Problem 1 - (20 points - This problem must be attempted)

The circuit shown is a $RC$ oscillator. Find the frequency of oscillation in Hertz and the voltage gain, $K$, of the voltage amplifiers necessary for oscillation. The voltage amplifiers have infinite input resistance and zero output resistance.

Solution

The loop gain can be found from the schematic shown:

$$T(s) = \frac{V_r}{V_x} = K^2 \left( \frac{1}{sRC+1} \right) \left( \frac{sRC}{sRC+1} \right)$$

$$= \frac{K^2 sRC}{s^2 R^2 C^2 + 2sRC + 1} \quad \rightarrow \quad T(j\omega) = \frac{K^2 j\omega RC}{1 - \omega^2 R^2 C^2 + j\omega 2RC} = 1 + j0$$

We see from this equation that for oscillation to occur, the following conditions must be satisfied:

$$1 - \omega^2 R^2 C^2 = 0 \quad \text{and} \quad K^2 = 2$$

or

$$\omega_{osc} = \frac{1}{RC} = \frac{1}{10^4 \cdot 10^{-9}} = 10^5 \text{ radians/sec.} \quad \rightarrow \quad f_{osc} = 15.9\text{kHz}$$

and

$$K = \sqrt{2} = 1.414$$
Problem 2 – (20 points – This problem must be attempted)

The simplified schematic of a feedback amplifier is shown. Assume that all transistors are matched and \( g_m = 1\text{mA/V} \) and \( r_{ds} = \infty \). (a.) Where should the switch be connected for negative feedback? (b.) Use the method of feedback analysis to find \( v_2/v_1 \), \( R_{in} = v_1/i_1 \), and \( R_{out} = v_2/i_2 \).

Solution

(a.) A quick check of the ac voltage changes around the loop show that the switch should be connected to B.

(b.) This feedback circuit is series-series. The units of \( A \) are A/V and the units of \( \beta \) are V/A.

\[ \beta = z_{12f} = \frac{v_{1f}}{i_{2f}}\big|_{i_{1f}=0} = R_3 = 1\text{k}\Omega \]

The circuit for calculating the small-signal open-loop gain is,

\[ A = \frac{i_o'}{v_s'} = \left( \frac{i_o'}{v_{gs3}'} \right) \left( \frac{v_{gs3}'}{v_{g3}'} \right) \left( \frac{v_{g3}'}{v_s'} \right) = \left( g_m \right) \left( \frac{1}{1 + g_m R_3} \right) \left( g_m R_2 \right) \]

\[ A = \frac{i_o'}{v_s'} = (1\text{mS})(0.5)(5)(2) = 2.5\text{mS} \rightarrow A_F = \frac{i_o}{v_s} = \frac{A}{1 + A\beta} = \frac{2.5\text{mS}}{1 + 2.5 \cdot 1} = 0.714 \text{mS} \]

\[ \frac{v_2}{v_1} = \frac{v_2}{v_s} = \left( \frac{i_o}{v_s} \right) = \left( \frac{v_o}{v_s} \right) = -R_4(0.714\text{mS}) = -7.14 \text{V/V} \]

\[ \frac{v_2}{v_s} = -7.14 \text{ V/V} \]

\( R_1 \) is not influenced by feedback so \( \frac{v_1}{i_1} = R_1 = 1\text{k}\Omega \)

\[ R_o = R_4 + \infty = \infty \rightarrow R_{oF} = \infty(1 + 2.5) = \infty \]

\[ R_{out} = \frac{v_2}{i_2} = \frac{v_2}{v_s} = \frac{R_{oF} - R_4}{R_4} = \infty \| 10\text{k}\Omega = 10\text{k}\Omega \]

\[ \frac{v_2}{i_2} = R_4 = 10\text{k}\Omega \]
Problem 3 - (20 points - This problem is optional)

An inverting and noninverting unity gain voltage amplifier are shown using op amps. If the differential voltage gain of each op amp is given as

\[ A_{vd}(s) = \frac{10^4}{s + 1} \]

find the closed loop –3dB bandwidth, \( \omega_{3dB} \) for each of the two op amp configurations. Assume the op amps have infinite differential input resistance and zero output resistance.

