LOW FREQUENCY RESPONSE ANALYSIS

Quiz 7 - Diff. Amps

Goal: Find ω_L (lower -3dB freq.) and A_0 (midband gain)

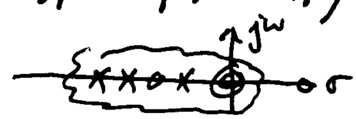
Methods for finding ω_L

1.) Direct analysis and numerically solve

2.) Approximation methods

a.) Dominant pole $\left(|\omega_p(\text{dominant})| \geq 4\,|\omega_p(\text{next})| \right)$

b.) No dominant pole



$$\omega_L \approx \sqrt{\sum_n p_n^2 - 2\sum_j z_j^2}$$

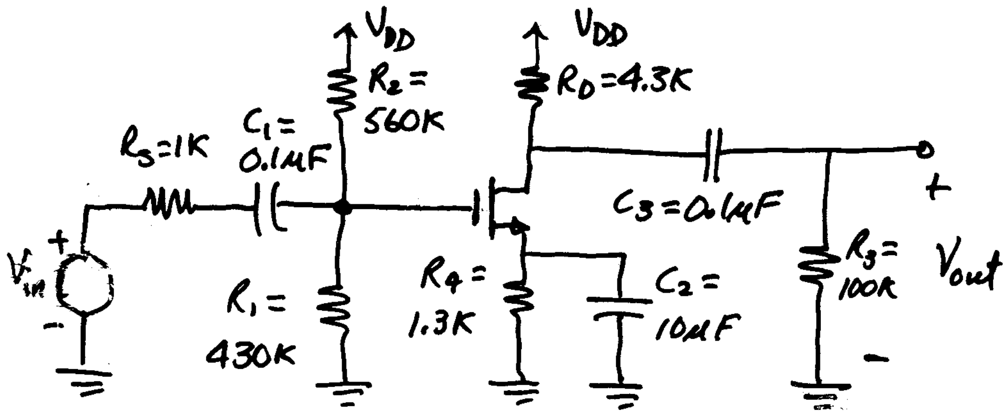
c.) Plot Bode plot and find ω_L

3.) Short-circuit time constant

$$\omega_L \approx \sum_{i=1}^{n} \frac{1}{R_{is} C_i}$$

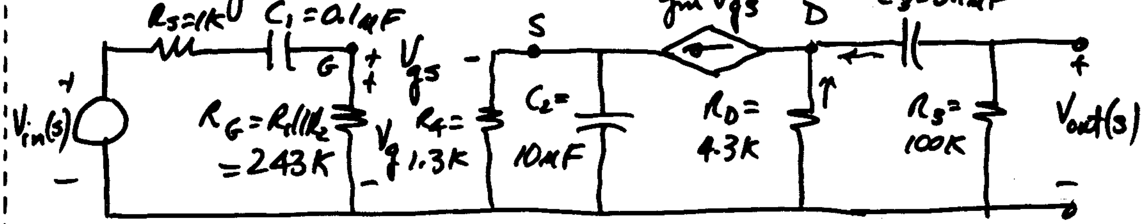
R_{is} is the Thevenin resistance seen by C_i with all other capacitors shorted.

MOSFET Low Frequency Example



Find ω_L if $g_m = 1.23\,mS$ and $r_o = \infty$ $(\lambda = 0)$

Small-signal model -



1.) Direct analysis

$$\frac{V_{out}}{V_{in}} = \left(\frac{V_{out}}{V_{gs}}\right)\left(\frac{V_{gs}}{V_g}\right)\left(\frac{V_g}{V_{in}}\right) \qquad Z_c(s) = \frac{1}{sC}$$

$$V_{out}(s) = -g_m\left(\frac{R_D R_3}{R_D + R_3 + \frac{1}{sC_3}}\right)V_{gs} = \left(-\frac{g_m R_D R_3}{R_D + R_3}\right)\left(\frac{s}{s + \frac{1}{C_3(R_D + R_3)}}\right)$$

