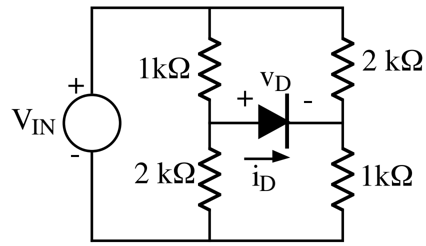
Homework Assignment No. 1 – Solutions

Problem 1

(a.) The first thing to do is to find Thevenin's equivalent circuit seen from the diode.



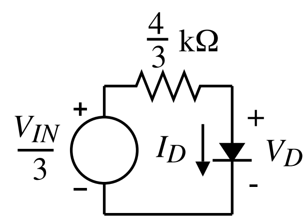
The Thevenin voltage is,

$$V_{TH} = V_{IN}\left(\frac{2}{3} - \frac{1}{3}\right) = \frac{V_{IN}}{3}$$

The Thevenin resistance is,

$$R_{TH} = 1k\Omega||2k\Omega + 1k\Omega||2k\Omega = \frac{4}{3}\ k\Omega$$

The equivalent circuit now becomes,



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Now , with V_{IN} = 10V, we know the diode is forward biased. Therefore, replacing it with a short-circuit gives,

$$V_D = \underline{\underline{0V}} \quad \text{and } I_D = \frac{10}{3}\ x\frac{3}{4k\Omega} = \underline{\underline{2.5\ mA}}$$

(b.) With V_{IN} = -10V, we know the diode is reverse biased. Therefore replacing it with an open-circuit gives,

$$V_D = \underline{\underline{-3.33V}} \quad \text{and } I_D = \underline{\underline{0\ mA}}$$

Problem 2

An enhancement NMOS amplifier is shown. The following questions are independent of each other (i.e. the answer of one is *not* used in the next question).

(a.) If $I_D = 0.5\text{mA}$, $V_T = 1\text{V}$, and $K = 0.5\text{mA/V}$, find g_m.

(b.) If $g_m = 0.5\text{mA/V}$ and $r_o = \infty$, find an algebraic expression for R_{out} and $A_v = v_{out}/v_{in}$.

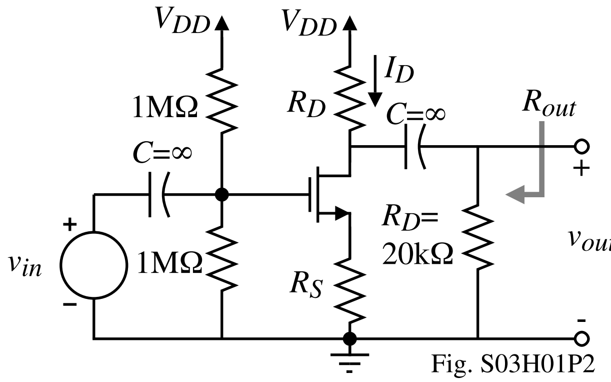


Fig. S03H01P2

(c.) Design R_D and R_S to give $R_{out} = 10\text{k}\Omega$ and $A_v = -10\text{V/V}$ if $g_m = 2\text{mA/V}$ and $r_o = \infty$.

Solution

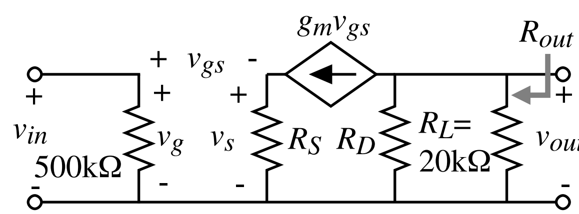
a.) $g_m = \sqrt{2KI_D} = \sqrt{2 \cdot 500 \cdot 500}\ \mu\text{A} = 707\mu\text{S} = \underline{0.707\text{mS}}$

b.) The corresponding small-signal model:

