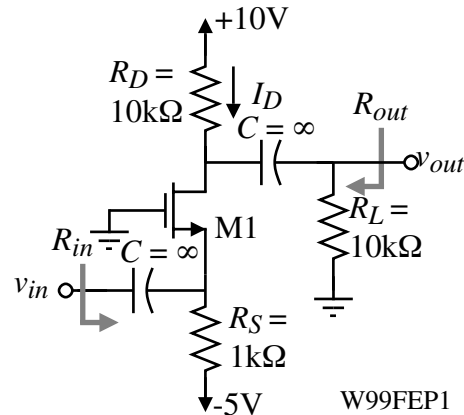


Homework Assignment No. 7 - Solutions

1.) If $K = 0.1\text{mA/V}^2$ and $V_t = 1\text{V}$ for the n-channel MOSFET shown, find the dc value of I_D , V_{GS} , and V_{DS} . If the dc value of $I_D = 1\text{mA}$ (not necessarily the answer above) find the value of the small-signal input resistance, R_{in} , voltage gain, v_{out}/v_{in} , and output resistance, R_{out} .



Solution

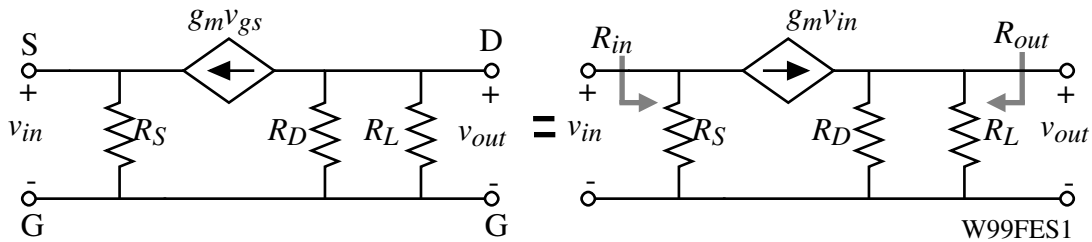
(a.) The key equations are $I_D = K(V_{GS} - V_t)^2$ and $5 = V_{GS} + I_D R_S$. Substituting the first into the second gives, $5 = V_{GS} + 0.1(V_{GS} - 1)^2 \rightarrow 5 = V_{GS} + 0.1V_{GS}^2 - 0.2V_{GS} + 0.1$

or $V_{GS}^2 + 8V_{GS} - 49 = 0 \rightarrow V_{GS} = -4 \pm \frac{1}{2} \sqrt{64 + 4 \cdot 49} = -4 \pm 8.0623 = 4.0623\text{V}$

$I_D = 0.1\text{mA}(4.0623 - 1)^2 = 0.9377\text{mA}$ and $V_{DS} = 15\text{V} - I_D(11\text{k}\Omega) = 4.68\text{V}$

$\therefore \boxed{V_{GS} = 4.0623\text{V}, I_D = 0.9377\text{mA} \text{ and } V_{DS} = 4.68\text{V}}$

(b.) The small signal model for this amplifier is shown below. r_{ds} has been neglected since λ was not given. The small signal parameter g_m is: $g_m = 2\sqrt{KI_D} = 0.632\text{mA/V}$



The voltage gain is found as,

$$v_{out} = +g_m(R_D \parallel R_L)v_{in} \rightarrow \frac{v_{out}}{v_{in}} = g_m(R_D \parallel R_L) = 0.632 \cdot 5 = 3.16\text{V/V}$$

$\therefore \boxed{\frac{v_{out}}{v_{in}} = 3.16\text{V/V}}$

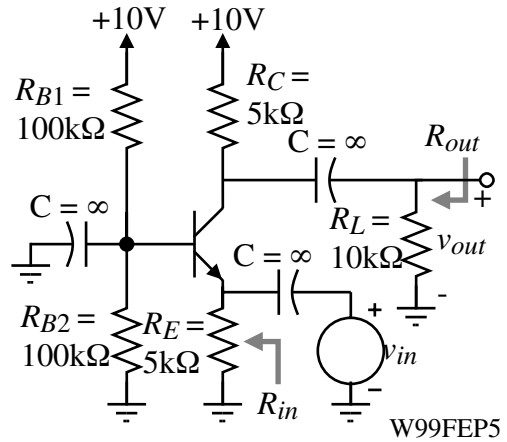
The output resistance is simply $R_D \parallel R_S$ and is $\boxed{R_{out} = 5\text{k}\Omega}$