$$z_3 = 0 \qquad p_3 = \frac{-1}{C_3(R_D + R_3)} = -95.9 \text{ rads/sec}$$

$$V_{gs} = V_g - V_s = +V_g - g_m V_{gs}\left(\frac{R_4 \frac{1}{sC_2}}{R_4 + \frac{1}{sC_2}}\right) = V_g - \frac{g_m R_4 V_{gs}}{1 + sC_2 R_4}$$

$$V_{gs}\left[1 + \frac{g_m R_4}{sC_2 R_4 + 1}\right] = V_g \rightarrow \frac{V_{gs}}{V_g} = \frac{sC_2 R_4 + 1}{sC_2 R_4 + 1 + g_m R_4}$$

$$\therefore \frac{V_{gs}}{V_g} = \frac{s + \frac{1}{R_4 C_2}}{s + \frac{1 + g_m R_4}{R_4 C_2}} \Rightarrow z_2 = -\frac{1}{R_4 C_2} = -76.9 \text{ rads/sec}$$

$$p_2 = -\frac{1 + g_m R_4}{R_4 C_2} = -200 \text{ rads/sec}$$

$$\frac{V_g}{V_{in}} = \frac{R_G}{R_G + R_S + \frac{1}{sC_1}} = \left(\frac{R_G}{R_G + R_S}\right)\left(\frac{1}{1 + \frac{1}{sC_1(R_G + R_S)}}\right)$$

$$= \left(\frac{R_G}{R_G + R_S}\right)\left(\frac{s}{s + \frac{1}{C_1(R_G + R_S)}}\right) \Rightarrow z_1 = 0$$

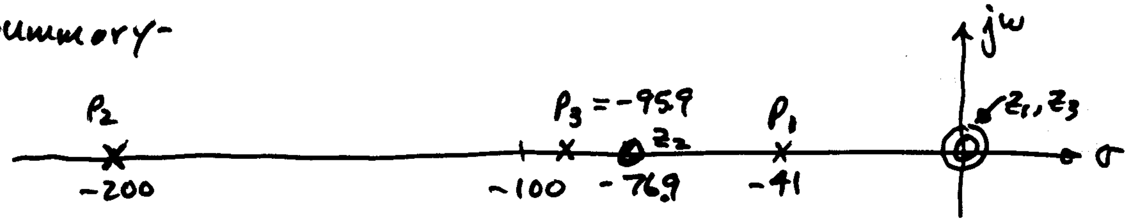
$$p_1 = \frac{-1}{C_1(R_G + R_S)} = -41 \text{ rads/sec}$$

$$\frac{V_{out}(s)}{V_{in}(s)} = \left(-\frac{g_m R_D R_3}{R_D + R_3}\right)\left(\frac{R_G}{R_G + R_S}\right)\left[\left(\frac{s}{s + p_1}\right)\left(\frac{s + z_2}{s + p_2}\right)\left(\frac{s}{s + p_3}\right)\right]$$

