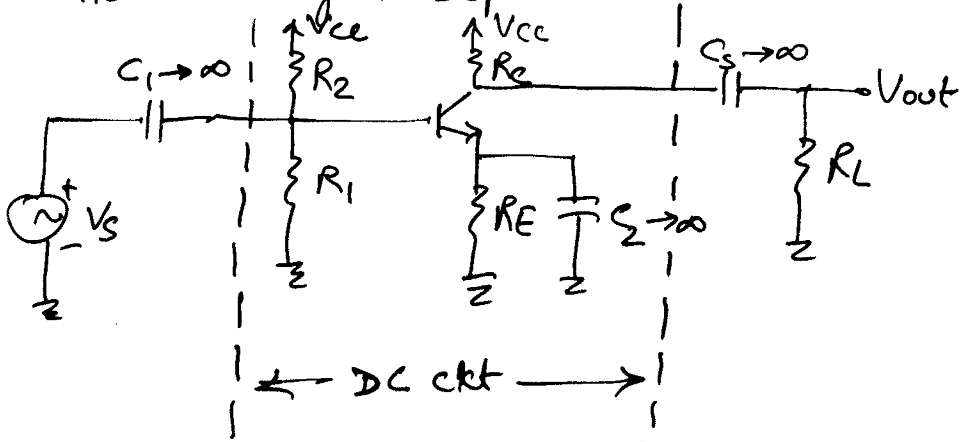
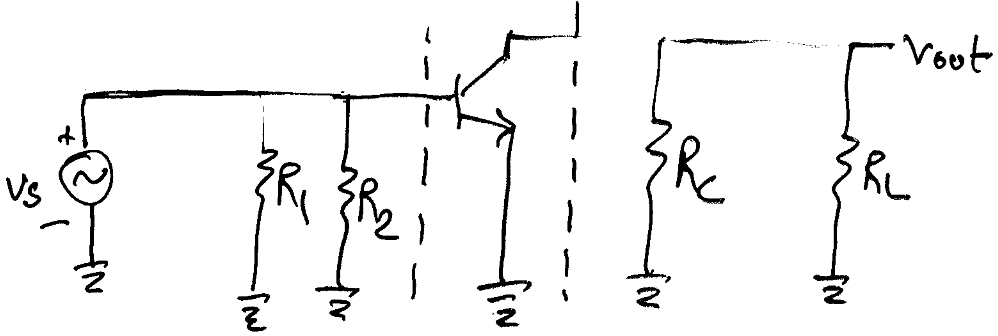
3) Coupling and Bypass Capacitors

How do you separate the ac and dc signals?

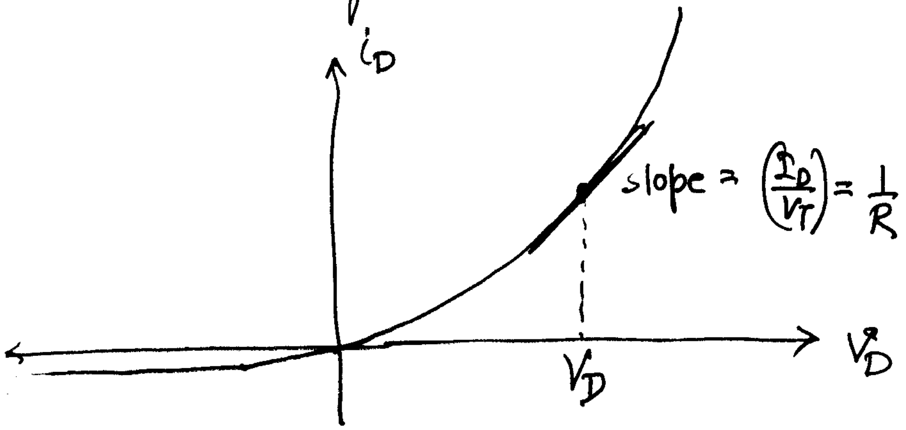


↓

AC ckt.
1. All ∞ caps are shorted
2. All dc voltages are shorted
3. All dc currents sources are opened



