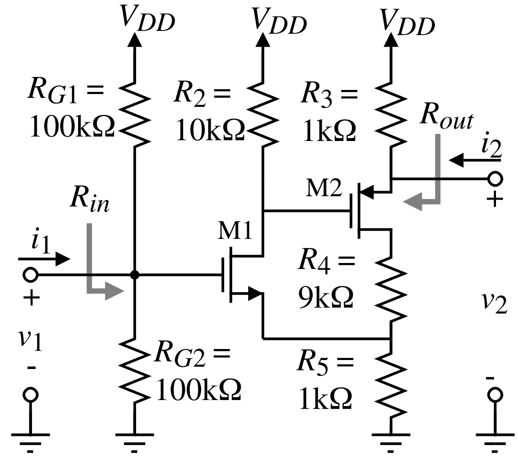
Homework Assignment No. 13 - Solutions

Problem 1

Use the method of feedback analysis to find v_2/v_1, $R_{in} = v_1/i_1$, and $R_{out} = v_2/i_2$. Assume that all transistors are matched and that $g_{m1} = g_{m2} =$ 1mS. Neglect r_{ds}.

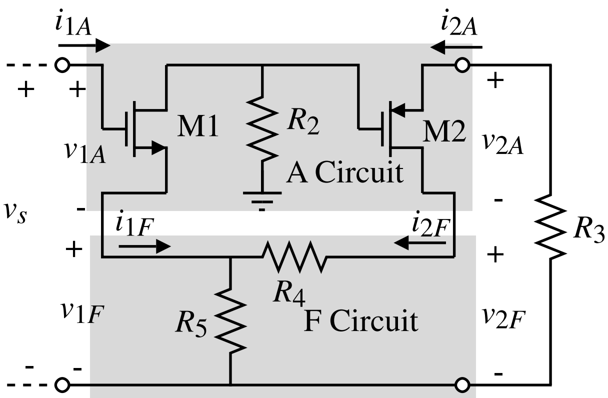


F02H12P4

Solution

The topology is series-series.

The circuit is redrawn below in order to help identify the various terms for the analysis:



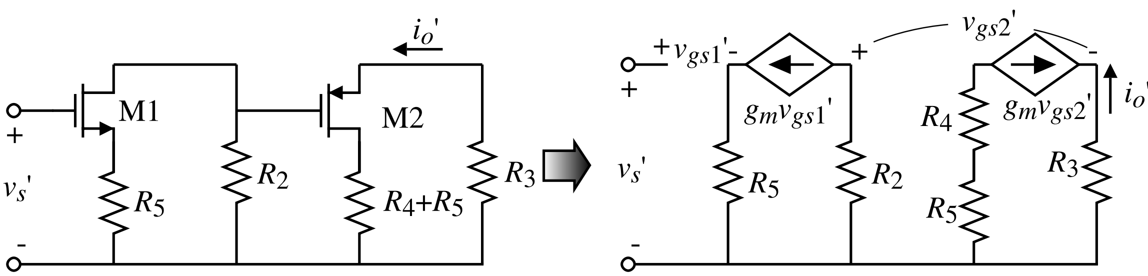
F02H12S4A

$$z_{11F} = \left.\frac{v_{1F}}{i_{1F}}\right|_{i_{2F}=0} = R_5 = 1\text{k}\Omega$$

$$z_{22F} = \left.\frac{v_{2F}}{i_{2F}}\right|_{i_{1F}=0} = R_4 + R_5 = 10\text{k}\Omega$$

$$z_{12F} = \beta = \left.\frac{v_{1F}}{i_{2F}}\right|_{i_{1F}=0} = R_5 = 1\text{k}\Omega$$

Calculation of the A circuit:



F02H12S4B

$$A = \frac{i_o'}{v_s'} = \left(\frac{i_o'}{v_{gs2}'}\right)\left(\frac{v_{gs2}'}{v_{gs1}'}\right)\left(\frac{v_{gs2}'}{v_s'}\right) = (-g_m)\left(\frac{-g_m R_2}{1+g_m R_3}\right)\left(\frac{1}{1+g_m R_5}\right) = (10^{-3})(-5)(-0.5) = 2.5 \text{ mS}$$

$$\therefore \frac{i_o}{v_s} = \frac{A}{1+A\beta} = \frac{2.5\text{mS}}{1+2.5} = 0.714\text{mS}$$