Solution

The best approach is to replace the op amp with its small-signal model and calculate the closed loop voltage gain. The model is,

\[ V_{out} = -A_{vd}(s)V_{in} = -A_{vd}(s)\left(\frac{R_2}{R_1+R_2}V_{in} + \frac{R_1}{R_1+R_2}V_{out}\right) = -\frac{A_{vd}(s)R_2}{R_1+R_2}V_{in} - \frac{A_{vd}(s)R_1}{R_1+R_2}V_{out} \]

\[ V_{out}\left(1 + \frac{A_{vd}(s)R_1}{R_1+R_2}\right) = -\frac{A_{vd}(s)R_2}{R_1+R_2}V_{in} \Rightarrow V_{out} = -\frac{A_{vd}(s)R_2}{R_1+R_2}V_{in} = -\frac{A_{vd}(s)R_2}{R_1+R_2}V_{in} = -\frac{A_{vd}(s)}{2}V_{in} \]

\[ V_{out} = \frac{0.5}{A_{vd}(s)+0.5} = \frac{0.5 \times 10^4}{s+0.5 \times 10^6} \Rightarrow \omega_{3dB} = 0.5 \times 10^6 \text{ radians/sec.} \]

For the noninverting unity gain amplifier, \( R_1 = \infty \) and \( R_2 = 0 \) to give,

\[ V_{out} = A_{vd}(s)V_{id} = A_{vd}(s)(V_{in} - V_{out}) \rightarrow V_{out} = \frac{A_{vd}(s)}{1+A_{vd}(s)} = \frac{10^4}{s+1+10^4} \approx \frac{10^6}{s+1} \]

\[ \therefore \omega_{3dB} = 10^6 \text{ radians/sec.} \]
Problem 4 - (20 points - This problem is optional)

1.) If $g_m = 1\text{mA/V}$, what is the midband voltage gain of the amplifier shown? Assume $r_d = \infty$.

2.) Find all poles and zeros of this amplifier in radians/sec.

Solution

The small-signal model for this problem is,

$$\frac{V_{out}}{V_{in}} = \frac{V_{gs}}{V_g} = \frac{R_G}{R_G + R_S + \frac{1}{sC_1}}$$

$$\therefore \frac{V_{out}}{V_{in}} = \left( -g_m R_3 \right) \left[ \frac{s + \frac{1}{R_4 C_2}}{s + \frac{1+g_m R_4}{R_4 C_2}} \right] \left[ \frac{R_G}{R_G + R_S} \right] \frac{s}{s + \frac{1}{C_1(R_G + R_S)}}$$

$$\text{MBG} = \frac{-g_m R_3 R_G}{R_G + R_S} = \frac{-1\text{mS}\cdot20\text{k}\Omega\cdot1\text{M}\Omega}{1\text{M}\Omega + 1\text{k}\Omega} = -19.98 \text{V/V}$$

Zeros at $s = 0$ and $s = -\frac{1}{R_4 C_2} = \frac{-1}{10^3.2\times10^{-6}} = -500 \text{ radians/sec}$.

Poles at $s = -\frac{1+g_m R_4}{R_4 C_2} = \frac{-2}{10^3.2\times10^{-6}} = -1000 \text{ radians/sec}$

and

$$s = -\frac{1}{C_1(R_G + R_S)} = \frac{-1}{10^{-8} \cdot 1.001 \times 10^6} = -99.9 \text{ radians/sec}.$$
Problem 5 - (20 points - This problem is optional)

The FET in the amplifier shown has $g_m = 1\text{mA/V}$, $r_d = \infty$, $C_{gd} = 0.5\text{pF}$, and $C_{gs} = 10\text{pF}$. (a.) Find the midband gain, $V_{out}/V_{in}$. (b.) Find the upper -3dB frequency, $f_H$, in Hz.

Solution

The high-frequency, small-signal model for this problem is shown below.

The MBG can be found as,

$$
\text{MBG} = (-g_m R_3) \left( \frac{R_G}{R_s + R_G} \right)
$$

$$
= (-20) \left( \frac{1\text{M}\Omega}{1\text{k}\Omega + 1\text{M}\Omega} \right) = -19.98 \text{V/V}
$$

The two approaches to working this problem are the OTC and Miller. Let us choose Miller since it is simpler and more direct.

$$
C_{total} = C_{gs} + C_{gd}(1+20) = 10\text{pF} + 0.5\text{pF}(21) = 20.5\text{pF}
$$

The resistance seen by $C_{total}$ is $R_s||R_G = 1\text{k}\Omega||1\text{M}\Omega = 0.999\text{k}\Omega$

$$
\therefore \omega_{3dB} = \frac{1}{999\times20.5\times10^{-12}} = 48.829 \text{ Mradians/sec.} \rightarrow f_{3dB} = 7.77\text{MHz}
$$

Check to see if the Miller approach is valid.

$$
\frac{1}{\omega_{3dB} C_{gd}} = \frac{1}{(48.829\times10^6)(0.5\text{pF})} = 40.96\text{k}\Omega \text{ which is twice } R_3
$$

