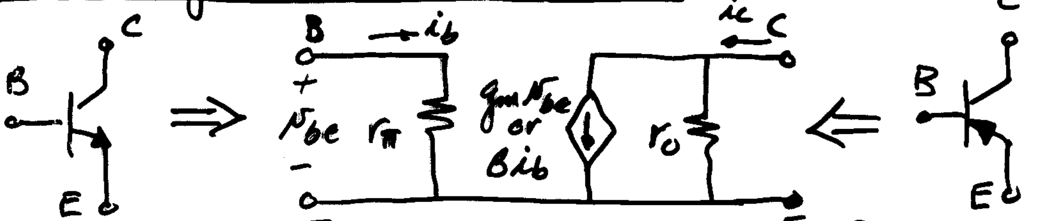
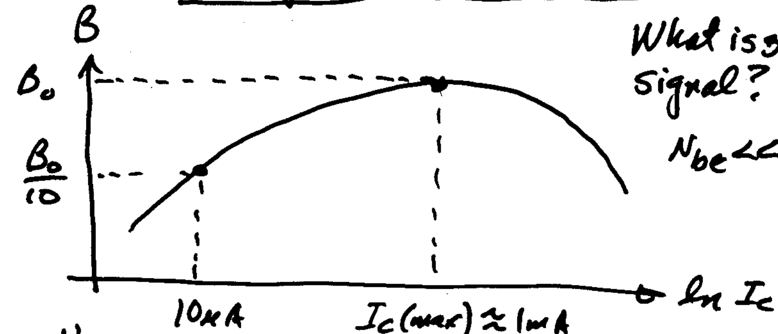
Small Signal Model for the BJT



$$g_m = \frac{I_{CQ}}{V_T} \qquad r_\pi = \frac{\beta}{g_m} \qquad r_o \cong \frac{V_A}{I_{CQ}} = \frac{V_A + V_{CEQ}}{I_{CQ}}$$

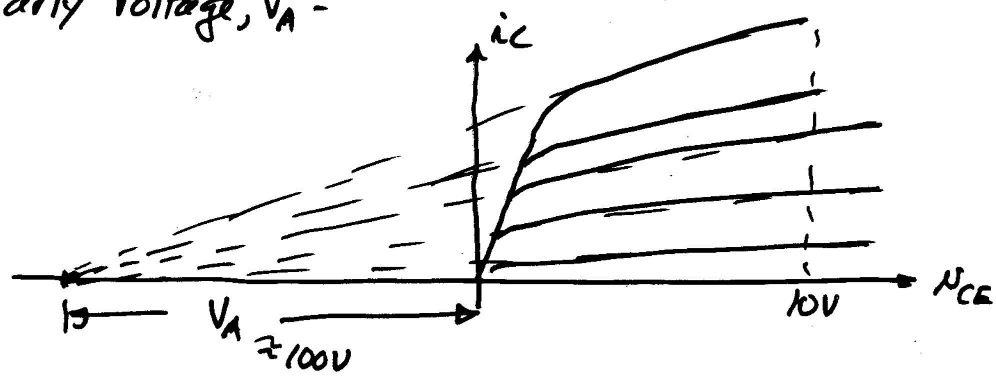
$\beta = f(I_C)$ -



What is small signal?

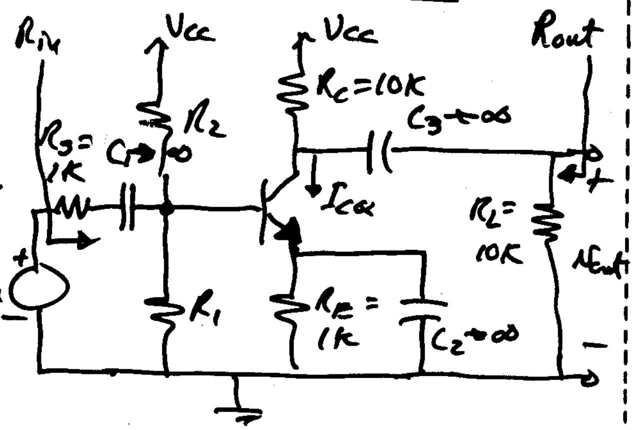
$v_{be} << \frac{kT}{q} \simeq 25mV$

Early Voltage, V_A -



BJT Common-Emitter Amplifier Example

Find the small-signal input resistance, R_{in}, the output resistance, R_{out}, and $\frac{v_{out}}{v_{in}}$ where $I_{CQ} = 1mA$, $\beta_o = 100$ and $V_A = 100V$. Assume $R_1 \| R_2 = R_B = 20K$

