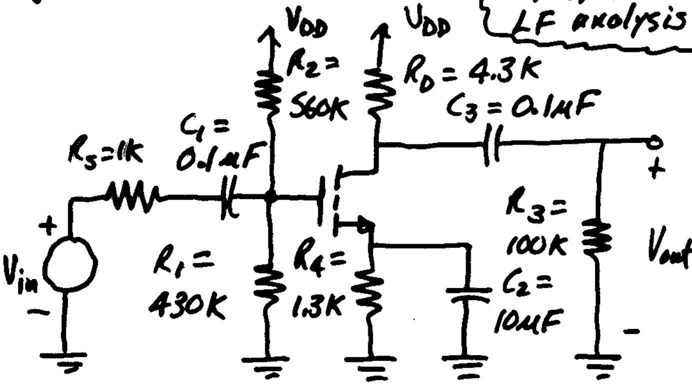
MOSFET Example of High Frequency Analysis

Quiz 8
10/29/04
LF analysis

Find ω_H of the MOSFET amplifier shown if $g_m = 1.23$ mS, $C_{gs} = 10$ pF, and $C_{gd} = 2$ pF

Note: This is the same circuit used for LF analysis (see p. 1 of 10/13/04 lecture)

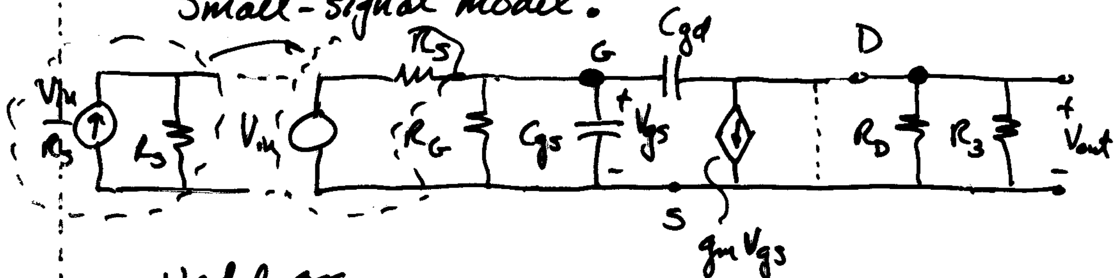


$R_G = R_1 \| R_2 = 243K$

$X_C = \frac{1}{\omega C}$

(1.) Direct Analysis

Small-signal model:



Nodal eqs.

$$\left(\frac{1}{R_s} + \frac{1}{R_G} + sC_{gs} + sC_{gd}\right)V_{gs} - sC_{gd}V_{out} = \frac{V_{in}}{R_s}$$

$\frac{1}{R_s} = G_s$

$$sC_{gd}(V_{out} - V_{gs}) + g_m V_{gs} + \left(\frac{1}{R_D} + \frac{1}{R_3}\right)V_{out} = 0$$

Two pages later

$$\frac{V_{out}}{V_{in}} = \frac{-G_s(g_m - sC_{gd})}{(G_s + G_G)(G_D + G_3) + s\left[C_{gs}(G_D + G_3) + C_{gd}(G_D + G_3 + G_s + G_G) + g_m C_{gd}\right] + s^2 C_{gd} C_{gs}}$$

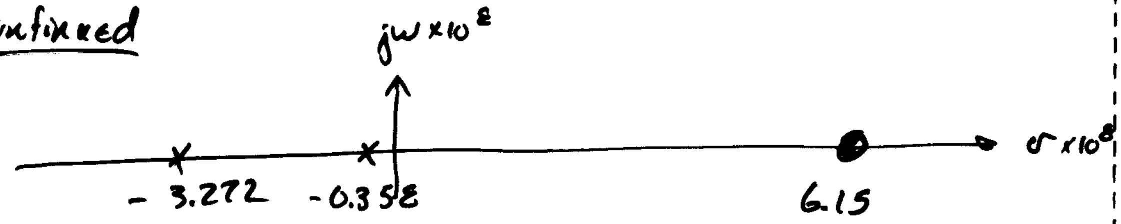
Substituting the values:

