 \text{mS})(0.5)(-20)(-0.5 \text{k}\Omega) = -5 \text{ A/A}
\]

Now, \( \frac{i_o}{i_1} = \frac{-5}{1+(-1)(-5)} = \frac{5}{6} \text{ A/A} \)

\[
R_{in}(\beta=0) = R_4||g_m = 0.5 \text{k}\Omega, \quad R_{inf} = \frac{0.5 \text{k}\Omega}{1+5} = \frac{1}{12} \text{k}\Omega = 83.33 \Omega, \quad \frac{v_1}{i_1} = \frac{1083.33}{5} \text{ A/A}
\]

\[
v_2 = \frac{i_o}{i_1} \left( \frac{-R_3}{v_1} \right) = \frac{5}{6} \times 1083.33 = \frac{10}{13} = 0.7692 \text{ V/V}
\]

\[
R_o(\beta=0) = R_3+(1/g_m) = 2 \text{k}\Omega, \quad R_{oF} = 2 \text{k}\Omega(1+5) = 12 \text{k}\Omega
\]

\[
\frac{v_2}{i_2} = (12 \text{k}\Omega -1 \text{k}\Omega)||1 \text{k}\Omega = 11 \text{k}\Omega||1 \text{k}\Omega = \frac{11}{12} \text{k}\Omega = 916.67 \Omega
\]
Problem 3 - The amplifier in the feedback circuit shown has a transfer function of
\[ A(s) = \frac{100}{s + \frac{10^5}{100} + 1} \]

What value of \( \beta \) will increase the upper –3db frequency by a factor of 10 for the closed loop gain? What is the closed loop, low frequency gain?

**Solution**

\[
AF = \frac{A}{1 + A\beta} = \frac{1}{\frac{s/10^5 + 1}{100} + \beta} = \frac{100}{1 + 100\beta} \frac{s}{s + 10^5(1 + 100\beta) + 1}
\]

\[
\therefore 10^5(1 + 100\beta) = 10^6 \rightarrow 1 + 100\beta = 10 \rightarrow \beta = 9/100 = 0.09
\]

The closed-loop, low frequency gain is,
\[
AF(0) = \frac{100}{1 + 100\beta} = \frac{100}{1 + 9} = 10 \rightarrow AF(0) = 10
\]

4.) Problem 18.40 of the text.

\[
T = \frac{v_o}{v_i} = \frac{g_m}{r_o + r_{\pi}} = \frac{(\beta_o + 1)R}{r_o + r_{\pi} + (\beta_o + 1)R} \quad | \quad g_m = 40(10^{-4}) = 4.00mS
\]

\[
r_o = 50 + 1.4 = 514k\Omega \quad | \quad r_{\pi} = 50 + 11.3 = 613k\Omega \quad | \quad r_{\pi} = \frac{100(0.025)}{12V/10k\Omega} = 2.08k\Omega
\]

\[
T = \frac{(4 \times 10^{-3})(280k\Omega)}{280k\Omega + 2.08k\Omega + 101(10k\Omega)} = 876 \quad (58.9 \text{ dB})
\]

5.) Problem 18.59 of the text.

\[
2 \times 10^{14} \pi^2
\]

\[A(s) = \frac{\left(\frac{2\pi \times 10^3}{s}\right)\left(\frac{2\pi \times 10^5}{s}\right)}{\left(1 + \frac{s}{2\pi \times 10^3}\right)\left(1 + \frac{s}{2\pi \times 10^5}\right)} = \frac{5 \times 10^5}{\left(1 + \frac{s}{2\pi \times 10^3}\right)\left(1 + \frac{s}{2\pi \times 10^5}\right)}
\]

\[A(s) \text{ represents a low - pass amplifier with two widely - spaced poles}
\]

**Open - loop** : \( A_o = 5 \times 10^5 = 114 dB \quad | \quad f_L = 0 \quad | \quad f_H = f_1 = 1000 \text{ Hz}

(b) A common mistake would be the following :

**Closed - loop** : \( f_H = 1000Hz\left[1 + 5 \times 10^5(0.01)\right] = 5MHz

**Oops!** - This exceeds \( f_2 = 100 \text{ kHz}! \) This is a two - pole low - pass amplifier.

\[2 \times 10^{14} \pi^2
\]

\[A_o(s) = \frac{\left(\frac{2\pi \times 10^3}{s}\right)^2 \left(\frac{2\pi \times 10^5}{s}\right)^2}{1 + \left(\frac{2\pi \times 10^3}{s}\right)^2 \left(\frac{2\pi \times 10^5}{s}\right)^2 (0.01)^2} = \frac{2 \times 10^{14} \pi^2}{s^2 + 1.01(2\pi \times 10^3)s + 2 \times 10^{12} \pi^2}
\]

Using dominant - root factorization : \( f_1 = 101 \text{ kHz}, \quad f_2 = 4.95 \text{ MHz}

So the closed - loop values are \( f_H = 101 \text{ kHz} \text{ and } f_L = 0.\)