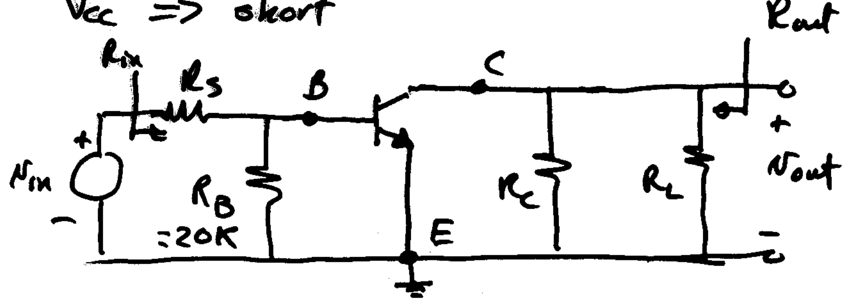


1.) AC equiv. ckt.

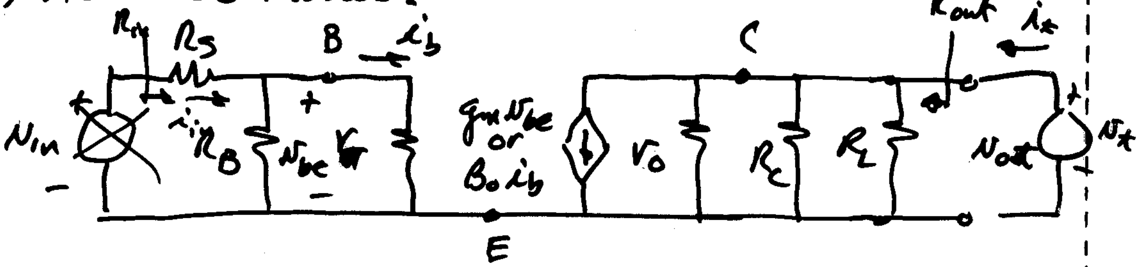
$C \to \infty \Rightarrow$ short

$V_{CC} \Rightarrow$ short

(current source) $\Rightarrow$ opens



2.) AC or SS model:



$R_{in} = R_S + R_B || r_\pi$

$R_{out} = r_o || R_L || R_C$

$g_m = \frac{I_{CQ}}{V_T} = \frac{1\,mA}{25\,mV} = \frac{1}{25}\,S$

$r_\pi = \frac{B_o}{g_m} = 25 \times 100 = 2.5\,k\Omega$

$r_o \cong \frac{V_A}{I_{CQ}} = \frac{100V}{1mA} = 100\,k\Omega$

$\therefore R_{in} = 1K + 20K || 2.5K = \underline{\underline{3.22K}}$

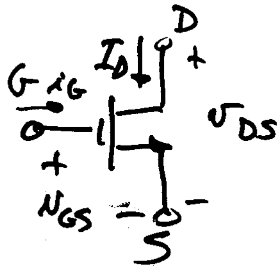
$R_{out} = 100K || 10K || 10K = \underline{\underline{4.7K}}$

$\frac{v_{out}}{v_{in}} = \left(\frac{v_{out}}{i_b}\right)\left(\frac{i_b}{v_{in}}\right) = \left(-B_o R_{out}\right)\left(\frac{R_B}{r_\pi + R_B}\frac{1}{R_{in}}\right)$

$i_{in} = \frac{v_{in}}{R_{in}} \rightarrow i_b = i_{in}\frac{R_B}{r_\pi + R_B} = \frac{R_B}{r_\pi + R_B}\frac{v_{in}}{R_{in}}$

$\frac{v_{out}}{v_{in}} = \frac{-B_o R_{out} R_B}{(r_\pi + R_B)[R_S + R_B || r_\pi]} = \frac{-100(4.7k)(20k)}{(22.5K)(3.22K)} \cong \underline{\underline{-131\,\frac{V}{V}}}$

