High Frequency Response

Methods of Finding $\omega_H$

1. Given the roots $\left( \frac{1}{\omega_i} \right)$
   a. Bode plot
   b. Dominant pole
      
      
      \[ \omega_H \approx \omega_p \text{ (dominant)} \text{ if } \omega_p \text{ (dominant)} \leq \frac{\omega_p \text{ (dominant)}}{4} \]
   
   c. \[ \omega_H = \frac{1}{\omega_p} \approx \frac{1}{\sqrt{\frac{1}{\omega_p^2} + \frac{1}{\omega_p^2}} + \ldots} \]

   d. Exact solution (quadratic)

2. Roots not Known
   a. Miller effect
      \[ Z_i = \frac{V_i}{I_i} = \frac{V_i}{V_1 - V_2} \]
      \[ Z_i = \frac{V_i}{sC_m(V_1 - V_2)} = \frac{1}{sC_m(1 + \frac{V_2}{V_1})} \]
      \[ Z_i = \frac{1}{sC_m(1 + AV)} \]
      
      But $\frac{V_2}{V_1} = AV$, so $Z_i = \frac{1}{sC_m(1 - AV)}$
      
      If $AV$ is negative, then $Z_i = \frac{1}{sC_m(1 + AV)}$
      
      \[ C_{eq} = (1 + AV)C_m \]
b.) Approximate solution for a quadratic (p. 1311)

\[ s^2 + \alpha s + b = (s + \rho_1)(s + \rho_2) = s^2 + \rho_1 s + \rho_2 s + \rho_1 \rho_2 \]

Assume \( \rho_1 < \rho_2 \)

\[ s^2 + \alpha s + b \approx s^2 + \rho_2 s + \rho_1 \rho_2 \Rightarrow \rho_2 = \frac{\alpha}{2}, \quad \rho_1 = \frac{b}{\alpha} \]

c.) Open-circuit Time Constant

\[ W_H = \frac{1}{\sum_{i=1}^{m} R_{i0} C_i} \]

Where \( R_{i0} \) is the Thevenin resistance seen from \( C_i \) with all other caps open-circuited.

Where does the high frequency response come from?

- Coupling caps and bypass caps are shorts.
- Active Transistor parasitics and other parasitics.

BJT High Frequency Model:

\[ C_m = \text{the reverse-bias BC junction capacitance} \]

\[ C_m = \frac{C_{40}}{\sqrt{1 + \frac{V_{CE}}{\Phi_{bc}}} \quad \text{Normal regime of operation}} \]
\[ C_T = g_m \tau_F + C_{be} \text{ where } \tau_F = \text{forward time constant} \]

**Unity-Gain Bandwidth** \( \omega_T \)

**CE configuration at high frequencies**

\[ \frac{I_{out}}{I_n} = \left( \frac{V_{be}}{V_{be}} \right) = \left( g_m \right) \left( \frac{V_{be}}{I_m} \right) = \frac{g_m}{\frac{1}{\tau_F + s(C_T + C_m)} + \frac{1}{s(C_T + C_m)}} \]

**Output impedance**

\[ \frac{I_{out}}{I_n} = \frac{g_m V_T}{s(C_T + C_m) V_T} = \frac{G_0}{s(C_T + C_m) V_T} \]

\[ W_T = \frac{\tau_F}{C_{pi} + C_{m}} \]

**Typically** \( C_T >> C_m \)
$V_x$: $V_x$ is a small resistance in series with base of about 100-500Ω. It varies with $I_c$ and is called "the base spreading resistance."

**MOSFET High Frequency Model**

$$W_T = \frac{g_m}{C_{gs} C_{gd}}$$

$C_{gs} > C_{gd}$

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**Circuit Diagram**

- $V_{in}$
- $R_c$
- $C_{gd}$
- $R_D$
- $R_L$
- $V_{out}$