QUIZ NO. 2

(Average score = 6.4/10 of those taking the quiz)

The $g$-parameters for a linear two-port network are given as

\[ i_1 = g_{11}v_1 + g_{12}i_2 \quad \text{and} \quad v_2 = g_{21}v_1 + g_{22}i_2 \]

Find $g_{11}$, $g_{21}$, $g_{12}$, and $g_{22}$ for the linear two-port network shown. (This is a model for an inverting op amp with a large but finite voltage gain, $A$.)

Solution

$i_2 = 0$: \[ g_{11} = \frac{i_1}{v_1} \quad \text{and} \quad g_{21} = \frac{v_2}{v_1} \]

Loop equation at the input gives \[ v_1 = i_1(R_1 + R_2) - Av_i \]

Another loop equation at the input gives \[ v_i = v_1 - i_1R_1 \]

\[ v_1 = i_1(R_1 + R_2 + A R_1) \rightarrow g_{11} = \frac{i_1}{v_1} = \frac{1+A}{R_1 + R_2 + A R_1} \]

At the output we can write, \[ v_2 = -Av_i \]

Replacing $v_i$ with $v_1 - i_1R_1$ gives

\[ v_2 = -A(v_1 - i_1R_1) = -Av_1 + AR_1 \left( \frac{v_1 - v_2}{R_1 + R_2} \right) = -Av_1 + \left( \frac{AR_1}{R_1 + R_2} \right) v_1 - \left( \frac{AR_1}{R_1 + R_2} \right) v_2 \]

or

\[ v_2 = -A \left( \frac{R_1 + R_2 + AR_1}{R_1 + R_2} \right) v_2 = A \left( \frac{R_1}{R_1 + R_2 - 1} \right) v_1 - \left( \frac{AR_1}{R_1 + R_2} \right) v_2 \]

Taking the ratio of the second and last terms above gives

\[ g_{21} = \frac{v_2}{v_1} = \frac{-AR_2}{R_1 + R_2 + AR_1} \]

$v_1 = 0$: \[ g_{12} = \frac{i_1}{i_2} \quad \text{and} \quad g_{22} = \frac{v_2}{i_2} \]

A model for the two port driven by $i_2$ with $v_1 = 0$ is shown. We see that all the current, $i_2$, will flow into the controlled voltage source because it has a zero resistance and none will flow through the $R_1 + R_2$ combination. Therefore, $i_1 = 0$ and thus $v_2 = 0$. Consequently, both $g_{12}$ and $g_{22}$ are zero.