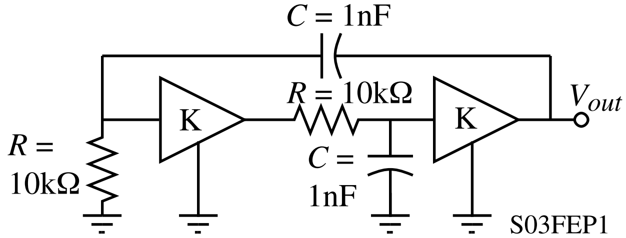
FINAL EXAMINATION – SOLUTION

(Average Score = 78/100)

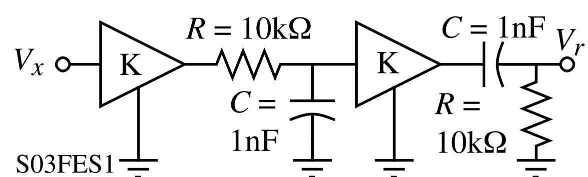
Problem 1 - (20 points - This problem must be attempted)

The circuit shown is a *RC* oscillator. Find the frequency of oscillation in Hertz and the voltage gain, *K*, of the voltage amplifiers necessary for oscillation. The voltage amplifiers have infinite input resistance and zero output resistance.



Solution

The loop gain can be found from the schematic shown:



$$T(s) = \frac{V_r}{V_x} = K^2\left(\frac{1}{sRC+1}\right)\left(\frac{sRC}{sRC+1}\right)$$

$$= \frac{K^2 sRC}{s^2R^2C^2 + 2sRC + 1} \quad \rightarrow \quad T(j\omega) = \frac{K^2 j\omega RC}{1-\omega^2R^2C^2 + j\omega\, 2RC} = 1 + j0$$

We see from this equation that for oscillation to occur, the following conditions must be satisfied:

$$1-\omega^2R^2C^2 = 0 \qquad \text{and} \qquad K^2 = 2$$

or

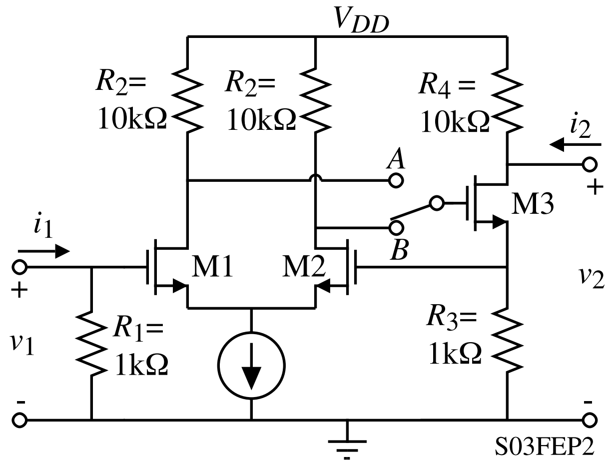
$$\omega_{osc} = \frac{1}{RC} = \frac{1}{10^4\cdot 10^{-9}} = 10^5 \text{ radians/sec.} \quad \rightarrow \quad f_{osc} = \underline{\underline{15.9\text{kHz}}}$$

and

$$K = \sqrt{2} = \underline{\underline{1.414}}$$

Problem 2 – (20 points – This problem must be attempted)

The simplified schematic of a feedback amplifier is shown. Assume that all transistors are matched and g_m = 1mA/V and $r_{ds} = \infty$. (a.) Where should the switch be connected for negative feedback? (b.) Use the method of feedback analysis to find v_2/v_1, $R_{in} = v_1/i_1$, and $R_{out} = v_2/i_2$.