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{-10^{-3}\left(1.23\times 10^{-3} - s\,2pF\right)}{20\times 10^{-24}\left[s^2 + 3.628\times 10^{8} s + 1.165\times 10^{16}\right]}$$

$z_1 = +6.15\times 10^{8}\ \frac{rads}{sec}$

$p_1 = -0.358\times 10^{8}\ \frac{rads}{sec}$ & $p_2 = -3.272\times 10^{8}\ \frac{rads}{sec}$

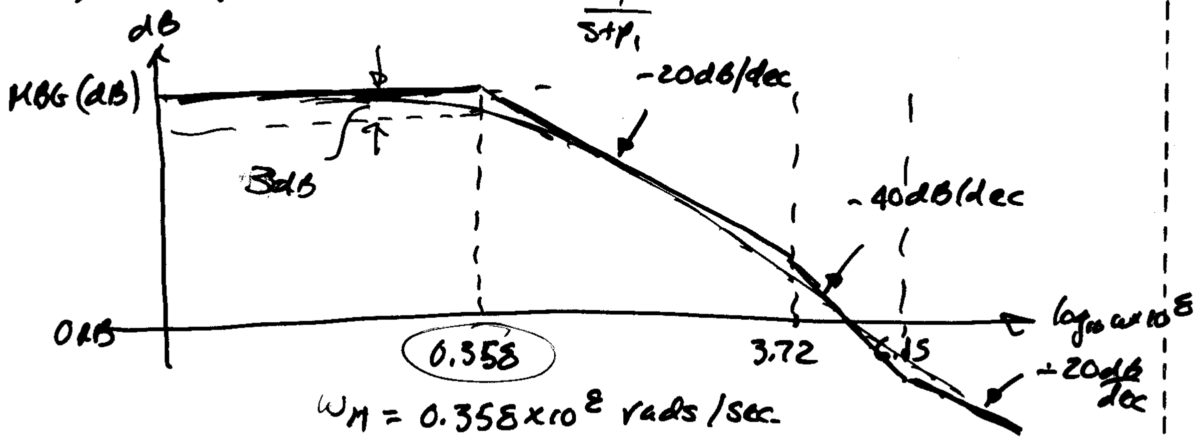
Continued



a.) Dominant pole

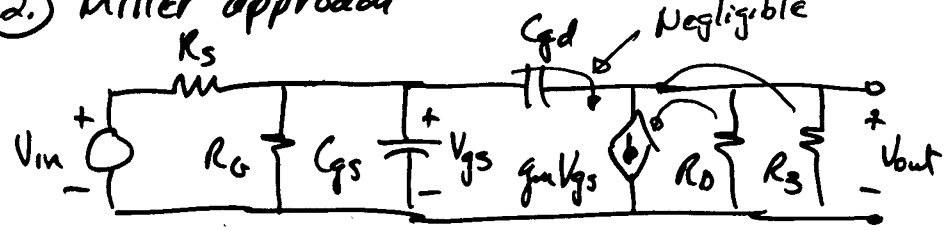
$\omega_H \approx |\text{dominant pole}| \simeq 0.358 \times 10^8 \frac{\text{rads}}{\text{sec}} \Rightarrow f_H = 5.67\,\text{MHz}$

b.) Bode plot



$\omega_H = 0.358 \times 10^8$ rads/sec.

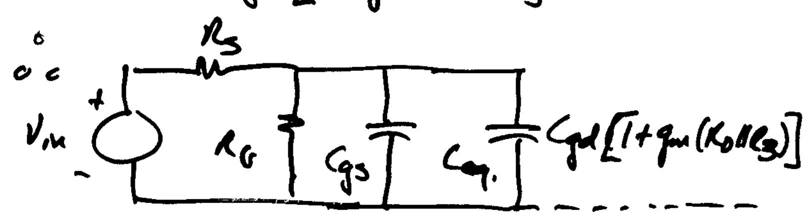
2.) Miller approach



$K = \frac{V_{out}}{V_{gs}} = ?$ $\quad C_{eq.} = C_{gd}(1-K)$

$V_{out} \approx -g_m(R_D \| R_3) V_{gs}$ if $\frac{1}{\omega_H C_{gd}} \gg R_D \| R_3$