Since, $z_{11A} = \infty$, then $R_{in} = 50\text{k}\Omega \| \infty = \underline{50\text{k}\Omega}$

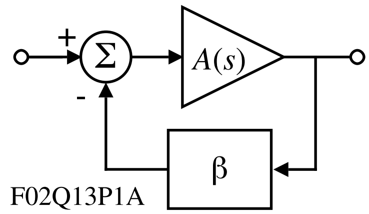
$$\frac{v_2}{v_1} = \frac{-i_o R_3}{v_s} = \frac{-i_o}{v_s} R_3 = -0.714\text{mS}(1\text{k}\Omega) = \underline{\underline{-0.714 \text{ V/V}}}$$

$$R_o = (z_{22T} + R_3)(1+A\beta) = \left(\frac{1}{g_m} + R_3\right)(1+A\beta) = 2\text{k}\Omega(3.5) = 7\text{k}\Omega$$

However, the output resistance specified by the problem, R_{out}, is found as

$$R_{out} = (R_o - R_3) \| R_3 = 6\text{k}\Omega \| 1\text{k}\Omega = \underline{\underline{857\Omega}}$$

2.) The amplifier in the feedback circuit shown has a transfer function of



$$A(s) = \frac{100}{\frac{s}{10^5} + 1}$$

What value of β will increase the upper –3db frequency by a factor of 10 for the closed loop gain? What is the closed loop, low frequency gain?

<u>*Solution*</u>

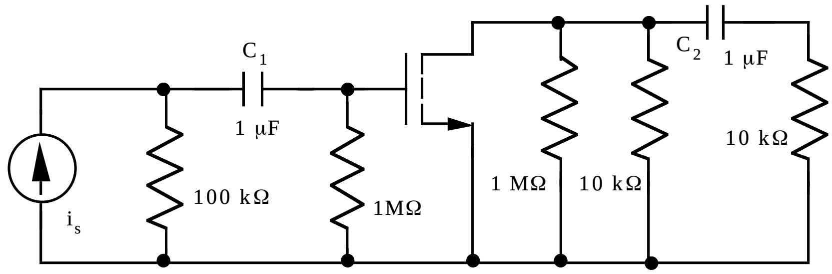
$$A_F = \frac{A}{1+A\beta} = \frac{1}{\frac{1}{A}+\beta} = \frac{1}{\frac{s/10^5+1}{100}+\beta} = \frac{100}{\frac{s}{10^5}+1+100\beta} = \left(\frac{100}{1+100\beta}\right)\frac{1}{\frac{s}{10^5(1+100\beta)}+1}$$

$\therefore\ 10^5(1+100\beta) = 10^6 \quad \rightarrow \quad 1+100\beta = 10 \quad \rightarrow \quad \underline{\underline{\beta = 9/100 = 0.09}}$

The closed-loop, low frequency gain is,

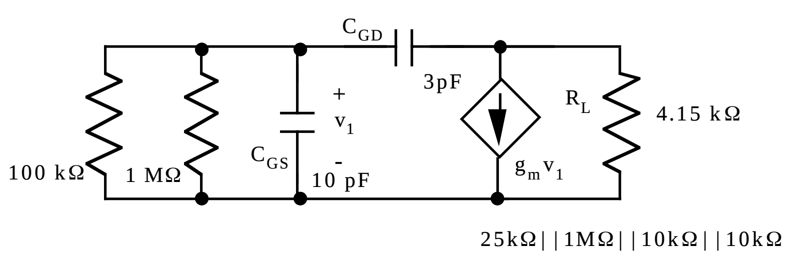
$$A_F(0) = \frac{100}{1+100\beta} = \frac{100}{1+9} = 10 \qquad \rightarrow \qquad \underline{\underline{A_F(0) = 10}}$$

3.) Problem 18.30 of the text.