Bypass Caps

Coupling Caps

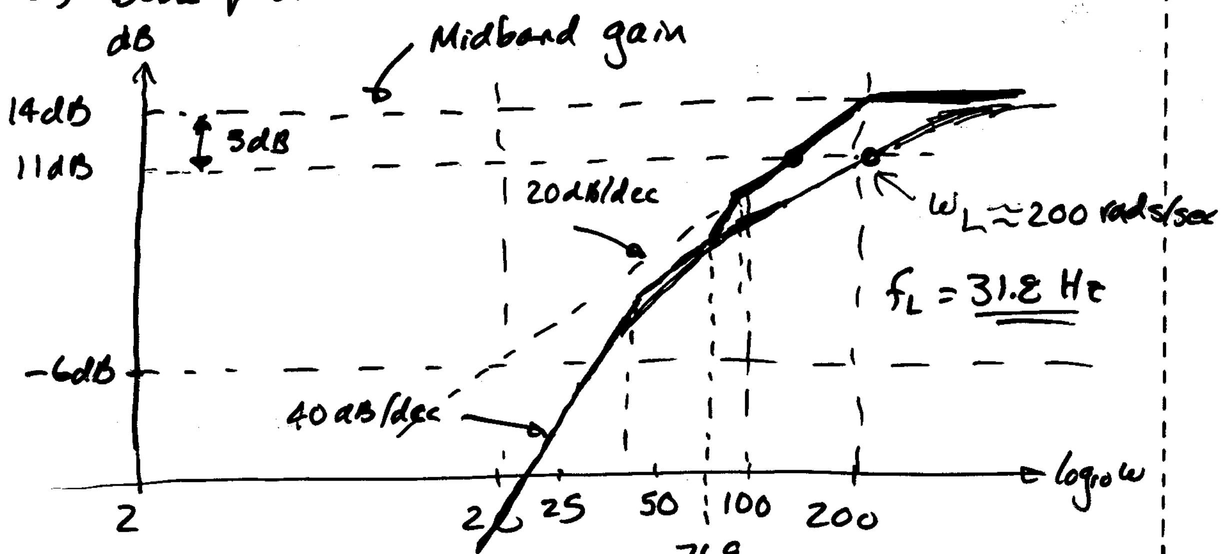
Summary -



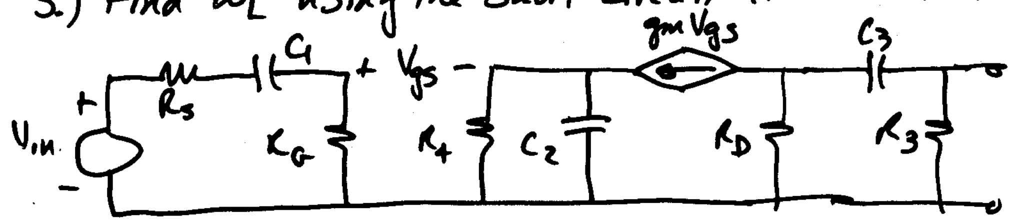
1.) $\omega_L \cong \sqrt{(200)^2 + (95.9)^2 + (41)^2 - 2(76.9)^2} = 2\pi \cdot 31.5\,Hz$

$f_L = \underline{31.5\,Hz}$

2.) Bode plot -



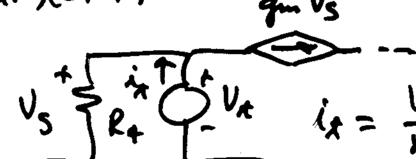
3.) Find ω_L using the short-circuit time constant method.



C_1: $R_{1s} = R_s + R_G = 244K$

$$\frac{1}{C_1 R_{1s}} = \frac{1}{(0.1\mu F)(244K)} = -41 \text{ rads/sec.}$$

C_2: $R_{2s} = R_4 \| \frac{1}{g_m}$



$$\frac{1}{C_2 R_{2s}} = \frac{1 + g_m R_4}{R_4 C_2} \simeq 200 \frac{\text{rads}}{\text{sec}}$$

$$i_x = \frac{V_x}{R_4} + g_m V_x$$

$$\frac{V_x}{i_x} = \frac{1}{g_m + \frac{1}{R_4}} = \frac{R_4}{1 + g_m R_4}$$

C_3: $R_{3S} = R_D + R_3$

$$\frac{1}{C_3 R_{3S}} = \frac{1}{C_3(R_D+R_3)} = 95.9 \text{ rads/sec}$$

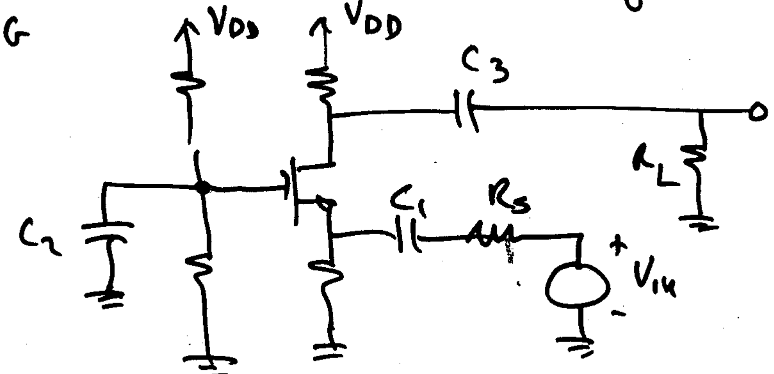
$$\therefore \quad \omega_L \approx \frac{1}{R_{1S}C_1} + \frac{1}{R_{2S}C_2} + \frac{1}{R_{3S}C_3} = 337 \text{ rads/sec}$$