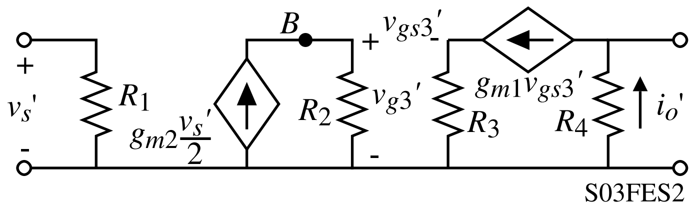
Solution

(a.) A quick check of the ac voltage changes around the loop show that the switch should be connected to B.

(b.) This feedback circuit is series-series. The units of A are A/V and the units of β are V/A.

$$\beta = z_{12f} = \frac{v_{1f}}{i_{2f}}\bigg|_{i_{1f}=0} = R_3 = 1\text{k}\Omega$$

The circuit for calculating the small-signal open-loop gain is,



$$A = \frac{i_o'}{v_s'} = \left(\frac{i_o'}{v_{gs3}'}\right)\left(\frac{v_{gs3}'}{v_{g3}'}\right)\left(\frac{v_{g3}'}{v_s'}\right) = (g_{m3})\left(\frac{1}{1+g_{m3}R_3}\right)\left(\frac{g_{m2}R_2}{2}\right)$$

$$A = \frac{i_o'}{v_s'} = (1\text{mS})(0.5)(5)(2) = 2.5\text{mS} \rightarrow A_F = \frac{i_o}{v_s} = \frac{A}{1+A\beta} = \frac{2.5\text{mS}}{1+2.5\cdot 1} = 0.714 \text{ mS}$$

$$\frac{v_2}{v_1} = \frac{v_2}{v_s} = \left(\frac{v_2}{i_o}\right)\left(\frac{i_o}{v_s}\right) = -R_4(0.714\text{mS}) = -7.14 \text{ V/V}$$

$$\boxed{\frac{v_2}{v_s} = -7.14 \text{ V/V}}$$

R_1 is not influenced by feedback so $\boxed{\frac{v_1}{i_1} = R_1 = 1\text{k}\Omega}$

$$R_o = R_4 + \infty = \infty \qquad \rightarrow \qquad R_{oF} = \infty(1+2.5) = \infty$$

$$R_{out} = \frac{v_2}{i_2} = (R_{oF} - R_4)||R_4 = \infty||10\text{k}\Omega = 10\text{k}\Omega$$

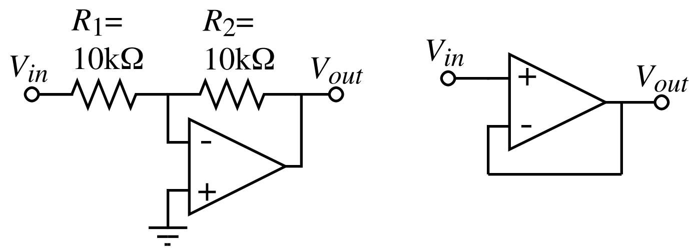
$$\boxed{\frac{v_2}{i_2} = R_4 = 10\text{k}\Omega}$$

Problem 3 - (20 points - This problem is optional)

An inverting and noninverting unity gain voltage amplifier are shown using op amps. If the differential voltage gain of each op amp is given as

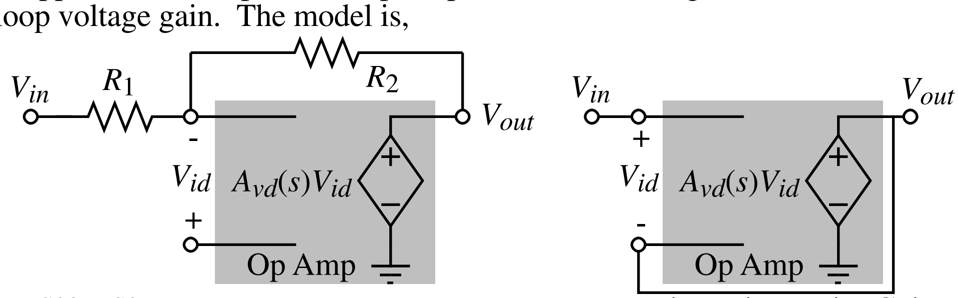
$$A_{vd}(s) = \frac{10^4}{\frac{s}{100}+1}$$

find the closed loop –3dB bandwidth, ω_{-3dB} for each of the two op amp configurations. Assume the op amps have infinite differential input resistance and zero output resistance.



