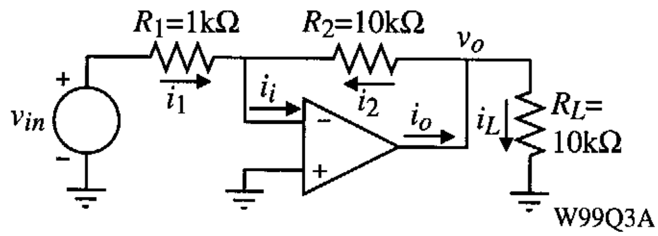
Homework Assignment No. 3 - Solution

1.) The op amps in this problem are ideal.

a.) If v_{in} = +1V, find the value the currents i_1, i_i, i_2, i_o, and i_L including the sign.



$$i_1 = \frac{1\text{V}}{1\text{k}\Omega} = 1\text{mA}$$

$$i_i = 0$$

$$i_2 = \frac{-10\text{V}}{10\text{k}\Omega} = -1\text{mA}$$

$$i_L = \frac{-10\text{V}}{1\text{k}\Omega} = -1\text{mA}$$

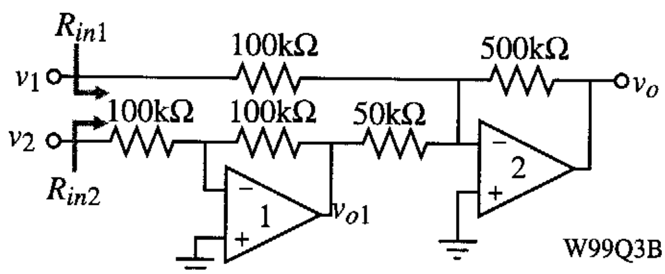
$$i_o = i_2 + i_L = -2\text{mA}$$

b.) Express v_o as a function of v_1 and v_2.

Note: $v_{o1} = -v_2$

And

$v_o = -5v_1 - 10v_{o1}$

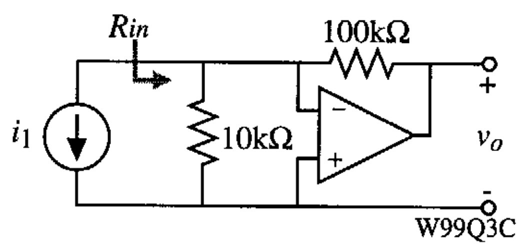


$\therefore\ v_o = -5v_1 - 10(-v_2) = -5v_1 + 10v_2$

$$v_o = -5v_1 + 10v_2$$

$$R_{in1} = R_{in2} = 100\text{k}\Omega$$

c.) Find R_{in} and v_o if $i_1 = 0.1$mA.



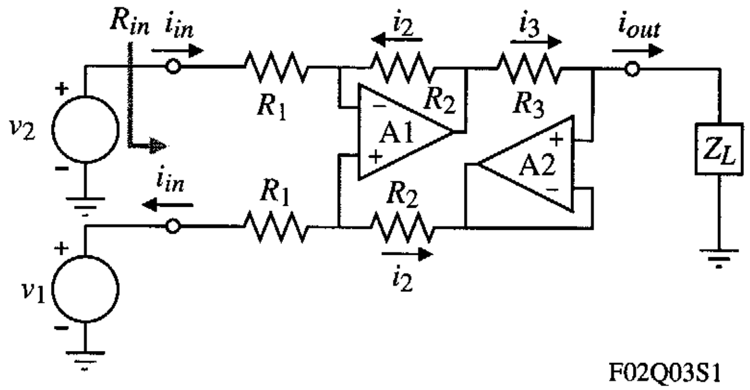
$$R_{in} = 0$$

and

$$v_o = 100\text{k}\Omega \cdot i_1 = 100\text{k}\Omega \cdot 0.1\text{mA} = 10\text{V}$$

R_{in} and v_o if $i_1 = 0.1$mA.

2.) Assume that the op amps are ideal and find i_{out} as a function of the inputs, v_1 and v_2. Find the input resistance defined as $R_{in} = (v_2 - v_1)/i_{in}$.