$$\omega_1 = \frac{1}{10^{-6}(100\text{k}\Omega + 1\text{M}\Omega)} = 0.909\frac{\text{rad}}{\text{s}} \quad | \quad \omega_2 = \frac{1}{10^{-6}(10\text{k}\Omega + 25\text{k}\Omega\|10\text{k}\Omega\|1\text{M}\Omega)} = 58.5\frac{\text{rad}}{\text{s}}$$

Separate widely spaced poles $\rightarrow f_L^A = f_2 = \frac{58.5}{2\pi} = 9.31$ Hz



Problem 18.30 - Continued

$$\omega_H^A = \frac{1}{r_{\pi o} C_T} = \frac{1}{\left(100\,k\Omega \| 1M\Omega\right)\left[10\,pF + 3\,pF\left(1 + 2\,mS\left(4.15\,k\Omega\right) + \frac{4.15\,k\Omega}{100\,k\Omega \| 1M\Omega}\right)\right]}$$

$$f_H^A = \frac{1}{2\pi}\frac{1}{\left(90.9\,k\Omega\right)\left(38.0\,pF\right)} = 46.1\ kHz$$

$$\mathbf{v_{gs}} = \mathbf{i_s}\left(100\,k\Omega \| 1M\Omega\right) = \left(90.9\,k\Omega\right)\mathbf{i_s} \quad | \quad \mathbf{v_o} = -\left(2x10^{-3}\right)\mathbf{v_{gs}}\left(25\,k\Omega \| 10\,k\Omega \| 10\,k\Omega \| 1M\Omega\right)$$

$$A = \frac{\mathbf{v_o}}{\mathbf{i}_s} = -\left(2\,mS\right)\left(4.15\,k\Omega\right)\left(90.9\,k\Omega\right) = -7.55x10^5\,\Omega \quad | \quad y_{12}^F = -10^{-5}\,S$$

$$1 + A\beta = 1 + \left(-7.55x10^5\,\Omega\right)\left(-10^{-6}\,S\right) = 1.76$$

$$f_L = \frac{9.31}{1.76} = 5.29\ Hz \qquad f_H = 46.1kHz\left(1.76\right) = 81.0\ kHz$$

4.) Problem 18.32 of the text.

$$\text{(a)}\quad A(s) = \frac{\dfrac{2x10^{14}\pi^2}{\left(2\pi\ x10^3\right)\left(2\pi\ x10^5\right)}}{\left(1 + \dfrac{s}{2\pi\ x10^3}\right)\left(1 + \dfrac{s}{2\pi\ x10^5}\right)} = \frac{5x10^5}{\left(1 + \dfrac{s}{2\pi\ x10^3}\right)\left(1 + \dfrac{s}{2\pi\ x10^5}\right)}$$

A(s) represents a low-pass amplifier with two widely-spaced poles

Open – loop: $A_o = 5x10^5 = 114dB \quad | \quad f_L = 0 \quad | \quad f_H \approx f_1 = 1000\ Hz$

(b)A common mistake would be the following:

Closed – loop: $f_H = 1000Hz\left[1 + 5x10^5\left(0.01\right)\right] = 5MHz$

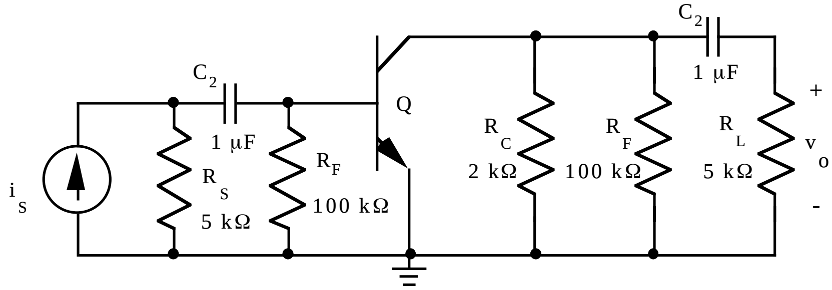
Oops! - This exceeds $f_2 = 100\ kHz$! This is a two-pole amplifier.