Solution

The best approach is to replace the op amp with its small-signal model and calculate the closed loop voltage gain. The model is,



Inverting unity gain amplifier:

$$V_{out} = -A_{vd}(s)V_{id} = -A_{vd}(s)\left(\frac{R_2}{R_1+R_2}V_{in} + \frac{R_1}{R_1+R_2}V_{out}\right) = -\frac{A_{vd}(s)R_2}{R_1+R_2}V_{in} - \frac{A_{vd}(s)R_1}{R_1+R_2}V_{out}$$

$$V_{out}\left(1 + \frac{A_{vd}(s)R_1}{R_1+R_2}\right) = -\frac{A_{vd}(s)R_2}{R_1+R_2}V_{in} \rightarrow \frac{V_{out}}{V_{in}} = -\frac{\frac{A_{vd}(s)R_2}{R_1+R_2}}{1+\frac{A_{vd}(s)R_1}{R_1+R_2}}V_{in} = -\frac{\frac{A_{vd}(s)}{2}}{1+\frac{A_{vd}(s)}{2}}V_{in}$$

$$\frac{V_{out}}{V_{in}} = -\frac{0.5}{\frac{1}{A_{vd}(s)}+0.5} = -\frac{0.5\text{x}10^4}{\frac{s}{100}+1+0.5\text{x}10^4} \approx \frac{0.5\text{x}10^6}{s+0.5\ \text{x}10^6} \rightarrow \omega_{-3dB} = \underline{\underline{0.5\text{x}10^6 \text{ radians/sec.}}}$$

For the noninverting unity gain amplifier, $R_1 = \infty$ and $R_2 = 0$ to give,

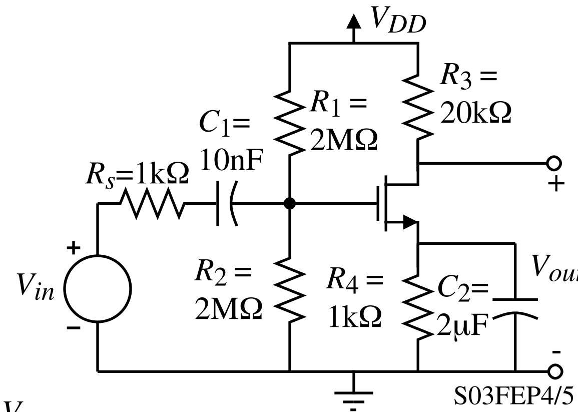
$$V_{out} = A_{vd}(s)V_{id} = A_{vd}(s)(V_{in} - V_{out}) \rightarrow \frac{V_{out}}{V_{in}} = \frac{A_{vd}(s)}{1+A_{vd}(s)} = \frac{10^4}{\frac{s}{100}+1+10^4} \approx \frac{10^6}{s+10^6}$$

$\therefore\ \omega_{-3dB} = \underline{\underline{10^6 \text{ radians/sec.}}}$

Problem 4 - (20 points - This problem is optional)

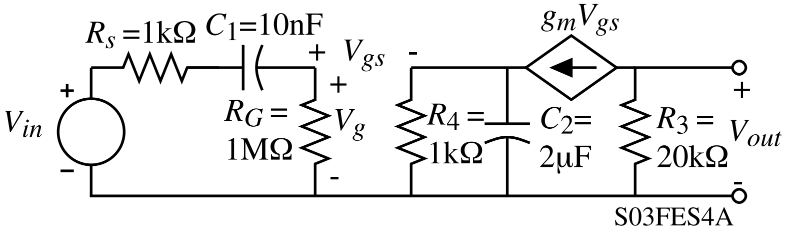
1.) If g_m = 1mA/V, what is the midband voltage gain of the amplifier shown? Assume $r_d = \infty$.

