## QUIZ NO. 13

(Average score $=7.0 / 10$ of those taking the quiz)
The circuit shown is to be an oscillator. The transistors are identical with $r_{d s}=\infty$. (a.) Should the switch at the gate of M1 be connected to point $A$ or $B$ in order to oscillate? (b.) Find the frequency of oscillation in Hertz and the value of $g_{m}$ necessary for oscillation.


## Solution

(a.) We see that the gain of each amplifier can be written as

$$
\frac{-g_{m} R}{s R C+1}
$$

and has a phase shift of $180^{\circ}-\tan ^{-1}(\omega R C)$. This phase shift per amplifier varies from $180^{\circ}$ to $90^{\circ}$. Three amplifiers cascaded give a phase shift of $540^{\circ}-3 \tan ^{-1}(\omega R C)$ or $180^{\circ}-$ $3 \tan ^{-1}(\omega R C)$. If the switch is at A, the total loop phase shift would be $720^{\circ}-3 \tan ^{-1}(\omega R C)$ or $3 \tan ^{-1}(\omega R C)$ and it would be impossible to get a loop phase shift of $360^{\circ}$. Therefore, the switch must be connected to B. (If you can't follow this, you could just pick a position and see if the equations for oscillation can be satisfied or not.)
(b.) With the switch is connected to B , the gain from the gate of M 1 to $V_{\text {out }}$ can be expressed as,

$$
\begin{aligned}
& \frac{V_{\text {out }}}{V_{g 1}}=T(s)=\left(\frac{-g_{m} R}{s R C+1}\right)^{3}=\frac{\left(-g_{m} R\right)^{3}}{(s R C)^{3}+3 s^{2} R^{2} C^{2}+3 s R C+1} \\
& T(j \omega)=\frac{-\left(g_{m} R\right)^{3}}{\left[1-3 \omega^{2} R^{2} C^{2}\right]+j \omega R C\left[3-\omega^{2} R^{2} C^{2}\right]}=1+\mathrm{j} 0 \\
& \omega_{\text {osc }}=\frac{\sqrt{3}}{R C}=\frac{1.732}{10 \times 10^{3} \cdot 1 \times 10^{-7}}=1.732 \mathrm{Krad} / \mathrm{sec} \quad \rightarrow \quad f_{\underline{\text { oss }}}=275.7 \mathrm{~Hz}
\end{aligned}
$$

Also, from the above equation, we get

$$
\begin{aligned}
&-\left(g_{m} R\right)^{3}=1-3 \omega^{2} R^{2} C^{2}=1-9=-8 \\
& \therefore \quad\left(g_{m} R\right)^{3}=8 \quad \rightarrow \quad g_{m} R=8^{0.33}=2 \quad \rightarrow \quad g_{m}=\frac{2}{10 \mathrm{~K} \Omega}=\underline{\underline{200 \mu S}}
\end{aligned}
$$