Solution

From the circuit we can write the following equations based on an ideal op amp:

$$i_{out} = i_3, \qquad v_2 - v_1 = 2R_1 i_{in}, \qquad i_2R_2 + i_2R_2 = i_3R_3, \qquad i_{in} = -i_2$$

$$\therefore \qquad i_{out} = i_3 = \frac{2R_2 i_2}{R_3} = \frac{2R_2}{R_3}(-i_{in}) = \frac{2R_2}{R_3}\left(-\frac{v_2 - v_1}{2R_1}\right) = \frac{R_2}{R_1R_3}(v_1 - v_2)$$

$$\boxed{i_{out} = \frac{R_2}{R_1R_3}(v_1 - v_2)}$$

The input resistance, R_{in} is seen to be equal to $2R_1$. $\boxed{R_{in} = 2R_1}$

12.21 (a) Note that voltages refer to the node numbers on the next page

Using voltage division since $\mathbf{i}_+ = 0$, $\mathbf{v}_2 = \mathbf{v}_4 + 6\dfrac{4.99\text{k}\Omega}{4.99\text{k}\Omega + 5.00\text{k}\Omega}$

$$\mathbf{v}_2 = \mathbf{v}_4 + (6 - \mathbf{v}_4)\frac{4.99\text{k}\Omega}{4.99\text{k}\Omega + 5.00\text{k}\Omega} = 0.5005\mathbf{v}_4 + 2.997\text{V}$$

Since $\mathbf{v}_{id} = 0$, $\mathbf{v}_1 = \mathbf{v}_2$ and $\mathbf{v}_5 = \mathbf{v}_1 - \dfrac{4 - \mathbf{v}_1}{5\text{k}\Omega}(5.01\text{k}\Omega)$

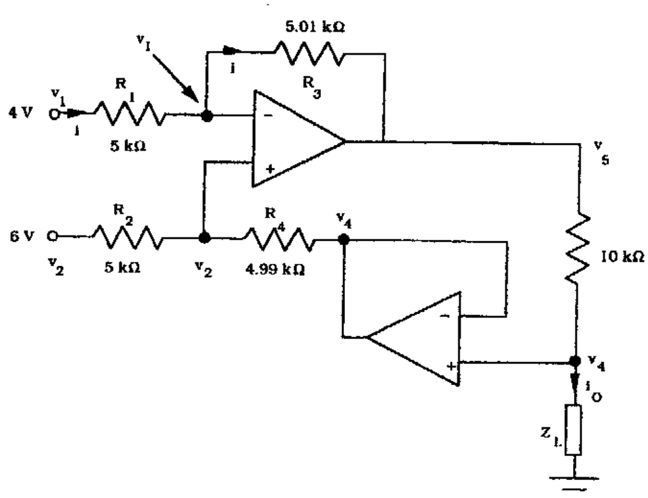
Solving for $\mathbf{v}_5$ yields $\mathbf{v}_5 = 1.992\text{V} + 1.002\mathbf{v}_4$

$$\mathbf{i}_o = \frac{\mathbf{v}_5 - \mathbf{v}_4}{10\text{k}\Omega} = 199\mu\text{A} + 2\text{x}10^{-7}\mathbf{v}_4$$

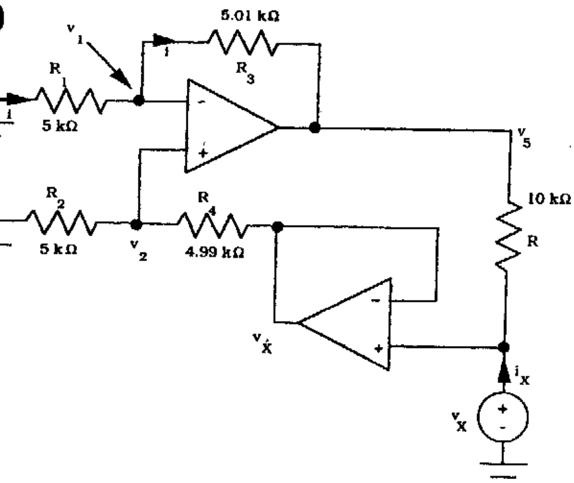
$\mathbf{v}_4$ is unknown; let us assume $2\text{x}10^{-7}\mathbf{v}_4 << 199\text{x}10^{-6}\text{A}$ which requires $\mathbf{v}_4 << 995\text{V}$. So for $\mathbf{v}_4 < 100\text{V}$, which should almost always be true in transistor circuits, $\mathbf{i}_O = 199\mu\text{A}$.

For $Z_L = 10$ kΩ, $v_4 = 1.99$ V, $v_2 = 3.99$ V, $v_1 = 3.99$ V, $v_5 = 3.99$ V

Note that $v_5 - v_4 = 2$ V = (6V - 4V)



(b.)