2.) Find all poles and zeros of this amplifier in radians/sec.



Solution

The small-signal model for this problem is,



$$\frac{V_{out}}{V_{in}} = \left(\frac{V_{out}}{V_{gs}}\right)\left(\frac{V_{gs}}{V_g}\right)\left(\frac{V_g}{V_{in}}\right)$$

$$\frac{V_{out}}{V_{gs}} = -g_mR_3$$

$$V_{gs} = V_g - V_s = V_g - g_mR_4 \| (1/sC_2)V_{gs} \;\rightarrow\; V_{gs}\left(1 + \frac{g_mR_4}{sR_4C_2+1}\right) = V_g$$

$$\frac{V_{gs}}{V_g} = \frac{sR_4C_2+1}{sR_4C_2+1+g_mR_4} = \frac{s + \frac{1}{R_4C_2}}{s + \frac{1+g_mR_4}{R_4C_2}} \qquad \frac{V_g}{V_{in}} = \frac{R_G}{R_G+R_S+\frac{1}{sC_1}} = \left(\frac{R_G}{R_G+R_S}\right)\frac{s}{s+\frac{1}{C_1(R_G+R_S)}}$$

$$\therefore\; \frac{V_{out}}{V_{in}} = (-g_mR_3)\left(\frac{s + \frac{1}{R_4C_2}}{s + \frac{1+g_mR_4}{R_4C_2}}\right)\left[\left(\frac{R_G}{R_G+R_S}\right)\frac{s}{s+\frac{1}{C_1(R_G+R_S)}}\right]$$

$$\text{MBG} = \frac{-g_mR_3R_G}{R_G+R_S} = \frac{-1\text{mS}\cdot 20\text{k}\Omega\cdot 1\text{M}\Omega}{1\text{M}\Omega+1\text{k}\Omega} = \underline{\underline{-19.98\text{V/V}}}$$

Zeros at $\underline{\underline{s = 0}}$ and $s = -\frac{1}{R_4C_2} = \frac{-1}{10^3\cdot 2\text{x}10^{-6}} = \underline{\underline{-500 \text{ radians/sec.}}}$

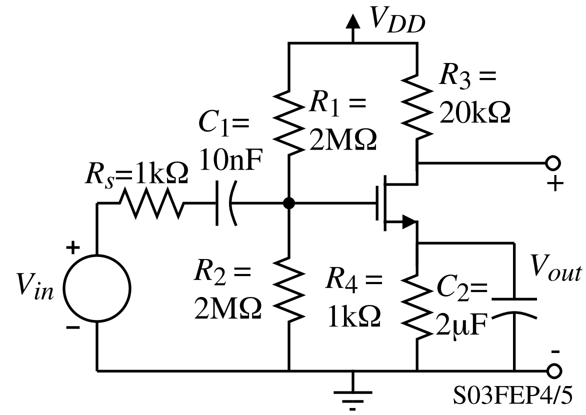
Poles at $s = -\frac{1+g_mR_4}{R_4C_2} = \frac{-2}{10^3\cdot 2\text{x}10^{-6}} = \underline{\underline{-1000 \text{ radians/sec}}}$

and

$$s = -\frac{1}{C_1(R_G+R_S)} = \frac{-1}{10^{-8}\cdot 1.001\text{x}10^6} = \underline{\underline{-99.9 \text{ radians/sec.}}}$$

Problem 5 - (20 points - This problem is optional)

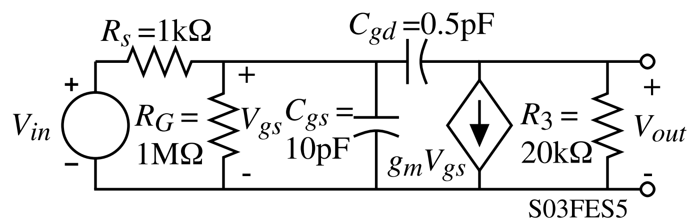
The FET in the amplifier shown has g_m = 1mA/V, $r_d = \infty$, C_{gd} = 0.5pF, and C_{gs} = 10pF. (a.) Find the midband gain, V_{out}/V_{in}. (b.) Find the upper -3dB frequency, f_H, in Hz.