$C_{eq.} = C_{gd}\left[1 + g_m(R_D \| R_3)\right]$



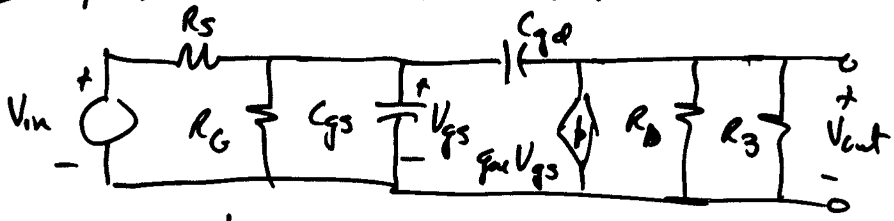
$$\omega_H \approx \frac{1}{[R_s \| R_G][C_{gs} + C_{eq.}]} = \frac{1}{(0.996k)[10pF + 2pF \cdot 6]} = 45.3 \times 10^6 \frac{\text{rads}}{\text{sec}}$$

Cont'd

$\frac{\omega_H}{2\pi} = f_H = 7.22\,MHz \qquad \frac{1}{\omega_H C_{gd}} = \frac{1}{45.3\times10^6 \cdot 2pF} = 11.037\times10^3$

$R_D \| R_3 \approx 4K \qquad \frac{1}{\omega_H C_{gd}} > 4K$

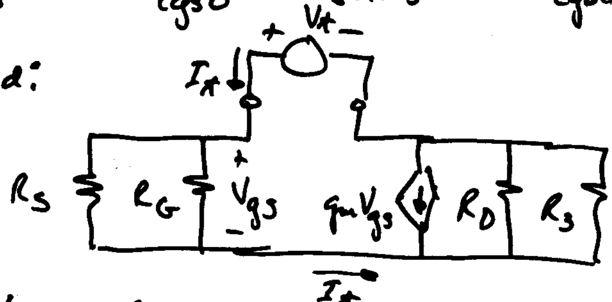
(3.) Open-Circuit Time Constant



$$\omega_H = \frac{1}{\sum_i R_{i0} C_i}$$

C_{gs}: $\quad R_{cgs0} = R_G \| R_S \rightarrow R_{cgs0} C_{gs} = 0.996K \times 10pF$

C_{gd}:



$g_m(R_D \| R_3)$

$$V_x = \underbrace{I_x(R_S \| R_G)}_{V_{gs}} + (I_x + g_m V_{gs})(R_D \| R_3) = I_x\left\{R_D \| R_3 + R_S \| R_G\left[1 + g_m(R_D \| R_3)\right]\right\}$$

$$R_{cgd0} = \frac{V_x}{I_x} = R_D \| R_3 + R_S \| R_G\left[1 + g_m(R_D \| R_3)\right]$$

$$= 4.122K + 0.996K\left[1+5\right] = 10.16K$$

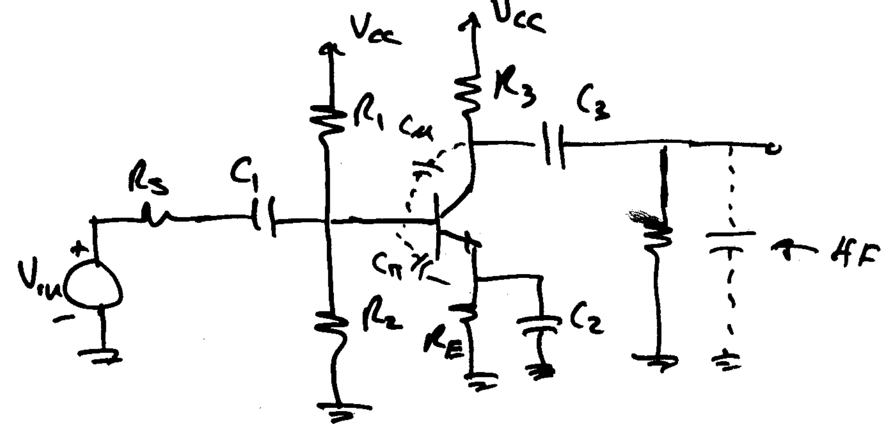
$$\omega_H \approx \frac{1}{R_{cgs0} C_{gs} + R_{cgd0} C_{gd}} = \frac{1}{(.966K)(10pF) + (10.16K)(2pF)}$$

$\doteq 33.33 \times 10^6 \frac{rads}{sec} \rightarrow \underline{f_H = 5.3\,MHz}$

There will be 2 examples of HF analysis on web site.

