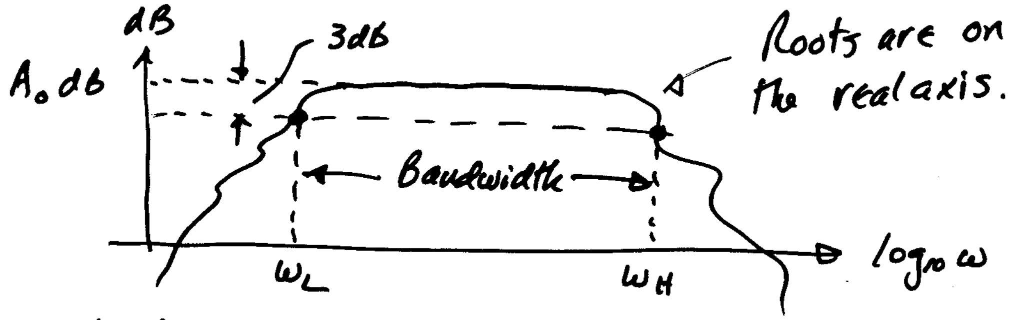
High Frequency Response



Methods of Finding ω_H

1.) Given the roots $\left(\frac{1}{s+p_i}\right)$

a.) Bode plot

b.) Dominant pole

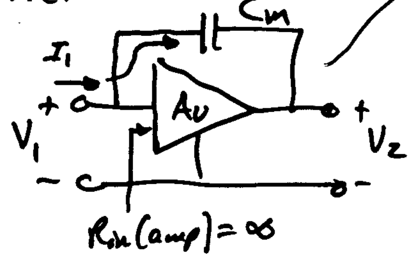
$$\omega_H \simeq \omega_p(\text{dominant}) \text{ if } \omega_p(\text{dominant}) \le \frac{\omega_p(\text{next})}{4}$$

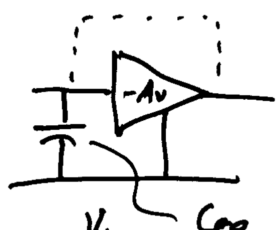
c.) $$\omega_H \simeq \frac{1}{\sqrt{\sum_n \frac{1}{\omega_{pn}^2} - 2\sum_n \frac{1}{\omega_{zn}^2}}} \simeq \frac{1}{\sqrt{\frac{1}{\omega_{p1}^2} + \frac{1}{\omega_{p2}^2} + \cdots}}$$

d.) Exact solution (quadratic)

2.) Roots not known

a.) Miller effect





$$Z_1 = \frac{V_1}{I_1} = \frac{V_1}{\frac{V_1 - V_2}{1/sC_m}}$$

$$Z_1 = \frac{V_1}{sC_m(V_1 - V_2)} = \frac{1}{sC_m\left(1 - \frac{V_2}{V_1}\right)}$$

But $\frac{V_2}{V_1} = A_v$, so $Z_1 = \frac{1}{sC_m(1 - A_v)}$

If A_v is negative, then $Z_1 = \frac{1}{sC_m(1+|A_v|)} = \frac{1}{sC_{eq.}}$

$$\boxed{C_{eq} = (1+|A_v|)\,C_m}$$

b.) Approximate solution for a quadratic (p. 1311)

$$s^2 + as + b = (s+p_1)(s+p_2) = s^2 + s(p_1+p_2) + p_1 p_2$$

Assume $p_1 < p_2$

$$s^2 + as + b \approx s^2 + p_2 s + p_1 p_2 \Rightarrow \underline{\underline{p_2 = a}} \; \& \; \underline{\underline{p_1 = \frac{b}{a}}}$$