Solution

The high-frequency, small-signal model for this problem is shown below.

The MBG can be found as,



$$\text{MBG} = (-g_m R_3)\left(\frac{R_G}{R_s + R_G}\right)$$

$$= (-20)\left(\frac{1\text{M}\Omega}{1\text{k}\Omega+1\text{M}\Omega}\right) = \underline{\underline{-19.98 \text{ V/V}}}$$

The two approaches to working this problem are the OTC and Miller. Let us choose Miller since it is simpler and more direct.

$$C_{total} = C_{gs} + C_{gd}(1+20) = 10\text{pf} + 0.5\text{pf}(21) = 20.5\text{pF}$$

The resistance seen by C_{total} is $R_s \| R_G$ = 1k‖1M = 0.999kΩ

$$\therefore \quad \omega_{-3\text{dB}} = \frac{1}{999\text{x}20.5\text{x}10^{-12}} = 48.829 \text{ Mradians/sec.} \rightarrow f_{-3\text{dB}} = \underline{\underline{7.77\text{MHz}}}$$

Check to see if the Miller approach is valid.

$$\frac{1}{\omega_{-3\text{dB}} C_{gd}} = \frac{1}{(48.829\text{x}10^6)(0.5\text{pF})} = 40.96\text{k}\Omega \text{ which is twice } R_3$$

We probably should try the OTC approach to see how it compares-

$$R_{Cgs} \approx 1\text{k}\Omega \rightarrow \omega_{Cgs} = \frac{1}{C_{gs} \cdot 1\text{k}\Omega} = 100 \text{ Mrads/sec.}$$

and

$$R_{Cgd} \approx R_s + (1+g_m R_s)R_3 = 1\text{k}\Omega + 2(20\text{k}\Omega) = 41\text{k}\Omega$$

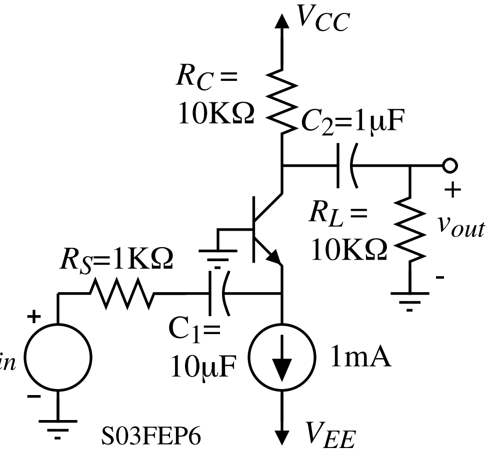
$$\omega_{Cgd} = \frac{1}{C_{gd} \cdot 41\text{k}\Omega} = 48.78 \text{ Mrads/sec.}$$

$$\therefore \omega_H \approx \frac{1}{\frac{1}{100\text{Mrads/sec}} + \frac{1}{48.78\text{Mrads/sec}}} = 32.75 \text{ Mrads/sec.} \rightarrow f_{-3\text{dB}} = \underline{\underline{5.21\text{MHz}}}$$

Using the more exact analysis of algebraically solving the nodal equations gives $f_{-3\text{dB}}$ = $\underline{\underline{5.95\text{MHz}}}$

Problem 6 - (20 points - This problem is optional).

A BJT amplifier is shown. Assume that the BJT has the small signal parameters of $\beta_F = 100$, V_t= 25mV, and $V_A = \infty$.

