QUIZ NO. 13
(Average score = 7.0/10 of those taking the quiz)

The circuit shown is to be an oscillator. The transistors are identical with $r_{ds} = \infty$. (a.) Should the switch at the gate of M1 be connected to point A or B in order to oscillate? (b.) Find the frequency of oscillation in Hertz and the value of $g_m$ necessary for oscillation.

Solution
(a.) We see that the gain of each amplifier can be written as

$$\frac{-g_m R}{sRC+1}$$

and has a phase shift of $180^\circ\cdot\tan^{-1}(\omega RC)$. This phase shift per amplifier varies from $180^\circ$ to $90^\circ$. Three amplifiers cascaded give a phase shift of $540^\circ\cdot\tan^{-1}(\omega RC)$ or $180^\circ-3\tan^{-1}(\omega RC)$ and it would be impossible to get a loop phase shift of $360^\circ$. Therefore, the switch must be connected to B. (If you can’t follow this, you could just pick a position and see if the equations for oscillation can be satisfied or not.)

(b.) With the switch is connected to B, the gain from the gate of M1 to $V_{out}$ can be expressed as,

$$\frac{V_{out}}{V_{g1}} = T(s) = \left(\frac{-g_m R}{sRC+1}\right)^3 = \frac{(-g_m R)^3}{(sRC)^3 + 3s^2R^2C^2 + 3sRC + 1}$$

$$T(j\omega) = \frac{(-g_m R)^3}{[1-3\omega^2R^2C^2]+j\omega RC [3-\omega^2R^2C^2]} = 1+j0$$

$$\omega_{osc} = \frac{\sqrt{3}}{RC} = \frac{1.732}{10x10^3 \cdot 1x10^{-7}} = 1.732 \text{Krad/sec} \quad \rightarrow \quad f_{osc} = 275.7 \text{Hz}$$

Also, from the above equation, we get

$$- (g_m R)^3 = 1 - 3\omega^2R^2C^2 = 1 - 9 = -8$$

$$\therefore \quad (g_m R)^3 = 8 \quad \rightarrow \quad g_m R = 8^{0.33} = 2 \quad \rightarrow \quad g_m = \frac{2}{10K\Omega} = 200 \mu S$$