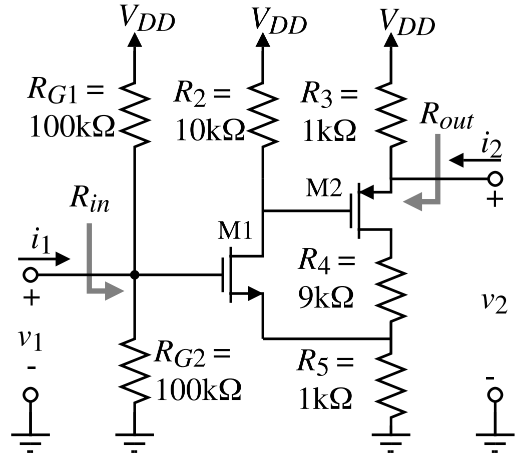
Homework Assignment No. 13 - Solutions

Problem 1

Use the method of feedback analysis to find v_2/v_1, $R_{in} = v_1/i_1$, and $R_{out} = v_2/i_2$. Assume that all transistors are matched and that $g_{m1} = g_{m2} =$ 1mS. Neglect r_{ds}.

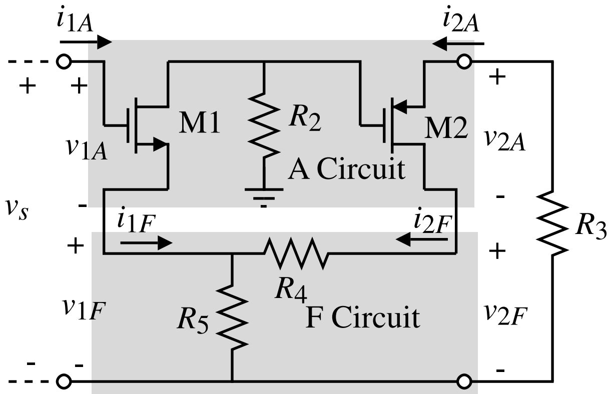


F02H12P4

Solution

The topology is series-series.

The circuit is redrawn below in order to help identify the various terms for the analysis:



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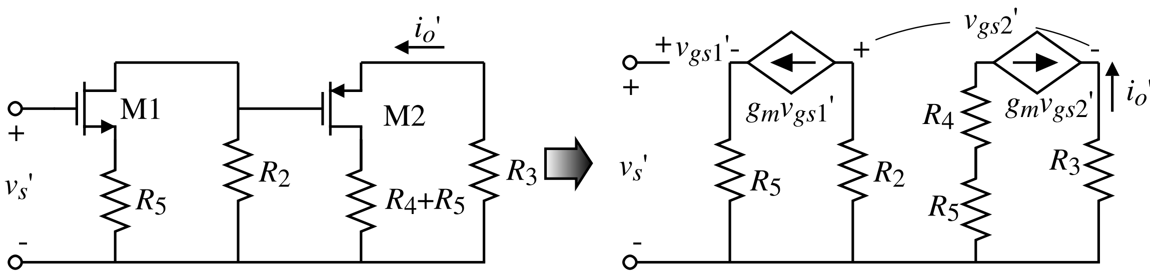
$$z_{11F} = \frac{v_{1F}}{i_{1F}}\bigg|_{i_{2F}=0} = R_5 = 1\text{k}\Omega$$

$$z_{22F} = \frac{v_{2F}}{i_{2F}}\bigg|_{i_{1F}=0} = R_4 + R_5 = 10\text{k}\Omega$$

$$z_{12F} = \beta = \frac{v_{1F}}{i_{2F}}\bigg|_{i_{1F}=0} = R_5 = 1\text{k}\Omega$$

Calculation of the A circuit:



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$$A = \frac{i_o'}{v_s'} = \left(\frac{i_o'}{v_{gs2}'}\right)\left(\frac{v_{gs2}'}{v_{gs1}'}\right)\left(\frac{v_{gs2}'}{v_s'}\right) = (-g_m)\left(\frac{-g_m R_2}{1+g_m R_3}\right)\left(\frac{1}{1+g_m R_5}\right) = (10^{-3})(-5)(-0.5) = 2.5\text{ mS}$$

$$\therefore \frac{i_o}{v_s} = \frac{A}{1+A\beta} = \frac{2.5\text{mS}}{1+2.5} = 0.714\text{mS}$$

Since, $z_{11A} = \infty$, then $R_{in} = 50\text{k}\Omega \| \infty =$ <u>50kΩ</u>

$$\frac{v_2}{v_1} = \frac{-i_o R_3}{v_s} = \frac{-i_o}{v_s} R_3 = -0.714\text{mS}(1\text{k}\Omega) = \underline{\underline{-0.714\text{ V/V}}}$$