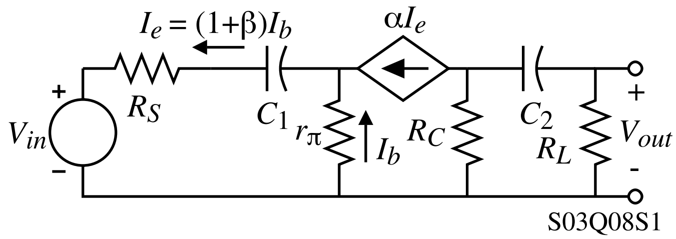


a.) Find the midband voltage gain of this amplifier, V_{out}/V_{in}.

b.) Find the value of the lower -3dB frequency, f_L, in Hz, using any method that is appropriate.

Solution

Small-signal model for this problem:



$$g_m = \frac{1\text{mA}}{25\text{mV}} = 40\text{mS}$$

$$r_\pi = \frac{\beta_F}{g_m} = 2.5\text{k}\Omega$$

Use the direct approach:

$$\frac{V_{out}}{V_{in}} = \left(\frac{V_{out}}{I_e}\right)\left(\frac{I_e}{V_{in}}\right) = \left(\frac{-\alpha_F R_C R_L}{R_C + R_L + \frac{1}{sC_2}}\right)\left(\frac{-1}{R_S + \frac{1}{sC_1} + \frac{1}{g_m}}\right)$$

$$= \left(\frac{\alpha_F R_C R_L}{R_C + R_L}\right)\left(\frac{g_m}{1 + g_m R_S}\right)\left(\frac{s}{s + \frac{1}{C_2(R_C + R_L)}}\right)\left(\frac{s}{s + \frac{1}{C_1\left(R_S + \frac{1}{g_m}\right)}}\right)$$

$$\therefore \qquad MBG = \left(\frac{\alpha_F R_C R_L}{R_C + R_L}\right)\left(\frac{g_m}{1 + g_m R_S}\right) = (5\text{K})\left(\frac{40\text{mS}}{1+40}\right) = \frac{200}{41} = \underline{\underline{+4.878\text{V/V}}}$$

$$\omega_1 = \frac{1}{C_1\left(R_S + \frac{1}{g_m}\right)} = \frac{10^6}{10(1000 + 25)} = 97.56 \text{ rads/sec.}$$

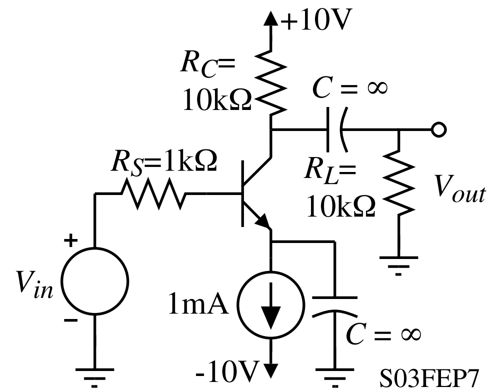
$$\omega_2 = \frac{1}{C_2(R_C + R_L)} = \frac{10^6}{1(20\text{K})} = 50 \text{ rads/sec.}$$

Since ω_1 and ω_2 are within an octave of each other then there is no dominant root so that we will simply sum the roots which is identical with the short-circuit time constant approach to give, ω_L as,

$$\omega_L \approx \omega_1 + \omega_2 = 147.56 \text{ rads/sec.} \quad \rightarrow f_L \approx \underline{\underline{23.5 \text{ Hz}}}$$

Problem 7 – (20 points, this problem is optional)

A common-emitter BJT amplifier is shown. Assume that the BJT has a $\beta = h_{fe} = 100$, $C_\mu = 2\text{pF}$, $V_t = 25\text{mV}$, $f_T = 500\text{MHz}$, $r_b = 0\Omega$, and $r_o = \infty$.



a.) Find the numerical values of r_π, g_m, and C_π.

b.) If $r_\pi = 1\text{k}\Omega$, $g_m = 0.01\text{A/V}$ and $C_\pi = 10\text{pF}$ for the above amplifier, find the value of the upper -3dB frequency, f_H, in Hz.