We probably should try the OTC approach to see how it compares-

$$
R_{Cgs} = 1\text{k}\Omega \rightarrow \omega_{Cgs} = \frac{1}{C_{gs} \cdot 1\text{k}\Omega} = 100 \text{ Mrads/sec.}
$$

and

$$
R_{Cgd} = R_s + (1+g_m R_s)R_3 = 1\text{k}\Omega + 2(20k\Omega) = 41\text{k}\Omega
$$

$$
\omega_{Cgd} = \frac{1}{C_{gd} \cdot 41\text{k}\Omega} = 48.78 \text{ Mrads/sec.}
$$

$$
\therefore \omega_H = \frac{1}{100\text{Mrads/sec} + 48.78\text{Mrads/sec}} = 32.75 \text{ Mrads/sec.} \rightarrow f_{3dB} = 5.21\text{MHz}
$$

Using the more exact analysis of algebraically solving the nodal equations gives $f_{3dB} = 5.95\text{MHz}$.
Problem 6 - (20 points - This problem is optional).

A BJT amplifier is shown. Assume that the BJT has the small signal parameters of $\beta_F = 100$, $V_t = 25\text{mV}$, and $V_A = \infty$.

a.) Find the midband voltage gain of this amplifier, $V_{out}/V_{in}$.

b.) Find the value of the lower -3dB frequency, $f_L$, in Hz, using any method that is appropriate.

**Solution**

Small-signal model for this problem:

$$ I_e = (1+\beta)I_b $$

$$ g_m = \frac{1mA}{25mV} = 40\text{mS} $$

$$ r_\pi = \frac{\beta_F g_m}{g_m} = 2.5k\Omega $$

Use the direct approach:

$$ \frac{V_{out}}{V_{in}} = \left(\frac{V_{out}}{I_e}\right) \left(\frac{I_e}{V_{in}}\right) = \left(\frac{-\alpha_F R_C R_L}{R_C + R_L + \frac{1}{sC_2}}\right) \left(\frac{-1}{\frac{1}{sC_1} + \frac{1}{g_m}}\right) $$

$$ = \left(\frac{\alpha_F R_C R_L}{R_C + R_L}\right) \left(\frac{g_m}{1+g_m R_S}\right) \left(\frac{s}{s + \frac{1}{C_2(R_C + R_L)}}\right) \left(\frac{s + \frac{1}{C_1(R_S + \frac{1}{g_m})}}{s + \frac{1}{C_1(R_S + \frac{1}{g_m})}}\right) $$

$$ \therefore \quad \text{MBG} = \left(\frac{\alpha_F R_C R_L}{R_C + R_L}\right) \left(\frac{g_m}{1+g_m R_S}\right) = 5K \times \frac{40\text{mS}}{41} = 200 \frac{4.878\text{V/V}}{10000 + 25} = 97.56 \text{rads/sec.} $$

$$ \omega_1 = \frac{1}{C_1(R_S + \frac{1}{g_m})} = 10^6 \frac{10(1000 + 25)}{100000} = 50 \text{rads/sec.} $$

$$ \omega_2 = \frac{1}{C_2(R_C + R_L)} = 10^6 \frac{1}{2(20K)} = 50 \text{rads/sec.} $$

Since $\omega_1$ and $\omega_2$ are within an octave of each other then there is no dominant root so that we will simply sum the roots which is identical with the short-circuit time constant approach to give, $\omega_L$ as,

$$ \omega_L = \omega_1 + \omega_2 = 147.56 \text{rads/sec.} \quad \rightarrow f_L = 23.5 \text{Hz} $$
Problem 7 – (20 points, this problem is optional)

A common-emitter BJT amplifier is shown. Assume that the BJT has a $\beta = hfe = 100$, $C_\mu = 2\text{pF}$, $V_t = 25\text{mV}$, $f_T = 500\text{MHz}$, $r_b = 0\Omega$, and $r_o = \infty$.

a.) Find the numerical values of $r_\pi$, $g_m$, and $C_\pi$.

b.) If $r_\pi = 1\text{k}\Omega$, $g_m = 0.01\text{A/V}$ and $C_\pi = 10\text{pF}$ for the above amplifier, find the value of the upper - 3dB frequency, $f_H$, in Hz.