$$\frac{v_{out}}{v_{in}} = \frac{v_{out}}{v_{gs}}\frac{v_{gs}}{v_{in}}$$

$$\frac{v_{out}}{v_{gs}} = -g_m(R_D \| R_L)$$

$$v_{gs} = v_g - v_s = v_{in} - (g_m R_S v_{gs})$$



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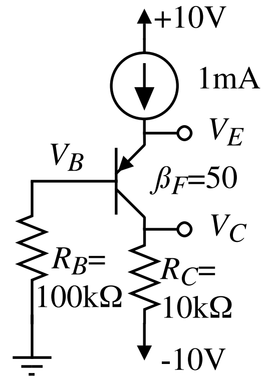
which gives $\dfrac{v_{gs}}{v_{in}} = \dfrac{1}{1+g_m R_S}$ $\rightarrow$ $\dfrac{v_{out}}{v_{gs}} = \boxed{\dfrac{-g_m(R_D \| R_L)}{1+g_m R_S}}$ and $R_{out} = \boxed{R_D \| R_L}$

c.) $R_{out} = 10\text{k}\Omega = R_D \| R_L = R_D \| 20\text{k}\Omega$ $\rightarrow$ $R_D = \underline{\underline{20\text{k}\Omega}}$

$$\frac{v_{out}}{v_{in}} = -10 = \frac{-g_m(R_D \| R_L)}{1+g_m R_S} = \frac{-20}{1+\dfrac{R_S}{500}} \rightarrow R_S = \underline{\underline{500\Omega}}$$

Problem 3

A pnp BJT circuit is shown. (a.) Find the dc values of I_E, I_C, I_B, V_E, V_C and V_B if $\beta = 50$ and $V_{EB}(\text{on}) = 0.65\text{V}$. (b.) For what value of R_C does the BJT become saturated? (Recall that saturation of a BJT corresponds to the *BE* and *BC* junctions forward biased.)



Solution

(a.) Note that $\boxed{I_E = 1\text{mA}}$ $\alpha_F = \frac{\beta_F}{1+\beta_F} = \frac{50}{51} = 0.98$

$\therefore I_C = \alpha_F I_E = 0.98 \cdot 1\text{mA} = 0.98\text{mA} \Rightarrow \boxed{I_C = 0.98\text{mA}}$

$$I_B = \frac{I_C}{\beta_F} = \frac{0.98\text{mA}}{50} = 19.6\mu\text{A} \Rightarrow \boxed{I_B = 19.6\mu\text{A}}$$

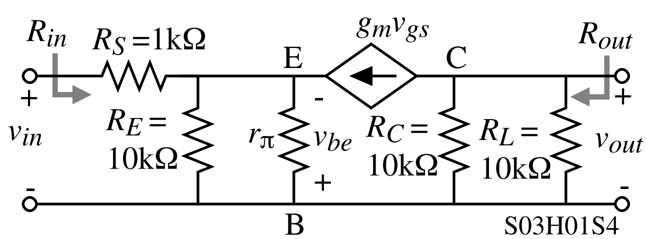
Now, $V_B = I_B \cdot 100\text{k}\Omega = 1.96\text{V} \Rightarrow \boxed{V_B = 1.96\text{V}}$

$V_E = V_B + V_{EB}(\text{on}) = 1.96\text{V} + 0.65\text{V} = 2.61\text{V} \Rightarrow \boxed{V_E = 2.61\text{V}}$

Finally, $V_C = -10\text{V} + I_C \cdot 10\text{k}\Omega = -10\text{V} + 0.98\text{mA} \cdot 10\text{k}\Omega = -0.2\text{V} \Rightarrow \boxed{V_C = -0.2\text{V} \approx 0\text{V}}$

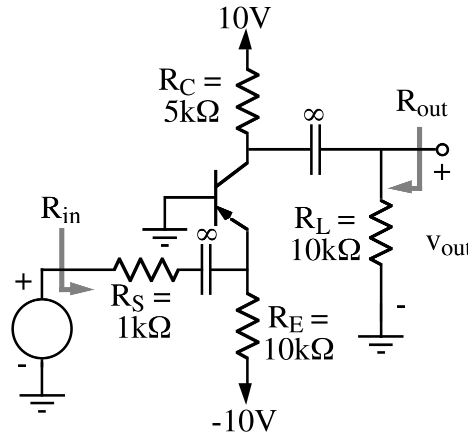
(b.) Saturation occurs when $V_{BC} = 0$ of $V_B = V_C$. Therefore, $V_C = 1.96\text{V}$. The current through R_C is still 0.98mA, so solving for R_C gives,

$$R_C = \frac{V_C + 10\text{V}}{I_C} = \frac{11.96\text{V}}{0.98\text{mA}} = 12.20\text{k}\Omega \Rightarrow \boxed{R_C = 12.2\text{k}\Omega}$$



