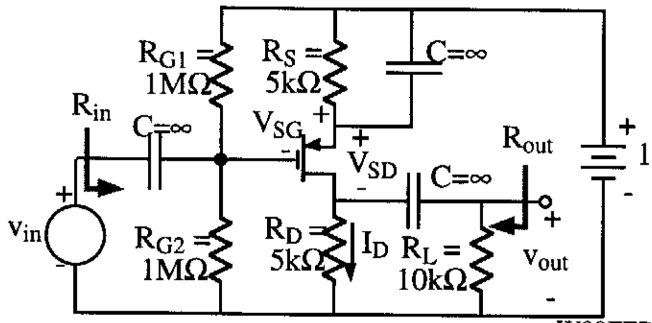
Homework Assignment No. 5 - Solutions

1.) Find the dc operating point, the small signal voltage gain, v_{out}/v_{in}, the small signal input resistance, R_{in}, and the small signal output resistance, R_{out}, if $K = 0.1\text{mA/V}^2$, $V_t = -1\text{V}$, and $\lambda = 0.01\text{V}^{-1}$.



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Solution

Finding the dc Thevenin equivalent circuit looking out the gate gives $V_{GG} = 7.5\text{V}$ and $R_G = 0.5\text{M}\Omega$. Assuming saturation gives $I_D = K(V_{GS}\text{-}V_t)^2$. Combining with

$$V_{GG} = 7.5\text{V} = V_{GS} + I_D R_S$$

gives $7.5 = V_{GS} + 5\text{k}\Omega\cdot 0.1\text{mA/V}^2(V_{GS}\text{-}1)^2 = V_{GS} + 0.5V_{GS}^2 - V_{GS} + 0.5$

which reduces to $V_{GS}^2 = 14 \quad \rightarrow \qquad V_{GS} = \sqrt{14} = 3.74\text{V}$

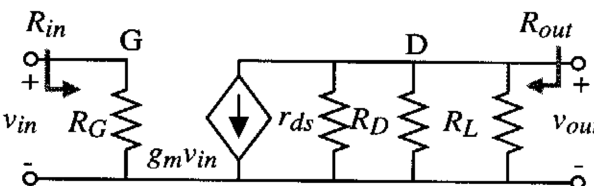
This gives $I_D = 0.1\text{mA}(3.75\text{-}1)^2 = 0.752\text{mA}$. Finally, $V_{DS} = 15 - I_D(R_D+R_S) = 7.48\text{V}$

$\therefore$ $V_{SG} = 3.74\text{V}, I_D = 0.752\text{mA}$ and $V_{SD} = 7.48\text{V}$

Note that the MOSFET is indeed saturated.

The small signal model parameters are $g_m = 2\sqrt{KI_D} = 2\sqrt{0.1\cdot 0.752} = 0.548\text{mA/V}$ and $r_{ds} = (\lambda\cdot I_D)^{-1} = 100/0.752 = 133\text{k}\Omega$.

The small-signal model for this problem is,



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$$\frac{v_{out}}{v_{in}} = -g_m(r_{ds}||R_D||R_L) = -0.548(133||5||10)$$

$$= -1.782\text{V/V},\ R_{in} = 500\text{k}\Omega$$

and $R_{out} = r_{ds}||R_D||R_L = 3.25\text{k}\Omega$.

$\therefore$ $R_{in} = 500\text{k}\Omega,\ R_{out} = 3.25\text{k}\Omega$ and $\frac{v_{out}}{v_{in}} = -1.782$ V/V

2.) Problem 13.91 of the text.

3.) Problem 13.100 of the text. [$A_v = -4.60$ V/V]

4.) Problem 13.108 of the text.

13.91

$$g_m = \sqrt{2\left(500\frac{\mu A}{V^2}\right)(100\mu A)(1+0.02(5))} = 332\mu S \quad | \quad r_o = \frac{50+5V}{100\mu A} = 550k\Omega$$

$$A_V = -\left(\frac{6.8M\Omega}{6.8M\Omega + 0.1M\Omega}\right)(332\mu S)\left(550k\Omega \| 50k\Omega \| 120k\Omega\right) = -10.9$$

13.100

$$g_m = \frac{2}{3}\sqrt{1mA(1mA)[1+0.015(9)]} = 710\mu S \quad | \quad r_o = \frac{\frac{1}{0.015}+9V}{1mA} = 75.7k\Omega$$

