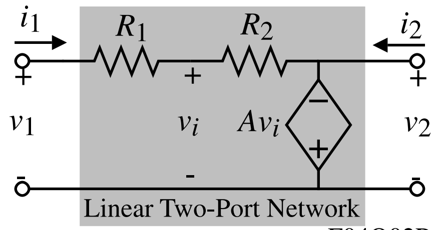
QUIZ NO. 2

(Average score = 6.4/10 of those taking the quiz)

The g-parameters for a linear two-port network are given as

$i_1 = g_{11}v_1 + g_{12}i_2$ and $v_2 = g_{21}v_1 + g_{22}i_2$

Find g_{11}, g_{21}, g_{12}, and g_{22} for the linear two-port network shown. (This is a model for an inverting op amp with a large but finite voltage gain, A.)



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<u>Solution</u>

$i_2 = 0$: $g_{11} = \frac{i_1}{v_1}$ and $g_{21} = \frac{v_2}{v_1}$

Loop equation at the input gives $v_1 = i_1(R_1 + R_2) - Av_i$

Another loop equation at the input gives $v_i = v_1 - i_1R_1$

$$\therefore \quad v_1 = i_1(R_1 + R_2) - A(v_1 - i_1R_1) = i_1(R_1 + R_2 + A\,R_1) - A\,R_1v_1$$

$$v_1(1+A) = i_1(R_1 + R_2 + A\,R_1) \rightarrow \boxed{g_{11} = \frac{i_1}{v_1} = \frac{1+A}{R_1 + R_2 + A\,R_1}}$$

At the output we can write, $v_2 = -Av_i$.

Replacing v_i with $v_1 - i_1R_1$ gives

$$v_2 = -A(v_1 - i_1R_1) = -Av_1 + AR_1\left(\frac{v_1 - v_2}{R_1 + R_2}\right) = -Av_1 + \left(\frac{AR_1}{R_1 + R_2}\right)v_1 - \left(\frac{AR_1}{R_1 + R_2}\right)v_2$$

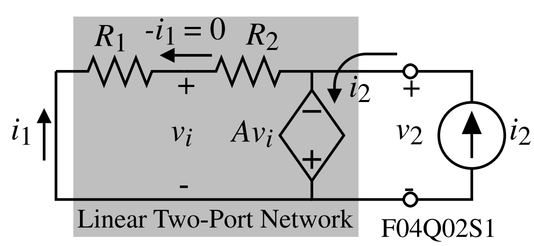
or

$$\left(1 + \frac{AR_1}{R_1 + R_2}\right)v_2 = \left(\frac{R_1 + R_2 + AR_1}{R_1 + R_2}\right)v_2 = A\left(\frac{R_1}{R_1 + R_2} - 1\right)v_1 = -A\left(\frac{R_2}{R_1 + R_2}\right)v_1$$

Taking the ratio of the second and last terms above gives $\boxed{g_{21} = \frac{v_2}{v_1} = \left(\frac{-AR_2}{R_1 + R_2 + AR_1}\right)}$

$v_1 = 0$: $g_{12} = \frac{i_1}{i_2}$ and $g_{22} = \frac{v_2}{i_2}$

A model for the two port driven by i_2 with $v_1 = 0$ is shown. We see that all the current, i_2, will flow into the controlled voltage source because it has a zero resistance and none will flow through the $R_1 + R_2$ combination. Therefore, $i_1 = 0$ and thus $v_2 = 0$. Consequently, both g_{12} and g_{22} are zero.



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$$\boxed{g_{12} = \frac{i_1}{i_2} = \frac{0}{i_2} = 0} \quad \text{and} \quad \boxed{g_{22} = \frac{v_2}{i_2} = \frac{0}{i_2} = 0}$$