$$f_L \approx 53.6 \text{ Hz}$$

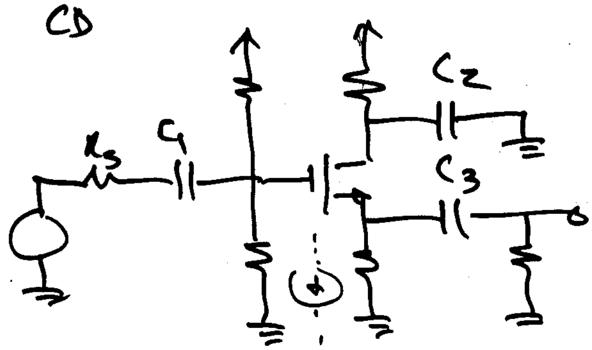
(Not all approximation methods are that good.)

Now work the CG & CD configurations

CG



CD



BJT Low Frequency Response

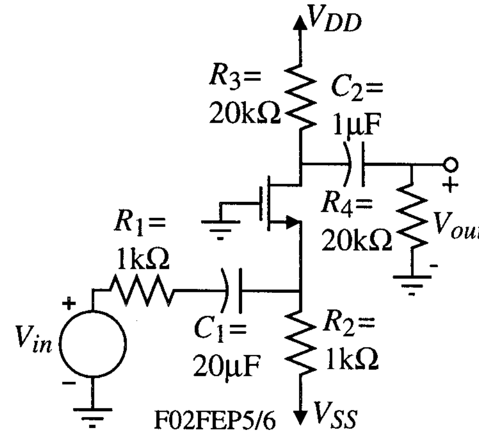
In the s.c. time constant method, if R_{iS} does not depend on whether the other capacitors are shorted or not, then

$$\frac{1}{R_{iS}C_i} = p_i$$

Problem 5 - (20 points - This problem is optional)

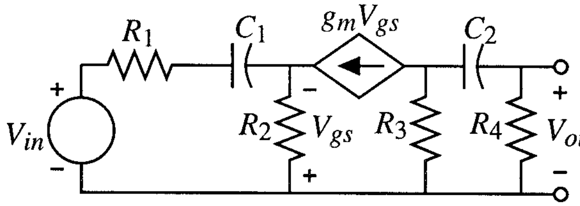
1.) If $g_m = 2\text{mA/V}$, what is the midband voltage gain of the amplifier shown? Assume $r_d = \infty$.

2.) Find the lower -3dB frequency (f_L) of the amplifier shown.



Solution

The small signal model for this problem is:



$$\frac{V_{out}(s)}{V_{in}(s)} = \left(\frac{V_{out}}{V_{gs}}\right)\left(\frac{V_{gs}}{V_{in}}\right)$$

$$\therefore\ \frac{V_{out}}{V_{gs}} = \frac{-g_m R_3 R_4}{R_3+R_4+\frac{1}{sC_2}} = \left(\frac{-g_m R_3 R_4}{R_3+R_4}\right)\left(\frac{s}{s + \frac{1}{C_2(R_3+R_4)}}\right) = (-20)\left(\frac{s}{s+25}\right)$$

Next, find V_{gs}/V_{in}:

$$\frac{V_{in}+V_{gs}}{R_1+\frac{1}{sC_1}} + \frac{V_{gs}}{R_2} + g_m V_{gs} = 0 \ \rightarrow\ \frac{-V_{in}}{R_1 + \frac{1}{sC_1}} = V_{gs}\left(\frac{1}{R_1 + \frac{1}{sC_1}} + \frac{1}{R_2 \| (1/g_m)}\right)$$

or

$$-V_{in}\left(R_2 \| \frac{1}{g_m}\right) = V_{gs}\left(R_2 \| \frac{1}{g_m} + R_1 + \frac{1}{sC_1}\right) \rightarrow \frac{V_{gs}}{V_{in}} = \left(\frac{R_2 \| \frac{1}{g_m}}{R_1 + R_2 \| \frac{1}{g_m}}\right)\left(\frac{s}{s + \frac{1}{C_1\left[R_1 + R_2 \| \frac{1}{g_m}\right]}}\right)$$

$$\frac{V_{gs}}{V_{in}} = \frac{-0.33}{1+0.33}\left(\frac{s}{s + 37.5}\right) = (-0.25)\left(\frac{s}{s+37.5}\right)$$