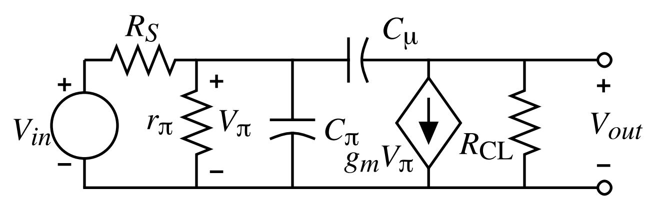
Solution

a.) $$g_m = \frac{I_C}{V_T} = \frac{1\text{mA}}{25\text{mV}} = 0.04 \text{ A/V}$$

$$r_\pi = \frac{\beta_o}{g_m} = \frac{100}{0.04} = 2500\Omega$$

$$C_\pi = \frac{g_m}{\omega_T} - C_\mu = \frac{0.04}{2\pi\cdot 500\text{x}10^6} - 2\text{pF} = 12.732\text{pF} - 2\text{pF} = 10.732 \text{ pF}$$

b.) The high-frequency, small-signal model for this problem is shown where $R_{CL} = R_C||R_L = 5\text{k}\Omega$.



The midband gain of this amplifier is given by

$$\frac{V_{out}}{V_{in}} = \left(\frac{V_{out}}{V_\pi}\right)\left(\frac{V_\pi}{V_{in}}\right) = -g_m R_C||R_L \left(\frac{r_\pi}{r_\pi + R_S}\right) = (-0.01\cdot 5\text{k}\Omega)(0.5) = -25\text{V/V}$$

$\therefore$ MBG = -25 V/V

Using Miller's theorem on this problem:

If $\frac{1}{\omega_H C} >> R_C||R_L$, then $C_{eq} \approx C_\pi + C(1 + g_m R_C||R_L) = 10\text{pf} + 2\text{pF}(1+50) = 112\text{pF}$

We know that, $\omega_H = \frac{1}{C_{eq}(r_\pi||R_S)} = \frac{1}{(112\text{pF}\cdot 500\Omega)} = 17.86$ Mrads/sec.

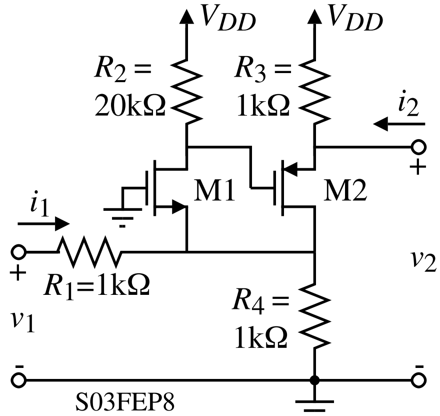
$$\therefore \quad f_H = \frac{\omega_H}{2\pi} = 2.842 \text{ MHz}$$

Note that:

$\frac{1}{\omega_H C} = \frac{10^6}{17.86\cdot 2} = 28.06\text{k}\Omega > 5\text{k}\Omega$ so the Miller approximation (neglecting C_μ) is valid.