The input resistance is found as,

$$i_{in} = \frac{v_{in}}{R_S} + g_m v_{in} \rightarrow R_{in} = \frac{v_{in}}{i_{in}} = \frac{1}{\frac{1}{R_S} + g_m} = \frac{1\text{k}\Omega}{1 + 0.632} = 613\Omega$$

$\therefore \boxed{R_{in} = 613\Omega}$

2.) (a.) If $\beta = 100$ and $V_T = 25\text{mV}$ of the NPN transistor shown, solve for the dc value of collector current. (b.) If this collector current is 1mA (so if you found I_C incorrectly, you will not be penalized on the rest of the problem) find the small signal values of the input resistance, R_{in} , output resistance, R_{out} , and voltage gain, v_{out}/v_{in} .



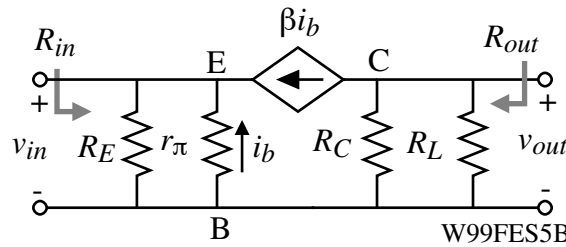
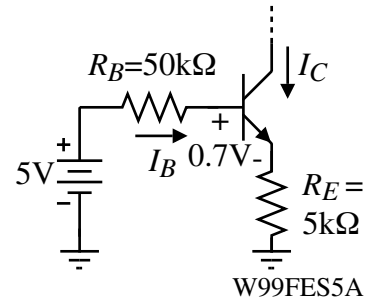
Solution

(a.) The circuit used to calculate the collector current is shown. The base current is

$$I_B = \frac{5 - 0.7}{50\text{k}\Omega + 101 \cdot 5\text{k}\Omega} = 7.75\mu\text{A}$$

$$\therefore I_C = 100 \cdot I_B = 0.775\text{mA}$$

(b.) The small signal parameters are $g_m = \frac{1\text{mA}}{25\text{mV}} = 0.04\text{mS}$ and $r_\pi = 101 \cdot 25\Omega = 2.525\text{k}\Omega$. The small-signal model for the above circuit is given as



$$i_{in} = \frac{v_{in}}{R_E} - (1+\beta)i_b = \frac{v_{in}}{R_E} + \frac{(1+\beta)v_{in}}{r_\pi} \rightarrow R_{in} = \frac{v_{in}}{i_{in}} = R_E \parallel \frac{1}{g_m} = 24.87\Omega$$

$$v_{out} = -\beta i_b (R_C \parallel R_L) = -\beta (R_C \parallel R_L) \frac{-v_{in}}{r_\pi} \rightarrow \frac{v_{out}}{v_{in}} = \frac{100 \cdot 3.33}{2.525} = 132\text{V/V}$$

Finally, $R_{out} = R_C \parallel R_L = 3.33\text{k}\Omega$

14.29

$$(a) r_{\pi} = \frac{100(0.025V)}{12.5\mu A} = 200k\Omega \quad | \quad g_m = 40(12.5\mu A) = 0.5mS$$

$$A_{V_{th}} = \frac{g_m R_L}{1 + g_m R_{th}} = \frac{0.5mS(100k\Omega)}{1 + 0.5mS(50\Omega)} = 48.8 \quad | \quad R_{IN} = \frac{r_{\pi}}{\beta_o + 1} = 1.98 k\Omega$$

$$R_{OUT} = r_o(1 + g_m R_{th}) = \frac{60V}{12.5\mu A} [1 + 0.5mS(50\Omega)] = 4.92 M\Omega \quad | \quad A_{I_{th}} = \alpha_o = 0.990$$