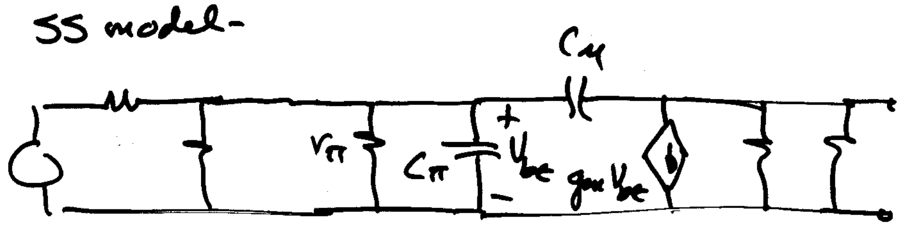
BJT HF Analysis

Generic ckt.



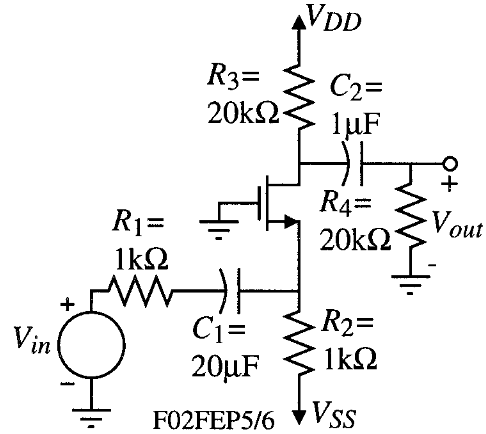
Given: g_m, r_π, C_π & C_μ

HF SS model-



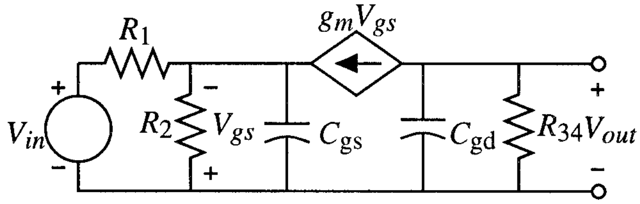
Problem 6 - (20 points - This problem is optional)

The FET in the amplifier shown has g_m = 1mA/V, $r_d = \infty$, C_{gd} = 0.5pF, and C_{gs} = 10pF. (a.) Find the midband gain, V_{out}/V_{in}. (b.) Find the upper -3dB frequency, f_H, in Hz. (Note: You cannot use the Miller's theorem on this problem because there is no bridging capacitor.)



Solution

The small signal model for the high frequency range is shown where $R_{34} = R_3||R_4$ = 10kΩ.



Because the capacitors are independent, probably the best way to work this problem is by superposition (open-circuit time constants). Therefore,

C_{gs}:

$$R_{Cgs} = R_1||R_2||(1/g_m) = 1\text{K}||1\text{K}||1\text{K} = 333\Omega \rightarrow \omega_{Cgs} = \frac{1}{C_{gs}\cdot 333\Omega} = 300 \text{ Mrads/sec.}$$

C_{gd}:

$$R_{Cgd} = R_{34} = 10\text{k}\Omega \rightarrow \omega_{Cgd} = \frac{1}{C_{gd}\cdot 10\text{k}\Omega} = 200 \text{ Mrads/sec.}$$

$$\therefore\ \omega_H \approx \frac{1}{\sqrt{\left(\frac{1}{300\text{Mrads/sec}}\right)^2 + \left(\frac{1}{200\text{Mrads/sec}}\right)^2}} = 166 \text{ Mrads/sec.}$$

