## QUIZ NO. 8 - SOLUTION

(Average score $=7.5 / 10$ of those taking the quiz.)
$r_{\pi}$ and $\beta$ of the BJT shown are $1 \mathrm{k} \Omega$ and 100 , respectively. Find the numerical value of all zeros, poles, and the midband gain for the voltage transfer function, $\mathrm{V}_{\text {out }}(\mathrm{s}) / \mathrm{V}_{\text {in }}(\mathrm{s})$. What is the lower -3 dB frequency?

## Solution

The small signal model for this problem is:


$$
\begin{aligned}
& \frac{V_{\text {out }}}{V_{\text {in }}}=\left(\frac{V_{\text {out }}}{I_{b}}\right)\left(\frac{I_{b}}{V_{\text {in }}}\right)=\left(\frac{-\beta R_{2} R_{3}}{R_{2}+R_{3}+\frac{1}{\mathrm{~s} C_{2}}}\right)\left(\frac{1}{r_{\pi}+(1+\beta)\left(\frac{R / \mathrm{s} C_{1}}{R_{1}+\frac{1}{\mathrm{~s} C_{1}}}\right)}\right) \\
& =\left(\frac{-\beta R_{2} R_{3}}{r_{\pi}\left(R_{2}+R_{3}\right)}\right)\left(\frac{\mathrm{s}}{\mathrm{~s}+\frac{1}{C_{2}\left(R_{2}+R_{3}\right)}}\right)\left(\frac{\mathrm{s}+\frac{1}{R_{1} C_{1}}}{\mathrm{~s}+\frac{r_{\pi}+(1+\beta) R_{1}}{r_{\pi} R_{1} C_{1}}}\right)=A_{0}\left(\frac{\mathrm{~s}}{\mathrm{~s}+\omega_{2}}\right)\left(\frac{\mathrm{s}+\omega_{1}}{\mathrm{~s}+\omega_{1}}\right)
\end{aligned}
$$

where

$$
A_{o}=\frac{-\beta \cdot R_{2} \| R_{3}}{r_{\pi}}=\frac{-100 \cdot 5 \mathrm{k} \Omega}{1 \mathrm{k} \Omega}=-500 \mathrm{~V} / \mathrm{V} \quad \omega_{1}=\frac{1}{R_{1} C_{1}}=\frac{1}{20 \mathrm{k} \Omega \cdot 100 \mu \mathrm{~F}}=0.5 \mathrm{rads} / \mathrm{sec}
$$

$$
\omega_{1^{\prime}}=\omega_{1}\left(1+\frac{(1+\beta) R_{1}}{r_{\pi}}\right)=0.5\left(1+\frac{101 \cdot 20}{1}\right)=1010 \mathrm{rads} / \mathrm{sec}
$$

and

$$
\omega_{2}=\frac{1}{\mathrm{C}_{2}\left(\mathrm{R}_{2}+\mathrm{R}_{3}\right)}=\frac{1}{20 \mathrm{k} \Omega \cdot 1 \mu \mathrm{~F}}=50 \mathrm{rads} / \mathrm{sec}
$$

Therefore,

$$
\text { Zeros at } \mathrm{s}=0 \text { and } \mathrm{s}=-0.5 \mathrm{rad} / \mathrm{sec}
$$

$$
\text { poles at } \mathrm{s}=-50 \mathrm{rads} / \mathrm{sec} \text { and } \mathrm{s}=-1010 \mathrm{rads} / \mathrm{sec}
$$

$$
\text { Midband gain }=A_{\mathrm{o}}=-500 \mathrm{~V} / \mathrm{V}
$$

The lower -3 dB frequency is

$$
\omega_{L}=\sqrt{50^{2}+1010^{2}-2\left(0.5^{2}\right)}=1018 \mathrm{rads} / \mathrm{sec} \text { or } f_{L}=161 \mathrm{~Hz}
$$