$$(b) A_{V_{th}} = \frac{0.5mS(100k\Omega)}{1 + 0.5mS(2.2k\Omega)} = 23.8 \quad | \quad R_{IN} = 1.98 k\Omega$$

$$R_{OUT} = \frac{60V}{12.5\mu A} [1 + 0.5mS(2.2k\Omega)] = 10.1 M\Omega$$

14.31

$$V_{SG} = 12 - (33k\Omega)I_{SD} \quad | \quad V_{SG} = 12 - \frac{(3.3 \times 10^4)(2 \times 10^{-4})}{2} (V_{SG} - 1)^2$$

$$V_{SG} = 2.68V \quad \& \quad I_{SD} = \frac{(2 \times 10^{-4})}{2} (V_{SG} - 1)^2 = 282\mu A$$

$$V_{SD} = 24 - I_{SD}(33k\Omega + 24k\Omega) = 7.93 V \quad - \text{ Saturation region operation is correct.}$$

$$g_m = \sqrt{2(2 \times 10^{-4})(2.82 \times 10^{-4})} = 3.36 \times 10^{-4} S \quad | \quad v_{th} = \frac{33k\Omega}{0.5k\Omega + 33k\Omega} v_s = 0.985 v_s$$

$$R_{th} = 0.5k\Omega \parallel 33k\Omega = 493\Omega$$

$$R_L = 24k\Omega \parallel 100k\Omega = 19.4k\Omega \quad | \quad A_V = 0.985 \frac{3.36 \times 10^{-4} S(19.4k\Omega)}{1 + 3.36 \times 10^{-4} S(493\Omega)} = 5.51$$

$$A_I = \frac{33k\Omega}{33k\Omega + \frac{1}{g_m}} \frac{24k\Omega}{24k\Omega + 100k\Omega} = 0.178 \quad | \quad R_{IN} = 33k\Omega \parallel \frac{1}{g_m} = 2.73k\Omega \quad | \quad R_{OUT} = R_D = 24k\Omega$$

$$v_{sg} = v_s \frac{R_{IN}}{R_S + R_{IN}} \leq 0.2(V_{SG} - 1) \quad | \quad v_s \frac{2.73k\Omega}{0.5k\Omega + 2.73k\Omega} \leq 0.2(2.68 - 1) \rightarrow v_s \leq 0.398 V$$

14.33

$$I_B = \frac{(9 - 0.7)V}{100k\Omega + (50 + 1)82k\Omega} = 1.94\mu A \quad | \quad I_C = 96.9 \mu A$$

$$V_{CE} = 18 - 82000I_E - 39000I_C = 6.12 V \quad | \quad \text{Forward - active region is correct.}$$

$$g_m = 40I_C = 3.88mS \quad | \quad r_{\pi} = \frac{\beta_o}{g_m} = 12.9k\Omega \quad | \quad r_o = \frac{(50 + 6.12)V}{96.9\mu A} = 579k\Omega \quad - \text{ neglected}$$

$$v_{th} = v_s \frac{82k\Omega}{0.5k\Omega + 82k\Omega} = 0.994 v_s \quad | \quad R_{th} = 0.5k\Omega \parallel 82k\Omega = 497\Omega \quad | \quad R_L = 39k\Omega \parallel 100k\Omega = 28.1k\Omega$$

$$A_V = 0.994 \frac{50(28.1k\Omega)}{12.9k\Omega + 51(0.497k\Omega)} = 36.5 \quad | \quad R_{IN} = 82k\Omega \parallel \frac{r_{\pi}}{\beta_o + 1} = 253\Omega$$

$$A_I = A_V \frac{R_S + R_{IN}}{R_3} = 36.5 \frac{500\Omega + 253\Omega}{100k\Omega} = 0.275 \quad | \quad R_{OUT} = R_C = 39.0 k\Omega$$

$$v_{eb} = v_s \frac{R_{IN}}{R_S + R_{IN}} \leq 5.00mV \quad | \quad 0.336 v_s \leq 5.00mV \quad | \quad v_s \leq 14.9 mV$$