MOSFET Small Signal Model



Assume saturation –

$$i_D = \frac{K_n}{2}(v_{GS} - V_{TN})^2(1 + \lambda v_{DS})$$

$i_G = 0$

Linearize $\quad i_D = f(v_{GS}, v_{DS})$

$$i_d = k_1 v_{gs} + k_2 v_{ds}$$

$$k_1 = \left.\frac{\partial i_D}{\partial v_{GS}}\right|_Q = K_n(v_{GS} - V_{TN})(1 + \lambda V_{DS}) \approx K_n(V_{GS} - V_{TN})$$

where $\lambda V_{DS} << 1$

$$k_1 = g_m = K_n(V_{GS} - V_t) = \sqrt{2K_n I_D}$$

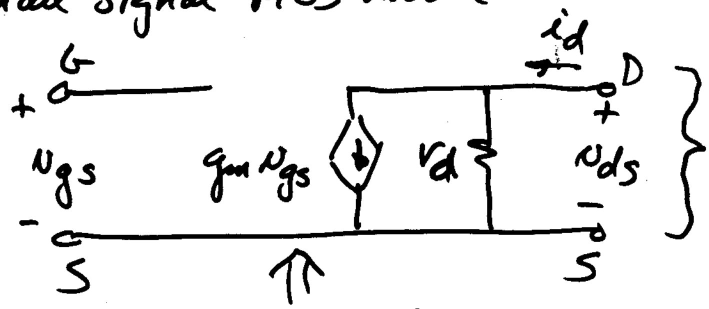
$$(V_{GS} - V_{TN})^2 \approx \frac{2I_D}{K_n} \rightarrow (V_{GS} - V_t) = \sqrt{\frac{2I_D}{K_n}}$$

$$k_2 = \left.\frac{\partial i_D}{\partial v_{DS}}\right|_Q \approx \frac{K_n}{2}(V_{GS} - V_{TN})^2 \lambda = ?$$

$$k_2 = \frac{K_n}{2}(V_{GS} - V_{TN})^2(1 + \lambda V_{DS})\frac{\lambda}{1 + \lambda V_{DS}} = \frac{\lambda I_D}{1 + \lambda V_{DS}}$$

$$k_2 \approx \lambda I_D = \frac{1}{r_d} = g_d$$

Small signal MOS model –

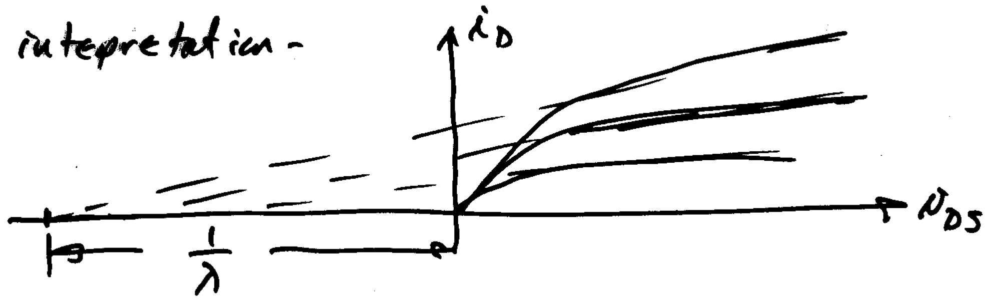


$$i_d = g_m v_{gs} + \frac{v_{ds}}{r_d}$$

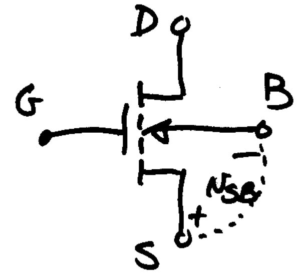
Same for PMOS or NMOS

Comments on MOS SS model -

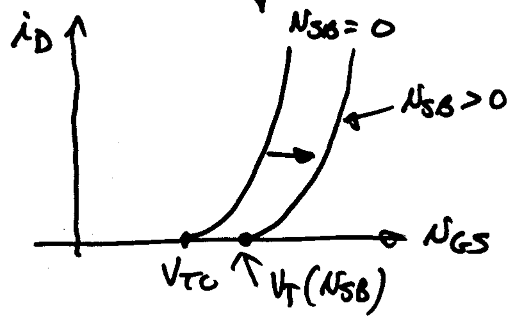
λ interpretation -



Bulk node -



$$V_{TN} = V_{TO} + \gamma\sqrt{v_{SB} + 2\phi_F} - \gamma\sqrt{2\phi_F}$$



$$i_d = g_m v_{gs} + g_d v_{ds} + k_3 v_{sb}$$

$$k_3 \equiv \left.\frac{\partial i_D}{\partial v_{SB}}\right|_Q = g_{mb} = \frac{\gamma K_n (V_{GS} - V_{TN})}{2\sqrt{V_{SB} + 2\phi_F}} = \eta\, g_m$$

$$\eta = \frac{\gamma}{2\sqrt{V_{SB} + 2\phi_F}}$$