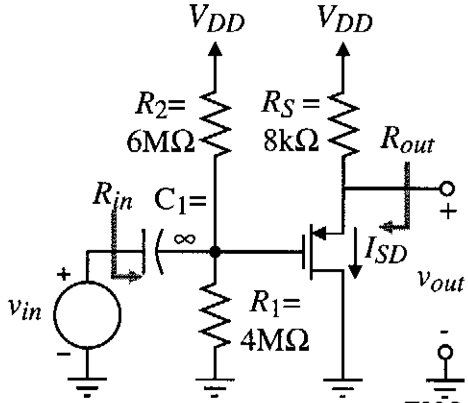
Homework Assignment No. 6 - Solution

1.) A PMOS common-drain amplifier is shown. Assume the parameters of the transistor are $k_F = 0.5\text{mA/V}^2$, $V_{TP} = -1\text{V}$, and $\lambda = 0$. (a.) If $I_{SD} = 0.5\text{mA}$, find the small signal model parameter values for g_m and r_o. (b.) Find an algebraic expression for the small signal input resistance, R_{in}, the output resistance, R_{out}, and the voltage gain, v_{out}/v_{in}. (c.) Numerically evaluate the small signal input resistance, R_{in}, the output resistance, R_{out}, and the voltage gain, v_{out}/v_{in}.



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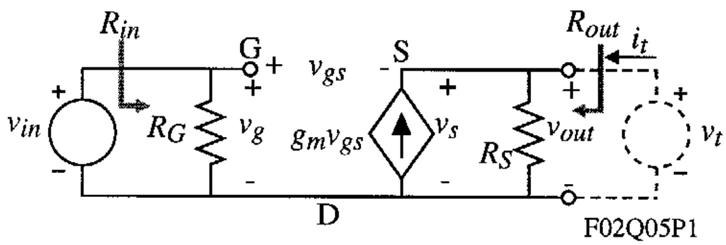
Solution

(a.) $g_m = \sqrt{2I_{SD}k_P} = \sqrt{2\cdot0.5\cdot0.5}$ mS = $\underline{0.707\text{mS}}$ and $r_o = \underline{\underline{\infty}}$

(b.) First we need a small signal model.



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Obviously, $\boxed{R_{in} = R_G = R_1 \| R_2}$. For R_{out} we apply the voltage source, v_t, and set v_{in} = 0 and solve for v_t/i_t which equivalent to R_{out}.

$$\therefore\ i_t = G_S v_t - g_m v_{gs} = G_S v_t - g_m(v_g - v_s) = G_S v_t - g_m(0 - v_s)$$

$$= G_S v_t + g_m v_s = G_S v_t + g_m v_t = (G_S + g_m)v_t$$

$$\therefore \quad R_{out} = \frac{v_t}{i_t} = \frac{1}{G_S + g_m} = \frac{R_S}{1 + g_m R_S} \quad \rightarrow \quad \boxed{R_{out} = \frac{R_S}{1 + g_m R_S}}$$

The output voltage can be expressed as,

$$v_{out} = g_m R_S v_{gs} = g_m R_S(v_g - v_s) = g_m R_S(v_{in} - v_{out})$$

$$\therefore \quad v_{out}(1 + g_m R_S) = g_m R_S v_{in} \quad \rightarrow \quad \boxed{\frac{v_{out}}{v_{in}} = \frac{g_m R_S}{1 + g_m R_S}}$$

(c.) $R_{in} = R_G = R_1 \| R_2 = \underline{2.4\text{M}\Omega}$, $R_{out} = \frac{8\text{k}\Omega}{1+0.707\cdot8} = \underline{1.2\text{k}\Omega}$ and $\frac{v_{out}}{v_{in}} = \frac{0.707\cdot8}{1+0.707\cdot8} = \underline{0.85\text{V/V}}$

14.13

$V_{EQ} = 18\frac{500k\Omega}{1.4M\Omega + 500k\Omega} = 4.74V$ | $R_{EQ} = 500k\Omega \| 1.4M\Omega = 368k\Omega$

$4.74 = V_{GS} + 27000I_{DS} = 1 + \sqrt{\frac{2I_{DS}}{250x10^{-6}}} + 27000I_{DS} \rightarrow I_{DS} = 104\mu A$

$V_{DS} = 18 - I_{DS}(75k\Omega + 27k\Omega) = 7.39V$ | Saturation region is correct.

$g_m = \sqrt{2(250x10^{-6})(104x10^{-6})} = 0.228mS$

$\mathbf{v_{th}} = \mathbf{v_s}\frac{368k\Omega}{1k\Omega + 368k\Omega} = 0.997\mathbf{v_s}$ | $R_{th} = 1k\Omega \| 368k\Omega = 0.997k\Omega$

$R_L = r_o \| 75k\Omega \| 470k\Omega \cong 75k\Omega \| 470k\Omega = 64.7k\Omega$ | $A_{Vth} = -(0.228mS)(64.7k\Omega) = -14.8$

$A_V = 0.997A_{Vth} = -14.7$ | $A_I = 368k\Omega(-g_m)\frac{75k\Omega}{75k\Omega + 470k\Omega} = -11.6$

$R_{IN} = 368\ k\Omega$ | $R_{OUT} = r_o \| 75k\Omega \cong 75k\Omega$

$\mathbf{v_{gs}} = 0.997\mathbf{v_s}$ | $V_{GS} - V_{TN} = \sqrt{\frac{2(104\mu A)}{250\mu A / V^2}} = 0.912V$ | $\mathbf{v_s} = 0.2\frac{0.912V}{0.997} = 0.183\ V$

$A_V \cong -\frac{V_{DD}}{V_{GS} - V_{TN}} = -\frac{18}{0.912} = -19.7$ | The rule-of-thumb estimate assumes $V_{R_L} = \frac{V_{DD}}{2}$.

