Assume that the small-signal parameters of the BJT amplifier shown are $g_m = 10 \text{mS}$, $r_\pi = 1 \text{k}\Omega$, $r_s = 0$, $r_o = \infty$, $C_\pi = 10 \text{pF}$, and $C_\mu = 1 \text{pF}$.

a.) Find the midband voltage gain of this amplifier, $V_{out}/V_{in}$.

b.) Find the value of the upper -3dB frequency, $f_H$, in Hz, first using the Miller approximation and secondly using the open-circuit time constant approach.

c.) Which of the two answers for $f_H$ in part b.) is the most accurate and why?

**Solution**

a.) The small-signal model for all three parts of this problem is shown.

The MBG is,

$$ \frac{V_{out}(0)}{V_{in}(0)} = -g_m(R_C || R_L) \left( \frac{r_\pi}{(r_\pi + R_S)} \right) = -50 \text{V/V} $$

b.) The Miller approximation gives the following capacitance between gate and source.

$$ C_{eq} = C_\pi + [1 - g_m (R_C || R_L)] C_\mu = 10 \text{pF} + (1+100)1 \text{pF} = 111 \text{pF}. $$

$$ \therefore \omega_H = \frac{1}{(R_S || r_\pi) C_{eq}} = 0.5 \text{K-111} \text{pF} = 18.02 \text{Mrads/sec.} \rightarrow f_H = \frac{18.02 \times 10^6}{2\pi} = 2.87 \text{MHz} $$

The OCTC approach requires finding $R_{c\pi O}$ and $R_{c\mu O}$.

These are found as,

$$ R_{c\pi O} = R_i = 500 \Omega $$

$$ R_{c\mu O} = ? $$

$$ V_I = V_{be} + (I_t + g_m V_{be})10k\Omega = I_t R_{eq} + (I_t + g_m I_t R_{eq})10k\Omega $$

$$ \therefore R_{c\mu O} = \frac{V_I}{I_t} = R_{eq} + (1 + g_m R_{eq})10k\Omega = 500\Omega + (1+5)10k\Omega = 60.5k\Omega $$

$$ \therefore \omega_H = \frac{1}{R_{c\pi O} C_\pi + R_{c\mu O} C_\mu} = \frac{500 \cdot 10\text{pF} + 60.5 \text{K} \cdot 1\text{pF}}{500 \cdot 10\text{pF} + 60.5 \text{K} \cdot 1\text{pF}} = 15.27 \times 10^6 \text{rads/sec.} $$

Thus, $f_H = \frac{15.27 \times 10^6}{2\pi} = 2.43 \text{MHz}$ However, since the RC products are poles we can solve for the poles as ($1/R_{c\pi O} C_\pi)^{-1}$=200 Mrads/sec and $1/R_{c\mu O} C_\mu)^{-1}$=16.53 Mrads/sec. The dominant pole is $-16.53 \text{Mrads/sec}$ which give $f_H = 2.63 \text{MHz}$

c.) The answer given by the OCTC method is probably more correct because the impedance of $C_\mu$ at $\omega_H$ for the Miller approach turns out to be $(1/18.02 \times 10^6 \cdot 10^{-12}) = 55.5k\Omega$ which is not all that much greater than $R_L || R_C = 10k\Omega$. 