Solution

a.) $g_m = \frac{I_C}{V_T} = \frac{1\text{mA}}{25\text{mV}} = 0.04 \text{ A/V}$

$$r_\pi = \frac{\beta_0}{g_m} = \frac{100}{0.04} = 2500\Omega$$

$$C_\pi = \frac{g_m}{\omega_T} - C_\mu = \frac{0.04}{2\pi\cdot500\times10^6} - 2\text{pF} = 12.732\text{pF} - 2\text{pF} = 10.732 \text{ pF}$$

b.) The high-frequency, small-signal model for this problem is shown where $R_{CL} = R_C||R_L = 5\text{k}\Omega$.

The midband gain of this amplifier is given by

$$\frac{V_{out}}{V_{in}} = \left(\frac{V_{out}}{V_{\pi}}\right)\left(\frac{V_{\pi}}{V_{in}}\right) = -g_m R_C||R_L \left(\frac{r_\pi}{r_\pi + R_S}\right) = (-0.01\cdot5\text{k}\Omega)(0.5) = -25\text{V/V}$$

∴ $\boxed{\text{MBG} = -25 \text{V/V}}$

Using Miller’s theorem on this problem:

If $\frac{1}{\omega_H C} \gg R_{C||L}$, then $C_{eq} = C_{\pi,=} + C (1 + g_m R_C||R_L) = 10\text{pf} + 2\text{pF}(1+50) = 112\text{pF}$

We know that, $\omega_H = \frac{1}{C_{eq} (r_\pi||R_S)} = \frac{1}{(112\text{pF}\cdot500\Omega)} = 17.86 \text{ Mrads/sec}$.

∴ $f_H = \frac{\omega_H}{2\pi} = 2.842 \text{ MHz}$

Note that:

$$\frac{1}{\omega_H C} = \frac{1}{17.86\times2} = 28.06\text{k}\Omega > 5\text{k}\Omega$$ so the Miller approximation (neglecting $C_\mu$) is valid.
Problem 8 – (20 points, this problem is optional)

A feedback amplifier is shown. Use the methods of feedback analysis to find the numerical values of $v_2/v_1$, $v_1/i_1$, and $v_2/i_2$. Assume that all transistors are matched and that $g_m = 1 \text{mS}$, and $r_{ds} = \infty$.

Solution

The feedback circuit is shunt-series. You should know by now not to include $R_1$ in the feedback circuit. The simplified transistor circuit and the open-loop equivalent is shown below. It is easy to see that $\beta = i_{1F}/i_{2F} (v_{1F} = 0) = -1$.

The small-signal, open-loop circuit is,

$$
A = i_{o'}/i_1 = \left(\frac{v_{gs2}}{v_{gs1}}\right) \left(\frac{v_{gs2}}{v_{gs1}}\right) \left(\frac{v_{gs1}}{i_1'}\right) = \left(\frac{v_{gs2}'}{v_{gs1}}\right) \left(\frac{v_{gs2}}{v_{gs1}}\right) \left(\frac{v_{gs1}}{i_1'}\right)
$$

Also, $i_1' + \frac{v_{gs1}'}{R_4} + g_m v_{gs1} = 0 \implies -g_m v_{gs1} = \frac{v_{gs1}}{R_4} + g_m v_{gs1} = 0 \implies -g_m v_{gs1} = \frac{v_{gs1}}{R_4} + g_m v_{gs1} = 0 \implies \frac{v_{gs1}}{R_4} + g_m v_{gs1} = 0 \implies \frac{v_{gs1}}{R_4} + g_m v_{gs1} = 0 \implies A/A

Now, $i_o/i_1 = \frac{-5}{1+(-1)/(-5)} = \frac{5}{6} \ A/A$

$$
R_{in}(\beta=0) = R_4 || (1/g_m) = 0.5 \text{k}\Omega, \quad R_{inF} = \frac{0.5 \text{k}\Omega}{1+5} = \frac{1}{12} \text{k}\Omega = 83.33 \text{\Omega} \quad \frac{v_1}{i_1} = 1083.33 \text{\Omega}
$$

$$
\frac{v_2}{v_1} = \frac{i_o}{i_1} \left(\frac{-R_3}{(v_1/i_1)}\right) = \frac{5}{6} \frac{-1000}{1083.33} = \frac{10}{13} = 0.7692 \ \text{V/V}
$$

$$
R_o(\beta=0) = R_3 + (1/g_m) = 2 \text{k}\Omega, \quad R_{of} = 2 \text{k}\Omega(1+5) = 12 \text{k}\Omega
$$

$$
\frac{v_2}{i_2} = (12 \text{k}\Omega - 1 \text{k}\Omega) || 1 \text{k}\Omega = 11 \text{k}\Omega || 1 \text{k}\Omega = \frac{11}{12} \text{k}\Omega = 916.67 \text{\Omega}
$$