$$\boxed{f_L = 26.48\text{MHz}}$$

The midband gain is given as

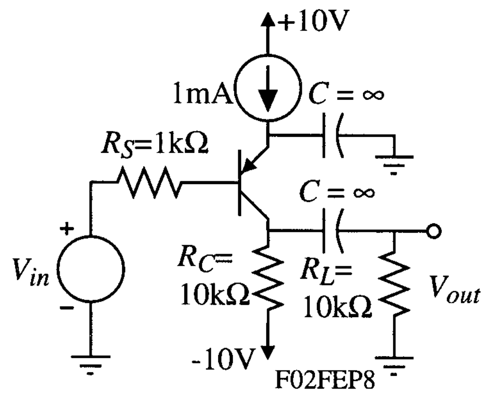
$$\boxed{\text{MBG} = \left(\frac{R_2||\frac{1}{g_m}}{R_1 + R_2||\frac{1}{g_m}}\right)\left(\frac{-g_m R_3 R_4}{R_3+R_4}\right) = \left(\frac{-0.5}{1.5}\right)(-10) = 3.33\text{V/V}}$$

Problem 8 – (20 points, this problem is optional)

A common-emitter BJT amplifier is shown. Assume that the BJT has a $\beta = h_{fe} = 100$, $C_\mu = 2\text{pF}$, $V_t = 25\text{mV}$, $f_T = 500\text{MHz}$, $r_b = 0\Omega$, and $r_o = \infty$.

a.) Find the numerical values of r_π, g_m, and C_π.

b.) If $r_\pi = 1\text{k}\Omega$, $g_m = 0.01\text{A/V}$ and $C_\pi = 10\text{pF}$ for the above amplifier, find the value of the upper -3dB frequency, f_H, in Hz.

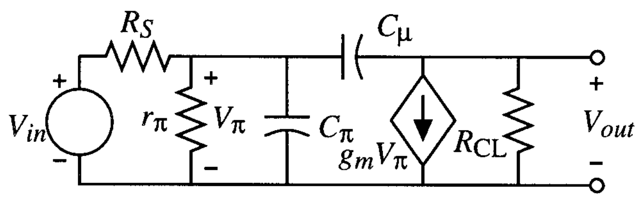


Solution

a.) $\boxed{g_m = \frac{I_C}{V_T} = \frac{1\text{mA}}{25\text{mV}} = 0.04 \text{ A/V}}$ $\boxed{r_\pi = \frac{\beta_o}{g_m} = \frac{100}{0.04} = 2500\Omega}$

$$\boxed{C_\pi = \frac{g_m}{\omega_T} - C_\mu = \frac{0.04}{2\pi\cdot 500\text{x}10^6} - 2\text{pF} = 12.732\text{pF} - 2\text{pF} = 10.732 \text{ pF}}$$

b.) The high-frequency, small-signal model for this problem is shown where $R_{CL} = R_C \| R_L = 5\text{k}\Omega$.
The midband gain of this amplifier is given by



$$\frac{V_{out}}{V_{in}} = \left(\frac{V_{out}}{V_\pi}\right)\left(\frac{V_\pi}{V_{in}}\right) = -g_m R_C \| R_L \left(\frac{r_\pi}{r_\pi + R_S}\right) = (-0.01\cdot 5\text{k}\Omega)(0.5) = -25\text{V/V}$$

$\therefore$ $\boxed{\text{MBG} = -25 \text{ V/V}}$

Using Miller's theorem on this problem:

If $\frac{1}{\omega_H C_\mu} >> R_C \| R_L$, then $C_{eq} \approx C_\pi + C_\mu(1 + g_m R_C \| R_L) = 10\text{pf} + 2\text{pF}(1+50) = 112\text{pF}$

We know that, $\omega_H = \frac{1}{C_{eq}(r_\pi \| R_S)} = \frac{1}{(112\text{pF}\cdot 500\Omega)} = 17.86$ Mrads/sec.

$\therefore$ $\boxed{f_H = \frac{\omega_H}{2\pi} = 2.842 \text{ MHz}}$

Note that:

$\frac{1}{\omega_H C_\mu} = \frac{10^6}{17.86\cdot 2} = 28.06\text{k}\Omega > 5\text{k}\Omega$ so that the Miller approximation (neglecting C_μ) is valid.