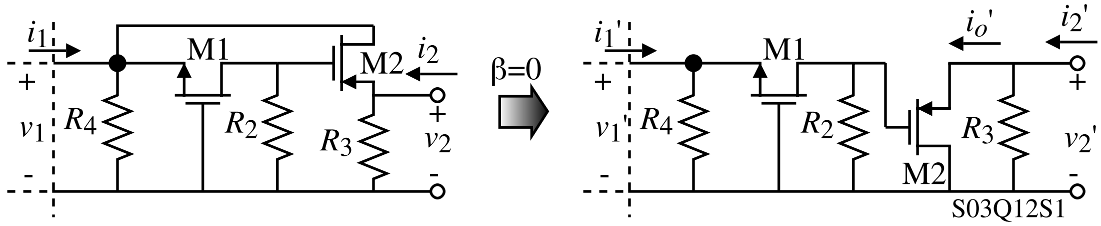
Problem 8 – (20 points, this problem is optional)

A feedback amplifier is shown. Use the methods of feedback analysis to find the numerical values of v_2/v_1, v_1/i_1, and v_2/i_2. Assume that all transistors are matched and that g_m= 1mS, and $r_{ds} = \infty$.

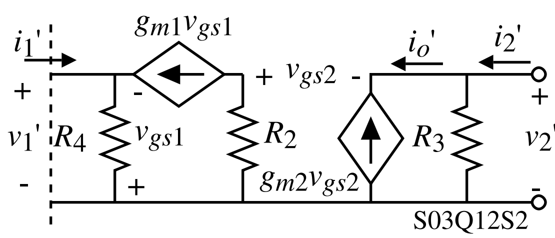


Solution

The feedback circuit is shunt-series. You should know by now not to include R_1 in the feedback circuit. The simplified transistor circuit and the open-loop equivalent is shown below. It is easy to see that $\beta = i_{1F}/i_{2F}\ (v_{1F}=0) = -1$.



The small-signal, open-loop circuit is,



$$A = \frac{i_o'}{i_1'} = \left(\frac{i_o'}{v_{gs2}'}\right)\left(\frac{v_{gs2}'}{v_{g2}'}\right)\left(\frac{v_{g2}'}{v_{gs1}'}\right)\left(\frac{v_{gs1}'}{i_1'}\right)$$

$$v_{gs2}' = v_{g2}' - v_{s2}' = v_{g2}' - g_{m2}R_3\, v_{gs2}'$$

$$\therefore \quad v_{gs2}'(1+ g_{m2}R_3) = v_{g2}'$$

Also, $i_1' + \frac{v_{gs1}'}{R_4} + g_{m1}v_{gs1}' = 0 \qquad \Rightarrow$

$$i_1' = -\left(\frac{1}{R_4} + g_{m1}\right) v_{gs1}'$$

$$\therefore \; A = \frac{i_o'}{i_1'} = (-g_{m2})\left(\frac{1}{1+g_{m2}R_3}\right)(-g_{m1}R_2)\left(\frac{-1}{\frac{1}{R_4}+g_{m1}}\right) = (\text{-1mS})(0.5)(\text{-20})(\text{-0.5k}\Omega) = -5$$

A/A

Now, $\frac{i_o}{i_1} = \frac{-5}{1+(-1)(-5)} = -\frac{5}{6}$ A/A

$R_{in}(\beta=0) = R_4 \| (1/g_{m1}) = 0.5\text{k}\Omega$, $R_{inF} = \frac{0.5\text{k}\Omega}{1+5} = \frac{1}{12}\text{k}\Omega = 83.33\Omega \quad \frac{v_1}{i_1} = \underline{\underline{1083.33\Omega}}$

$$\frac{v_2}{v_1} = \frac{i_o}{i_1}\left(\frac{-R3}{(v_1/i_1)}\right) = -\frac{5}{6}\,\frac{-1000}{1083.33} = \frac{10}{13} = \underline{\underline{0.7692 \text{ V/V}}}$$

$R_o(\beta=0) = R_3+(1/g_{m2}) = 2\text{k}\Omega$, $\quad R_{oF} = 2\text{k}\Omega(1+5) = 12\text{k}\Omega$

$$\frac{v_2}{i_2} = (12\text{k}\Omega - 1\text{k}\Omega) \| 1\text{k}\Omega = 11\text{k}\Omega \| 1\text{k}\Omega = \frac{11}{12}\text{k}\Omega = \underline{\underline{916.67\Omega}}$$