$$A_V(s) = \frac{\dfrac{2x10^{14}\pi^2}{\left(s + 2\pi\ x10^3\right)\left(s + 2\pi\ x10^5\right)}}{1 + \dfrac{2x10^{14}\pi^2}{\left(s + 2\pi\ x10^3\right)\left(s + 2\pi\ x10^5\right)}\left(0.01\right)} = \frac{2x10^{14}\pi^2}{s^2 + 1.01\left(2\pi\ x10^5\right)s + 2x10^{12}\pi^2}$$

Using dominant-root factorization: $f_1 = 101\ kHz, \quad f_2 = 4.95\ MHz$

So the closed-loop values are $f_H = 101\ kHz$ and $f_L = 0$.

5.) Problem 18.35 of the text.



From the Exercise: $g_m = 40.3$ mS | $r_\pi = 3.72$ kΩ | $r_o = 50.8$ kΩ | $1 + A\beta = 2.19$

$$r_{o2} = \frac{50 + 1.4}{10^{-4}} = 514\text{k}\Omega \;\;|\;\; r_{o4} = \frac{50 + 11.3}{10^{-4}} = 613\text{k}\Omega \;\;|\;\; r_{\pi 5} = \frac{100(0.025)}{0.0012} = 2.08\text{k}\Omega$$

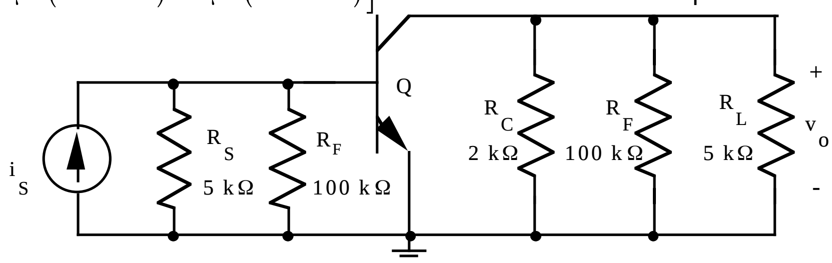
$$C_{\pi 1} = \frac{40.3\text{mS}}{2\pi(500\text{MHz})} - 0.75\text{pF} = 12.1\text{pF}$$

Using the open - circuit time constant approach with $C_1 = C_2 = 1$ μF:

$$R_{1O} = 5\text{k}\Omega + 100\text{k}\Omega \| r_\pi 5\text{k}\Omega + 100\text{k}\Omega \| 3.72\text{k}\Omega = 8.59\text{k}\Omega$$

$$R_{2O} = 5\text{k}\Omega + 50.8\text{k}\Omega \| 2\text{k}\Omega \| 100\text{k}\Omega = 6,89\text{k}\Omega$$

$$f_L = \frac{1}{2\pi}\left[\frac{1}{1\mu\text{F}(8.59\text{k}\Omega)} + \frac{1}{1\mu\text{F}(6.89\text{k}\Omega)}\right] = 41.6 \text{ Hz} \;\;|\;\; f_L^F = \frac{f_L}{1 + A\beta} = 19.0 \text{ Hz}$$



$$r_{\pi o} = 3.72\text{k}\Omega \| 100\text{k}\Omega \| 5\text{k}\Omega = 2.09\text{k}\Omega \;\;|\;\; R_L = 50.8\text{k}\Omega \| 2\text{k}\Omega \| 100\text{k}\Omega \| 5\text{k}\Omega = 1.37\text{k}\Omega$$

$$C_T = 12.1\text{pF} + 0.75\text{pf}\left[1 + 40.3\text{mS}(1.37\text{k}\Omega) + \frac{1.37\text{k}\Omega}{2.09\text{k}\Omega}\right] = 54.8\text{pF}$$

$$f_H = \frac{1}{2\pi r_{\pi o} C_T} = \frac{1}{2\pi(1.37\text{k}\Omega)(54.8\text{pF})} = 1.39 \text{ MHz} \;\;|\;\; f_H^F = f_H(1 + A\beta) = 3.04 \text{ MHz}$$