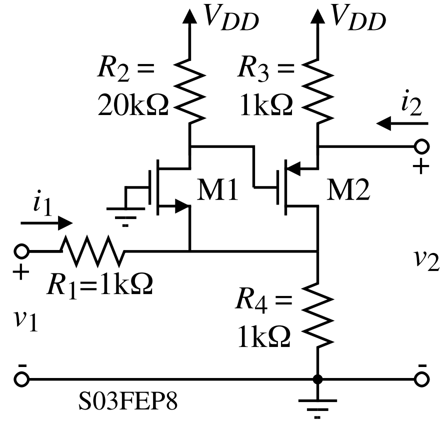
$$R_o = (z_{22T} + R_3)(1+A\beta) = \left(\frac{1}{g_m} + R_3\right)(1+A\beta) = 2\text{k}\Omega(3.5) = 7\text{k}\Omega$$

However, the output resistance specified by the problem, R_{out}, is found as

$$R_{out} = (R_o - R_3) \| R_3 = 6\text{k}\Omega \| 1\text{k}\Omega = \underline{857\Omega}$$

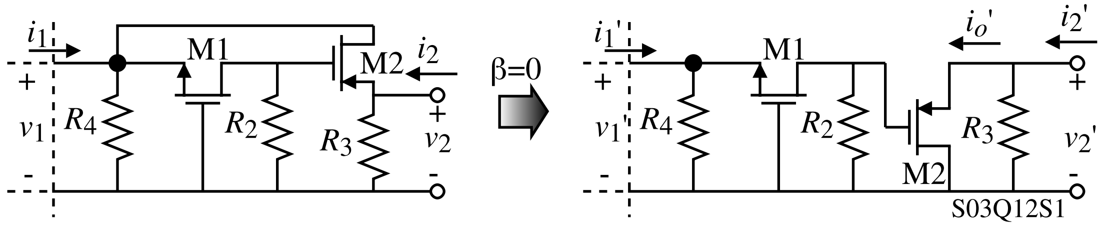
Problem 2

A feedback amplifier is shown. Use the methods of feedback analysis to find the numerical values of v_2/v_1, v_1/i_1, and v_2/i_2. Assume that all transistors are matched and that g_m= 1mS, and $r_{ds} = \infty$.

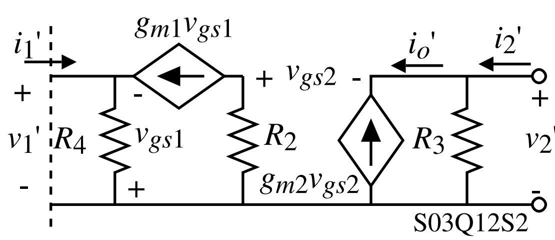


Solution

The feedback circuit is shunt-series. You should know by now not to include R_1 in the feedback circuit. The simplified transistor circuit and the open-loop equivalent is shown below. It is easy to see that $\beta = i_{1F}/i_{2F}\ (v_{1F}=0) = -1$.



The small-signal, open-loop circuit is,



$$A = \frac{i_o'}{i_1'} = \left(\frac{i_o'}{v_{gs2}'}\right)\left(\frac{v_{gs2}'}{v_{g2}'}\right)\left(\frac{v_{g2}'}{v_{gs1}'}\right)\left(\frac{v_{gs1}'}{i_1'}\right)$$

$$v_{gs2}' = v_{g2}' - v_{s2}' = v_{g2}' - g_{m2}R_3\, v_{gs2}'$$

$$\therefore \quad v_{gs2}'(1+ g_{m2}R_3) = v_{g2}'$$

Also, $i_1' + \frac{v_{gs1}'}{R_4} + g_{m1}v_{gs1}' = 0 \qquad \Rightarrow \qquad i_1' = -\left(\frac{1}{R_4} + g_{m1}\right) v_{gs1}'$

$$\therefore \ A = \frac{i_o'}{i_1'} = (-g_{m2})\left(\frac{1}{1+g_{m2}R_3}\right)(-g_{m1}R_2)\left(\frac{-1}{\frac{1}{R_4}+g_{m1}}\right) = (-1\text{mS})(0.5)(-20)(-0.5\text{k}\Omega) = -5 \text{ A/A}$$

Now, $\frac{i_o}{i_1} = \frac{-5}{1+(-1)(-5)} = -\frac{5}{6}$ A/A

$$R_{in}(\beta=0) = R_4 \| (1/g_{m1}) = 0.5\text{k}\Omega,\ R_{inF} = \frac{0.5\text{k}\Omega}{1+5} = \frac{1}{12}\text{k}\Omega = 83.33\Omega \qquad \frac{v_1}{i_1} = \underline{\underline{1083.33\Omega}}$$