4) Diode small-signal model → ac model



Large signal: $i_D = I_S \left[\exp\left(\frac{v_D}{V_T}\right) - 1\right]$

$$\underset{(ss)}{i_d} + \underset{(dc)}{I_D} = I_S \left[\exp\left(\frac{\overset{ss}{v_d} + \overset{dc}{V_D}}{V_T}\right) - 1\right]$$

$$= I_S \left[\exp\left(\frac{v_d}{V_T}\right) \cdot \exp\left(\frac{V_D}{V_T}\right) - 1\right]$$

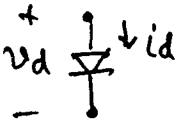
When $v_d \ll V_T$, ~~then~~ $\exp\left(\frac{v_d}{V_T}\right) \cong 1 + \overset{\checkmark}{\frac{v_d}{V_T}} + \overset{\times}{\frac{1}{2}\frac{v_d^2}{V_T^2}} + \cdots$

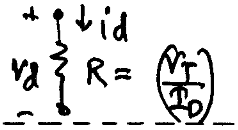
$$i_d + I_D = I_S \left[\exp\left(\frac{V_D}{V_T}\right)\left(1 + \frac{v_d}{V_T}\right) - 1\right]$$

$$i_d + I_D = \underbrace{I_S \left[\exp\left(\frac{V_D}{V_T}\right) - 1\right]}_{I_D} + \left[I_S \exp\left(\frac{V_D}{V_T}\right)\right]\left(\frac{v_d}{V_T}\right)$$

$$SS \rightarrow i_d = \left[I_S \exp\left(\frac{V_D}{V_T}\right)\right]\left(\frac{v_d}{V_T}\right) \cong \left(\frac{I_D + I_S}{V_T}\right) v_d \cong \left(\frac{I_D}{V_T}\right) v_d$$

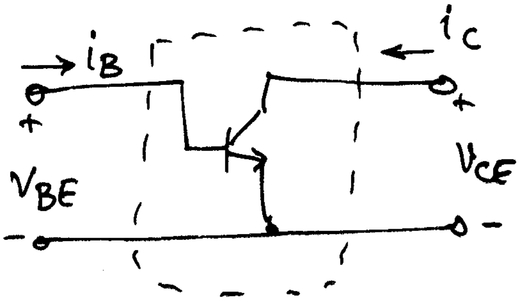
$$i_d \cong \left(\frac{I_D}{V_T}\right) v_d$$





Small-signal BJT model

CE BJT



Forward active
- BE is forward biased
- CB is reversed biased

$$\text{Large-signal}\begin{cases} i_C = I_S\left[\exp\left(\frac{V_{BE}}{V_T}\right)\right]\left[1+\frac{V_{CE}}{V_A}\right] \\ i_B = \frac{I_S}{\beta_F}\left[\exp\left(\frac{V_{BE}}{V_T}\right)\right]\end{cases}$$

$$\downarrow$$

Linearize

$$i_C = K_1 V_{BE} + K_2 V_{CE}$$

$$i_b = K_3 V_{BE}$$

$$K_1 = \left.\frac{i_c}{V_{BE}}\right|_{V_{CE}=0} = \left.\frac{\partial i_c}{\partial V_{BE}}\right|_{V_{CE}=0} = \left(\frac{I_c}{V_T}\right) = g_m$$

$$K_2 = \left.\frac{i_C}{V_{CE}}\right|_{V_{BE}=0} = \left.\frac{\partial i_c}{\partial V_{CE}}\right|_{V_{BE},Q} = \frac{I_c}{(V_A+V_{CE})} \simeq \frac{I_c}{V_A} \simeq g_o$$

$$K_3 = \left.\frac{i_b}{V_{BE}}\right|_{Q} = \left.\frac{\partial i_B}{\partial V_{BE}}\right|_{Q} = \frac{I_c}{\beta_F V_T} = g_\pi = \frac{1}{r_\pi}$$

$i_c = g_m v_{BE} + g_o v_{CE}$

$i_b = g_\pi v_{BE}$