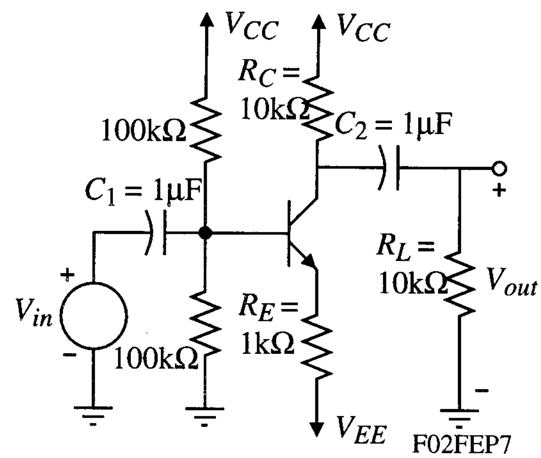
Thus, $\dfrac{V_{out}(s)}{V_{in}(s)} = 5\left(\dfrac{s}{s+25}\right)\left(\dfrac{s}{s+37.5}\right)$

$$\therefore\ \boxed{\text{MBG} = 5,\ \omega_L \approx \sqrt{(25)^2+(37.5)^2} = 45.07 \text{ rads/sec} \rightarrow f_L = 7.17 \text{ Hz}}$$

Low Frequency

Problem 7 - (20 points - This problem is optional).

A BJT amplifier is shown. Assume that the BJT has the small signal parameters of g_m = 50mA/V, r_π = 2kΩ, and $r_o = \infty$.



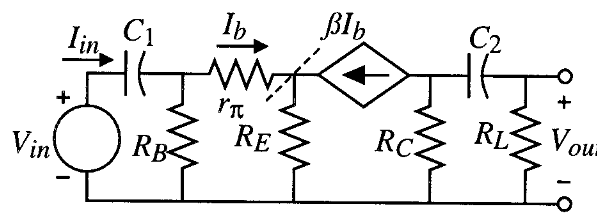
a.) Find the midband voltage gain of this amplifier, V_{out}/V_{in}.

b.) Find the numerical value of all poles and zeros of the low frequency response.

c.) Find the value of the lower -3dB frequency, f_L, in Hz.

Solution

The low-frequency, small signal model for this problem is shown where R_B = 50kΩ.



The algebraic approach to this problem is:

$$\frac{V_{out}}{V_{in}} = \left(\frac{V_{out}}{I_b}\right)\left(\frac{I_b}{I_{in}}\right)\left(\frac{I_{in}}{V_{in}}\right) = \left(\frac{-\beta R_L R_C}{R_C + R_L + \frac{1}{sC_2}}\right)\left(\frac{R_B}{R_B + r_\pi + (1+\beta)R_E}\right)\left(\frac{1}{\frac{1}{sC_1} + R_B \| [r_\pi + (1+\beta)R_E]}\right)$$

$$= \left(\frac{-\beta R_L R_C}{(R_C+R_L)[r_\pi + (1+\beta)R_E]}\right)\left(\frac{s}{s + \frac{1}{C_2(R_C+R_L)}}\right)\left(\frac{s}{s + \frac{1}{C_1(R_B \| [r_\pi + (1+\beta)R_E])}}\right)$$

$$= \frac{-100\cdot 10K\cdot 10K}{20K\cdot 103K}\left(\frac{s}{s+50}\right)\left(\frac{s}{s+29.7}\right) = -4.854\left(\frac{s}{s+50}\right)\left(\frac{s}{s+29.7}\right)$$

The midband gain is $\boxed{MBG = 4.854 \text{ V/V}}$

$\therefore \quad \omega_L \approx \sqrt{(29.7)^2 + (50)^2} = 58.2$ rads/sec. $\rightarrow$ $\boxed{f_L = 9.26\text{Hz}}$

The poles and zeros are,

$\boxed{\text{Two zeros at s = 0, a pole at s = -29.7 rads/sec. and a pole at s = -50 rads/sec.}}$