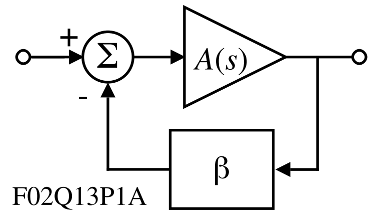
$$\frac{v_2}{v_1} = \frac{i_o}{i_1}\left(\frac{-R3}{(v_1/i_1)}\right) = -\frac{5}{6}\,\frac{-1000}{1083.33} = \frac{10}{13} = \underline{\underline{0.7692 \text{ V/V}}}$$

$$R_o(\beta=0) = R_3 + (1/g_{m2}) = 2\text{k}\Omega, \qquad R_{oF} = 2\text{k}\Omega(1+5) = 12\text{k}\Omega$$

$$\frac{v_2}{i_2} = (12\text{k}\Omega - 1\text{k}\Omega)\|1\text{k}\Omega = 11\text{k}\Omega\|1\text{k}\Omega = \frac{11}{12}\text{k}\Omega = \underline{\underline{916.67\Omega}}$$

Problem 3 - The amplifier in the feedback circuit shown has a transfer function of

$$A(s) = \frac{100}{\frac{s}{10^5}+1}$$



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What value of β will increase the upper –3db frequency by a factor of 10 for the closed loop gain? What is the closed loop, low frequency gain?

Solution

$$A_F = \frac{A}{1+A\beta} = \frac{1}{\frac{1}{A}+\beta} = \frac{1}{\frac{s/10^5+1}{100}+\beta} = \frac{100}{\frac{s}{10^5}+1+100\beta} = \left(\frac{100}{1+100\beta}\right)\frac{1}{\frac{s}{10^5(1+100\beta)}+1}$$

$$\therefore\ 10^5(1+100\beta) = 10^6 \quad\rightarrow\quad 1+100\beta = 10 \quad\rightarrow\quad \underline{\underline{\beta = 9/100 = 0.09}}$$

The closed-loop, low frequency gain is,

$$A_F(0) = \frac{100}{1+100\beta} = \frac{100}{1+9} = 10 \quad\rightarrow\quad \underline{\underline{A_F(0) = 10}}$$

4.) Problem 18.40 of the text.

$$T = \frac{v_o}{v_x} = g_{m2}\left(r_{o2}\|r_{o4}\right)\frac{(\beta_o+1)R}{\left(r_{o2}\|r_{o4}\right)+r_{\pi 3}+(\beta_o+1)R} \quad | \quad g_{m1} = 40\left(10^{-4}\right) = 4.00mS$$

$$r_{o2} = \frac{50+1.4}{10^{-4}} = 514k\Omega \quad | \quad r_{o4} = \frac{50+11.3}{10^{-4}} = 613k\Omega \quad | \quad r_{\pi 3} = \frac{100(0.025)}{(12V/10k\Omega)} = 2.08k\Omega$$

$$T = \left(4x10^{-3}\right)(280k\Omega)\frac{(101)10k\Omega}{280k\Omega+2.08k\Omega+101(10k\Omega)} = 876 \quad (58.9\ dB)$$

5.) Problem 18.59 of the text.

$$(a)\ \ A(s) = \frac{\frac{2x10^{14}\pi^2}{\left(2\pi x10^3\right)\left(2\pi x10^5\right)}}{\left(1+\frac{s}{2\pi x10^3}\right)\left(1+\frac{s}{2\pi x10^5}\right)} = \frac{5x10^5}{\left(1+\frac{s}{2\pi x10^3}\right)\left(1+\frac{s}{2\pi x10^5}\right)}$$

$A(s)$ represents a low-pass amplifier with two widely-spaced poles

Open-loop: $A_o = 5x10^5 = 114dB \quad | \quad f_L = 0 \quad | \quad f_H \cong f_1 = 1000\ Hz$

(b) A common mistake would be the following:

Closed-loop: $f_H = 1000Hz\left[1+5x10^5(0.01)\right] = 5MHz$

Oops! - This exceeds $f_2 = 100\ kHz$! This is a two-pole low-pass amplifier.

$$A_v(s) = \frac{\frac{2x10^{14}\pi^2}{\left(s+2\pi x10^3\right)\left(s+2\pi x10^5\right)}}{1+\frac{2x10^{14}\pi^2}{\left(s+2\pi x10^3\right)\left(s+2\pi x10^5\right)}(0.01)} = \frac{2x10^{14}\pi^2}{s^2+1.01\left(2\pi x10^5\right)s+2x10^{12}\pi^2}$$

Using dominant-root factorization: $f_1 = 101\ kHz,\ \ f_2 = 4.95\ MHz$

So the closed-loop values are $f_H = 101\ kHz$ *and* $f_L = 0$.