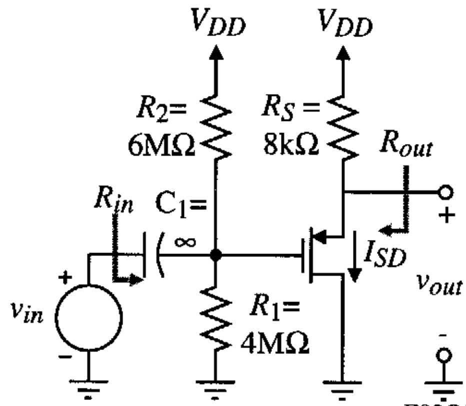
$$A_V = -\left(\frac{1M\Omega}{1M\Omega + 10k\Omega}\right)(710\mu S)\left(75.7k\Omega \| 7.5k\Omega \| 160k\Omega\right) = -4.60$$

13.108

$$R_{IN} = R_G = 6.8M\Omega \quad | \quad R_{OUT} = 50k\Omega \| r_o$$

$$r_o = \frac{(50+5)V}{0.1mA} = 550k\Omega \quad | \quad R_{OUT} = 50k\Omega \| 550k\Omega = 45.8k\Omega$$

5.) A PMOS common-drain amplifier is shown. Assume the parameters of the transistor are k_F = 0.5mA/V^2, V_{TP} = -1V, and λ = 0. (a.) If I_{SD} = 0.5mA, find the small signal model parameter values for g_m and r_o. (b.) Find an algebraic expression for the small signal input resistance, R_{in}, the output resistance, R_{out}, and the voltage gain, v_{out}/v_{in}. (c.) Numerically evaluate the small signal input resistance, R_{in}, the output resistance, R_{out}, and the voltage gain, v_{out}/v_{in}.



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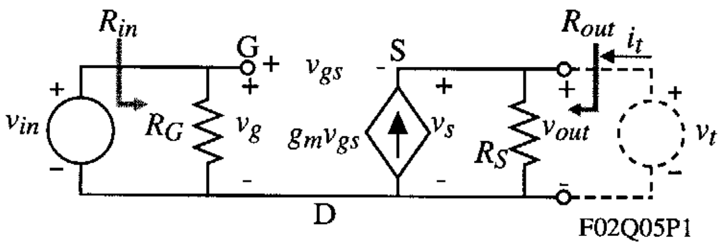
Solution

(a.) $g_m = \sqrt{2I_{SD}k_P} = \sqrt{2\cdot 0.5\cdot 0.5}$ mS = $\underline{0.707\text{mS}}$ and $r_o = \underline{\underline{\infty}}$

(b.) First we need a small signal model.



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Obviously, $\boxed{R_{in} = R_G = R_1 || R_2}$. For R_{out} we apply the voltage source, v_t, and set v_{in} = 0 and solve for v_t/i_t which equivalent to R_{out}.

$$\therefore\ i_t = G_S\, v_t - g_m v_{gs} = G_S\, v_t - g_m(v_g - v_s) = G_S\, v_t - g_m(0 - v_s)$$

$$= G_S\, v_t + g_m\, v_s = G_S\, v_t + g_m\, v_t = (G_S + g_m)v_t$$

$$\therefore \quad R_{out} = \frac{v_t}{i_t} = \frac{1}{G_S + g_m} = \frac{R_S}{1 + g_m R_S} \quad \rightarrow \quad \boxed{R_{out} = \frac{R_S}{1 + g_m R_S}}$$

The output voltage can be expressed as,

$$v_{out} = g_m R_S v_{gs} = g_m R_S(v_g - v_s) = g_m R_S(v_{in} - v_{out})$$

$$\therefore \quad v_{out}(1 + g_m R_S) = g_m R_S\, v_{in} \quad \rightarrow \quad \boxed{\frac{v_{out}}{v_{in}} = \frac{g_m R_S}{1 + g_m R_S}}$$

(c.) $R_{in} = R_G = R_1 || R_2 = \underline{\underline{2.4\text{M}\Omega}}$, $R_{out} = \frac{8\text{k}\Omega}{1+0.707\cdot 8} = \underline{\underline{1.2\text{k}\Omega}}$ and $\frac{v_{out}}{v_{in}} = \frac{0.707\cdot 8}{1+0.707\cdot 8} = \underline{\underline{0.85\text{V/V}}}$