c.) Open-circuit Time Constant

$$\omega_H \approx \frac{1}{\sum_{i=1}^{m} R_{io} C_i}$$

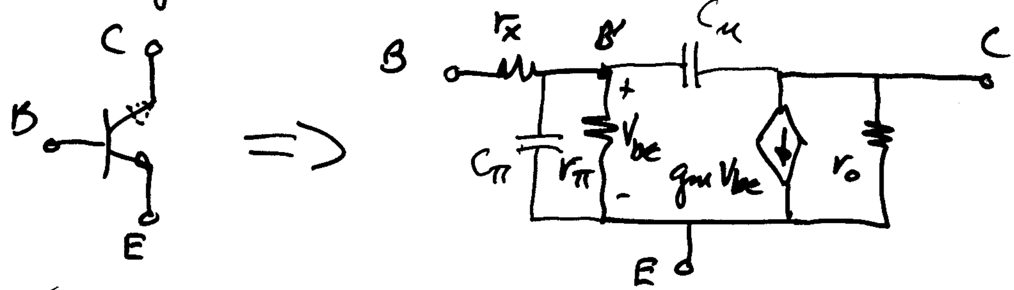
where R_{io} is the Thevenin resistance seen from C_i with all other caps open-circuited.

Where does the high frequency response come from?

Coupling caps and bypass caps are shorts.

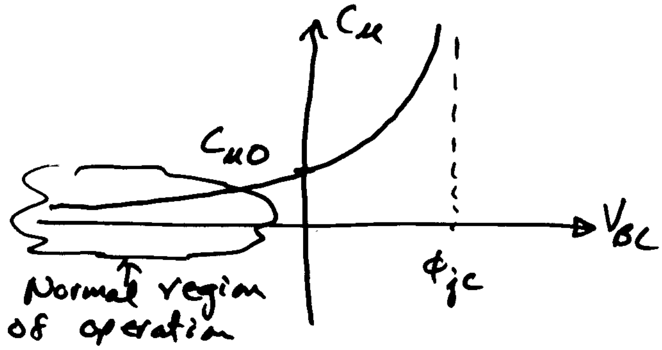
Ans. Transistor parasitics and other parasitics.

BJT High Frequency Model –

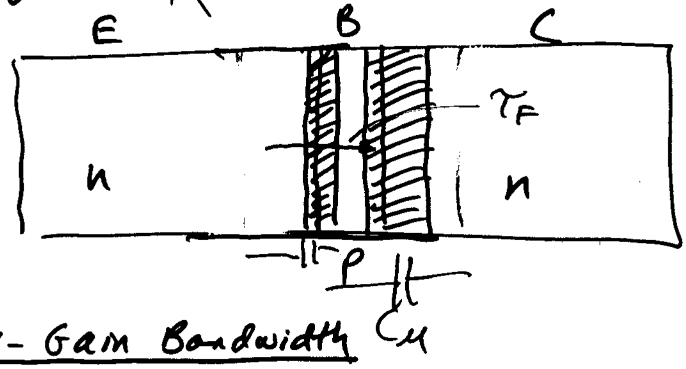


C_μ = the reverse-biase BC junction capacitance

$$C_\mu = \frac{C_{\mu 0}}{\sqrt{1 + \frac{V_{CB}}{\phi_{jc}}}}$$



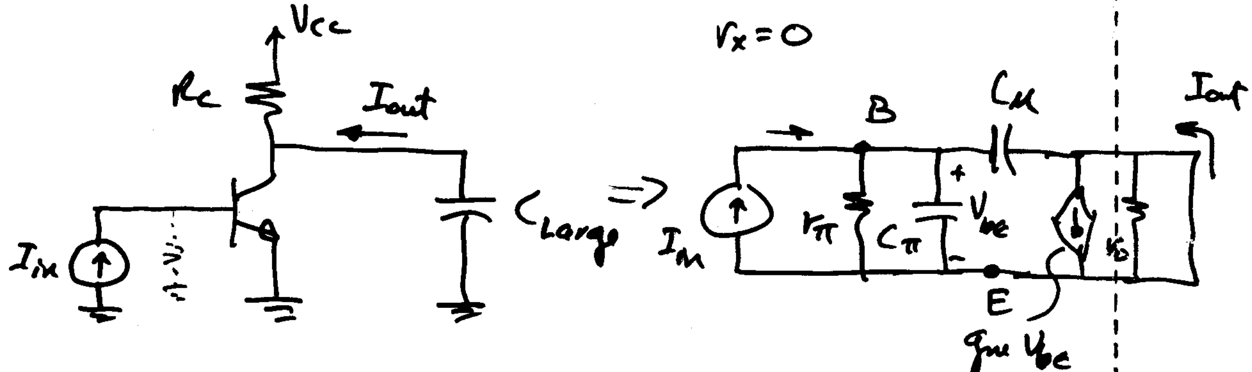
$C_\pi = g_m \tau_F + \cancel{C_{jbe}}$ where τ_F = forward time constant