We have $V_{R_L} = 104\mu A(75k\Omega) = 7.80V = 0.433V_{DD}$

The estimate also doesn't account for the presence of R_3.

14.15

$V_{GS} = -(11k\Omega)I_{DS} = -(11k\Omega)(20mA)\left(1 - \frac{V_{GS}}{-4}\right)^2 \rightarrow V_{GS} = -3.50V,\ I_{DS} = -\frac{V_{GS}}{11k\Omega} = 318\ \mu A$

$V_{DS} = 20 - I_{DS}(11k\Omega + 39k\Omega) = 4.10V$ | Saturation region is correct.

$g_m = \frac{2}{|-4|}\sqrt{20mA(318\mu A)} = 1.26mS$ | $\mathbf{v_{th}} = \mathbf{v_s}\frac{1M\Omega}{0.5k\Omega + 1M\Omega} = 1.00\mathbf{v_s}$

$R_{th} = 0.5k\Omega \| 1M\Omega = 0.500k\Omega$ | $R_L = 39k\Omega \| 500k\Omega = 36.2k\Omega$

$A_V = A_{Vth} = -\frac{1.26mS(36.2k\Omega)}{1 + 1.26mS(11k\Omega)} = -3.07$ | $R_{IN} = 1.00\ M\Omega$ | $R_{OUT} = 39k\Omega$

$A_I = -R_G\frac{g_m}{1 + g_m R_1} = -(10^6)\frac{1.26mS}{1 + 1.26mS(11k\Omega)} = -84.8$

$\mathbf{v_{th}} = 1.00\mathbf{v_s}$ | $V_{GS} - V_P = -3.5 - (-4) = 0.500V$ | $\mathbf{v_s} = 0.2(0.5)[1 + 1.26mS(11k\Omega)] = 1.49\ V$

14.21

$V_{EQ} = 18\frac{51k\Omega}{51k\Omega + 100k\Omega} = 6.08V$ | $R_{EQ} = 51k\Omega \| 100k\Omega = 33.8k\Omega$

$I_B = \frac{(6.08 - 0.7 + 18)V}{33.8k\Omega + (126)(4.7k\Omega)} = 37.3\mu A$ | $I_C = 4.67$ mA | $V_{CE} = 36 - 2000I_C - 4700I_E = 4.54$ V

Forward - active region is correct. | $r_\pi = \frac{125(0.025V)}{4.67mA} = 669\Omega$ | $r_o = \frac{(50 + 4.54)V}{4.67mA} = 11.7k\Omega$

$\mathbf{v_{th}} = \mathbf{v_s}\frac{33.8\Omega}{500\Omega + 33.8k\Omega} = 0.985\mathbf{v_s}$ | $R_{th} = 33.8k\Omega \| 500\Omega = 493\Omega$

$R_L = 24k\Omega \| 4.7k\Omega \| 11.7k\Omega = 2.94k\Omega$ | $A_{Vth} = -\frac{126(2.94k\Omega)}{0.493k\Omega + 0.669k\Omega + 126(2.94k\Omega)} = 0.997$

$A_V = 0.985A_{Vth} = 0.982$ | $R_{IN} = 33.8k\Omega \| [0.669k\Omega + 126(2.94k\Omega)] = 31.0$ kΩ

$A_I = A_V\frac{R_S + R_{IN}}{R_3} = 0.982\frac{0.5k\Omega + 31.0k\Omega}{24.0k\Omega} = 1.29$ | $R_{OUT} = \frac{493\Omega + 669\Omega}{126} \| 2.94k\Omega = 9.19\ \Omega$

$\mathbf{v_{be}} = 0.982\mathbf{v_s}\frac{0.669k\Omega}{0.493k\Omega + 0.669k\Omega + 126(2.94k\Omega)} = 0.00177\mathbf{v_s}$ | $\mathbf{v_s} = \frac{5.00mV}{0.00177} = 2.83$ V

14.23

$V_{GS} = 5V$ | $I_{DS} = \frac{4x10^{-4}}{2}(5-1)^2 = 3.2mA$ | $V_{DS} = 5 - (-5) = 10V$ - Saturation region operation is correct. | $g_m = \sqrt{2(4x10^{-4})(3.2mA)[1 + 0.02(10)]} = 1.75mS$

$r_o = \frac{\frac{1}{0.02} + 10}{3.2mA} = 18.8k\Omega$ – Cannot neglect! | $R_L = 18.8k\Omega \| 100k\Omega = 15.8k\Omega$

$A_V = \frac{10^6}{10^6 + 10^4}\frac{1.75mS(15.8k\Omega)}{1 + 1.75mS(15.8k\Omega)} = 0.956$ | $A_I = 10^6\frac{1.75mS(15.8k\Omega)}{1 + 1.75mS(15.8k\Omega)}\frac{1}{10^5} = 9.56$

$R_{IN} = R_G = 1\ M\Omega$ | $R_{OUT} = \frac{1}{g_m} \| r_o = 555\ \Omega$

$\mathbf{v_{gs}} = \mathbf{v_s}\frac{10^6}{10^6 + 10^4}\frac{1}{1 + 1.75mS(15.8k\Omega)} = 0.0346\mathbf{v_s}$ | $\mathbf{v_s} \le \frac{0.2(5-1)}{0.0346} = 23.2$ V But, v_{DS} must exceed $v_{GS} - V_{TN} \cong V_{GS} - V_{TN} = 4V$ for saturation.

$V_{DS} = 10 - \mathbf{v_o} = 10 - 0.956\mathbf{v_s} \ge 4 \rightarrow \mathbf{v_s} \le 6.28$ V – Limited by the Q - point voltages

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