$R_{OUT} = \dfrac{\mathbf{v}_X}{\mathbf{i}_X}$ and $\mathbf{i}_X = \dfrac{\mathbf{v}_X - \mathbf{v}_5}{10\text{k}\Omega}$ So we need to find i_X, and hence v_5, in terms of v_X

$$\mathbf{v}_1 = \mathbf{v}_2 = \mathbf{v}_X\frac{5.00\text{k}\Omega}{4.99\text{k}\Omega + 5.00\text{k}\Omega} = 0.5005\mathbf{v}_X$$

$$\mathbf{v}_5 = \mathbf{v}_1 + \mathbf{i}\,(5.01\text{k}\Omega) = \mathbf{v}_1 + \frac{\mathbf{v}_1}{5\text{k}\Omega}(5.01\text{k}\Omega) = 2.002\mathbf{v}_1 = 1.002\mathbf{v}_x$$

$\mathbf{i}_X = \dfrac{\mathbf{v}_X - \mathbf{v}_5}{10\text{k}\Omega} = \dfrac{\mathbf{v}_X - 1.002\mathbf{v}_X}{10\text{k}\Omega} = -\dfrac{0.002\mathbf{v}_X}{10\text{k}\Omega}$ and $R_{OUT} = -5$ MΩ ! A negative output resistance!

12.24 Applying op-amp assumption 1 to the circuit on the next page, the voltage at the top of R_2 is v_{O2}, and applying op-amp assumption 2,

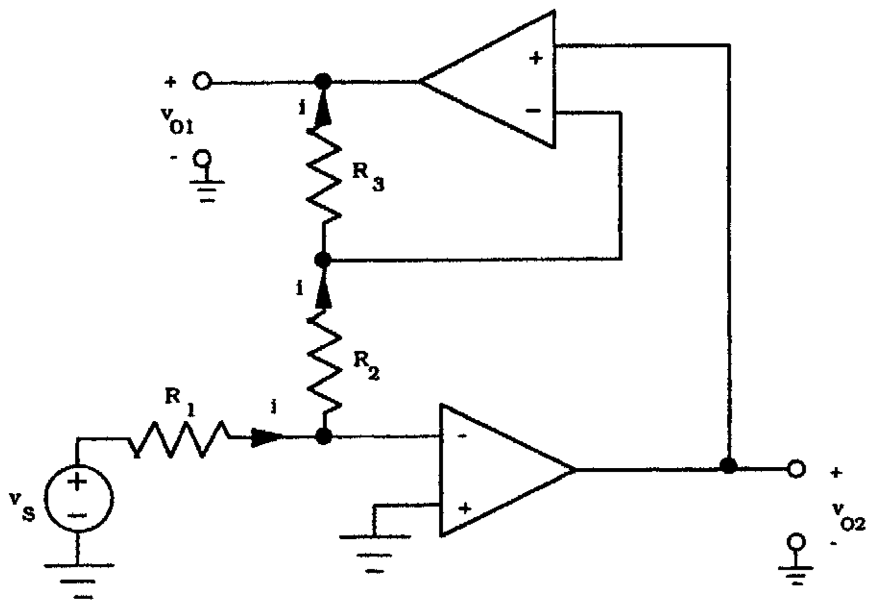
$$\frac{v_S}{R_1} = -\frac{v_{O2}}{R_2} \quad \text{or} \quad v_{O2} = -v_S\frac{R_2}{R_1}$$

Since the op-amp input currents are zero, and

$$i = \frac{v_S}{R_1}, \quad v_{O1} = -iR_2 - iR_3 = -\left(\frac{R_2}{R_1} + \frac{R_3}{R_1}\right)v_S$$

Alternatively, the voltage at the bottom of R_2 is zero, so

$$v_{O1} = \left(1 + \frac{R_3}{R_2}\right)v_{O2} = \left(1 + \frac{R_3}{R_2}\right)\left(-\frac{R_2}{R_1}\right)v_S = -\left(\frac{R_2}{R_1} + \frac{R_3}{R_1}\right)v_S$$



12.74

$$\beta = \frac{2k\Omega}{2k\Omega + 40k\Omega} = \frac{1}{21} \quad | \quad A\beta = \frac{10^5}{21} = 4760 >> 1$$

(a) $A_V = -\frac{R_2}{R_1} = -\frac{40k\Omega}{2k\Omega} = -20 \quad | \quad f_H = \beta f_T = \frac{3x10^6 Hz}{21} = 143kHz$

(b) $A_V = (-20)^3 = -8000\ (78dB) \quad | \quad f_{H3} = 0.51 f_H = 72.9kHz$