<u>Unity-Gain Bandwidth</u>

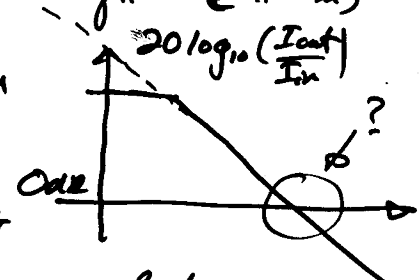
ω_T (f_T)

CE configuration at high frequencies -



$$\frac{I_{out}}{I_{in}} = \left(\frac{I_{out}}{V_{be}}\right)\left(\frac{V_{be}}{I_{in}}\right) = (g_m)\left(\quad\right) = \frac{g_m}{g_\pi + s(C_\pi + C_\mu)}$$

$$V_{be} = I_{in}\,\frac{1}{g_\pi + sC_\pi + sC_\mu}$$



$$\frac{I_{out}}{I_{in}} = \frac{g_m r_\pi}{1 + s(C_\pi + C_\mu) r_\pi} = \frac{\beta_o}{1 + s(C_\pi + C_\mu) r_\pi}$$

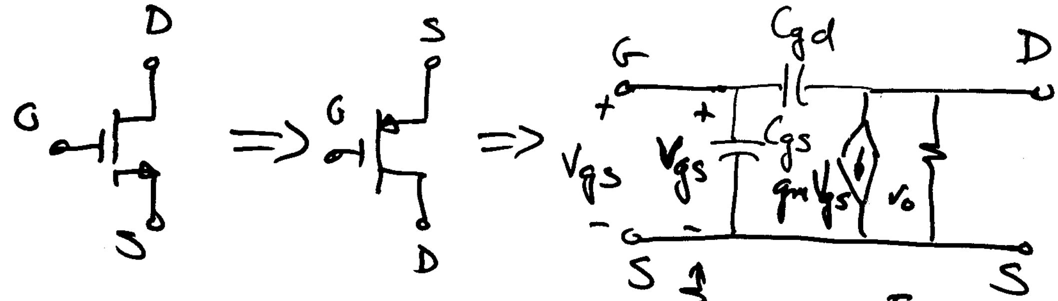
$$\frac{I_{out}}{I_{in}} \approx \frac{\beta_o}{s(C_\pi + C_\mu) r_\pi} = 1 \rightarrow \omega_T = \frac{\beta_o / r_\pi}{(C_\pi + C_\mu)} = \frac{g_m}{C_\pi + C_\mu}$$

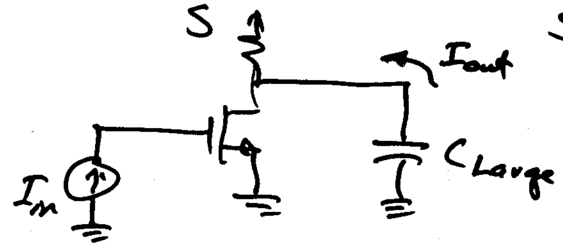
$$\boxed{\omega_T = \frac{g_m}{C_\pi + C_\mu}} \rightarrow C_\pi + C_\mu = \frac{g_m}{\omega_T} \qquad \boxed{C_\pi = \frac{g_m}{\omega_T} - C_\mu} \qquad \omega_T \approx \frac{1}{\tau_F}$$

Typically $C_\pi >> C_\mu$

r_x: r_x is a small resistance in series with base of about 100–500 Ω. It varies with I_c and is called "the base spreading resistance."

MOSFET High Frequency Model





$$\omega_T = \frac{g_m}{C_{gs} + C_{gd}}$$

$C_{gs} > C_{gd}$