Problem 4

For the transistor shown, ß = 100, r_π = 2.5kΩ, and g_m = 0.04S. Draw the small signal model and find the numerical values of the small signal voltage gain, v_{out}/v_{in}, the input resistance, R_{in}, and the output resistance, R_{out}.



Solution

Small-signal model:

$$R_{in} = R_S + R_E || \frac{r_\pi}{1+\beta} \; + R_S + R_E || \frac{1}{g_m}$$

$$R_{in} = 1k\Omega + 10k\Omega || 25\Omega = \underline{\underline{1024.9\Omega}}$$

$$R_{out} = R_C || R_L = \underline{\underline{5k\Omega}}$$

$$\frac{v_{out}}{v_{in}} = \frac{v_{out}}{v_{be}} \frac{v_{be}}{v_{in}} = [-g_m(R_C || R_L)] \left(\frac{-\left(\frac{r_\pi}{1+\beta}\right) || R_E}{R_S + \left(\frac{r_\pi}{1+\beta}\right)_\pi || R_E} \right) = [-g_m(R_C || R_L)] \left(\frac{-\left(\frac{1}{g_m}\right) || R_E}{R_S + \left(\frac{1}{g_m}\right)_\pi || R_E} \right)$$

$$= (-40 \cdot 3.33)\left(\frac{-24.9}{1024.9}\right) = \underline{\underline{+3.24V/V}}$$

Problem 5

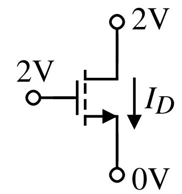
The following questions give the dc voltages at the terminals of an active device. You are to calculate the designated dc current.

a.) Find the diode current, I_D, where I_S = 100fA and V_T = 0.025V (2 pts).

Obviously, the diode is forward biased. Therefore,

$$I_D = I_S \exp\left(\frac{V_D}{V_T}\right) = 10^{-13} \exp\left(\frac{0.6}{0.025}\right) = \underline{\underline{2.65 \text{ mA}}}$$

b.) Find the drain-source current, I_{DS}, where K_n' = 25μA/V^2, V_{TN} = 1V and W/L = 10 (2 pts).



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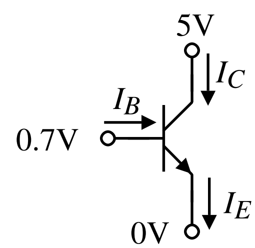
We see that the enhancement, n-channel MOSFET is in the saturation region. Therefore,

$$I_{DS} = \frac{K_n'W}{2L}(V_{GS} - V_{TN})^2 = \frac{25\cdot 10}{2}(2-1)^2 = \underline{\underline{125\ \mu\text{A}}}$$

c.) Find the collector, emitter, and base currents, I_C, I_E, and I_B if I_S = 100fA, V_T = 0.025V and β_F = 100 (4 pts).



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We see that the npn BJT is in the forward active region. Therefore,

$$I_C = I_S \exp\left(\frac{V_{BE}}{V_T}\right) = 10^{-13} \exp\left(\frac{0.7}{0.025}\right) = \underline{\underline{144.6 \text{ mA}}}$$

$$I_B = \frac{I_C}{\beta_F} = \underline{\underline{1.446 \text{ mA}}} \qquad \text{and} \qquad I_E = I_C + I_B = \underline{\underline{146 \text{ mA}}}$$

d.) Repeat (b.) if V_D = 1V and V_G = 3V (2 pts).

We see that the enhancement, n-channel MOSFET is in the linear region. Therefore,

$$I_{DS} = K_n'\frac{W}{L}\left(V_{GS} - V_{TN} - \frac{V_{DS}}{2}\right)V_{DS} = 25\cdot 10(3-1-0.5)(1) = \underline{\underline{375\ \mu\text{A}}}$$