

Bode Plots

Plotting a transfer function $A(j\omega)$ as

- 1.) $20 \log_{10}(|A(j\omega)|)$ as a function of $\log_{10} \omega$ or $\log_{10} f$.
- 2.) $\text{Arg}[A(j\omega)]$ as a function of $\log_{10} \omega$ or $\log_{10} f$.

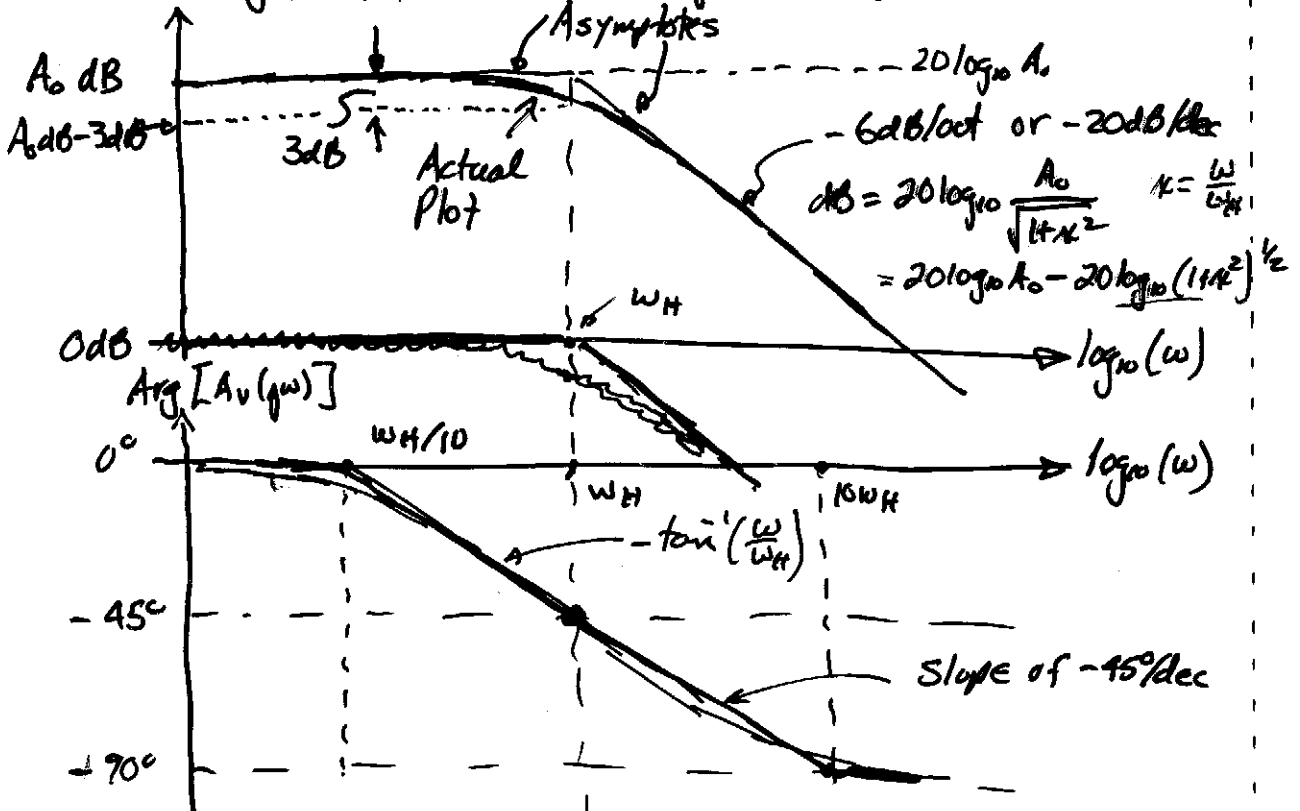
First-Order, Low-Pass Amplifier

$$A_V(s) = \frac{A_0 \omega_H}{s + \omega_H} \xrightarrow{s=j\omega} A_V(j\omega) = \frac{A_0 \omega_H}{j\omega + \omega_H} = \frac{A_0}{j\frac{\omega}{\omega_H} + 1} = \frac{A_0}{1 + j\frac{\omega}{\omega_H}}$$

$$|A_V(j\omega)| = \frac{A_0}{\sqrt{1 + (\frac{\omega}{\omega_H})^2}}$$

$$\text{Arg}[A_V(j\omega)] = -\tan^{-1}\left(\frac{\omega}{\omega_H}\right)$$

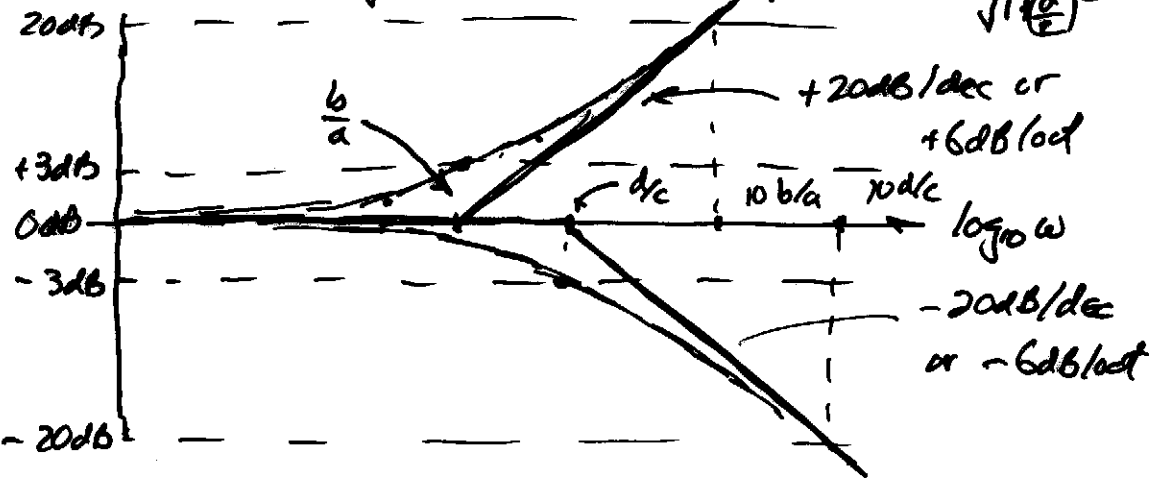
$$dB = 20 \log_{10} |A_V(j\omega)| \quad A = \frac{a+jb}{c+jd} \rightarrow \text{Arg}[A] = \tan^{-1}\left(\frac{b}{a}\right) - \tan^{-1}\left(\frac{d}{c}\right)$$



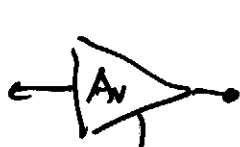
$$A(j\omega) = k \frac{a+jb}{c+jd} \rightarrow |A(j\omega)| = k \frac{\sqrt{a^2+b^2}}{\sqrt{c^2+d^2}} \rightarrow |A(j\omega)| \text{ dB} = 20 \log_{10} k + 20 \log_{10} \sqrt{a^2+b^2} - 20 \log_{10} \sqrt{c^2+d^2}$$

Example-

$$A(j\omega) = K \frac{a+jb}{c+jd} \rightarrow |A(j\omega)| = K \frac{\sqrt{a^2+b^2}}{\sqrt{c^2+d^2}} = K \frac{\sqrt{1+(b/a)^2}}{\sqrt{1+(d/c)^2}}$$

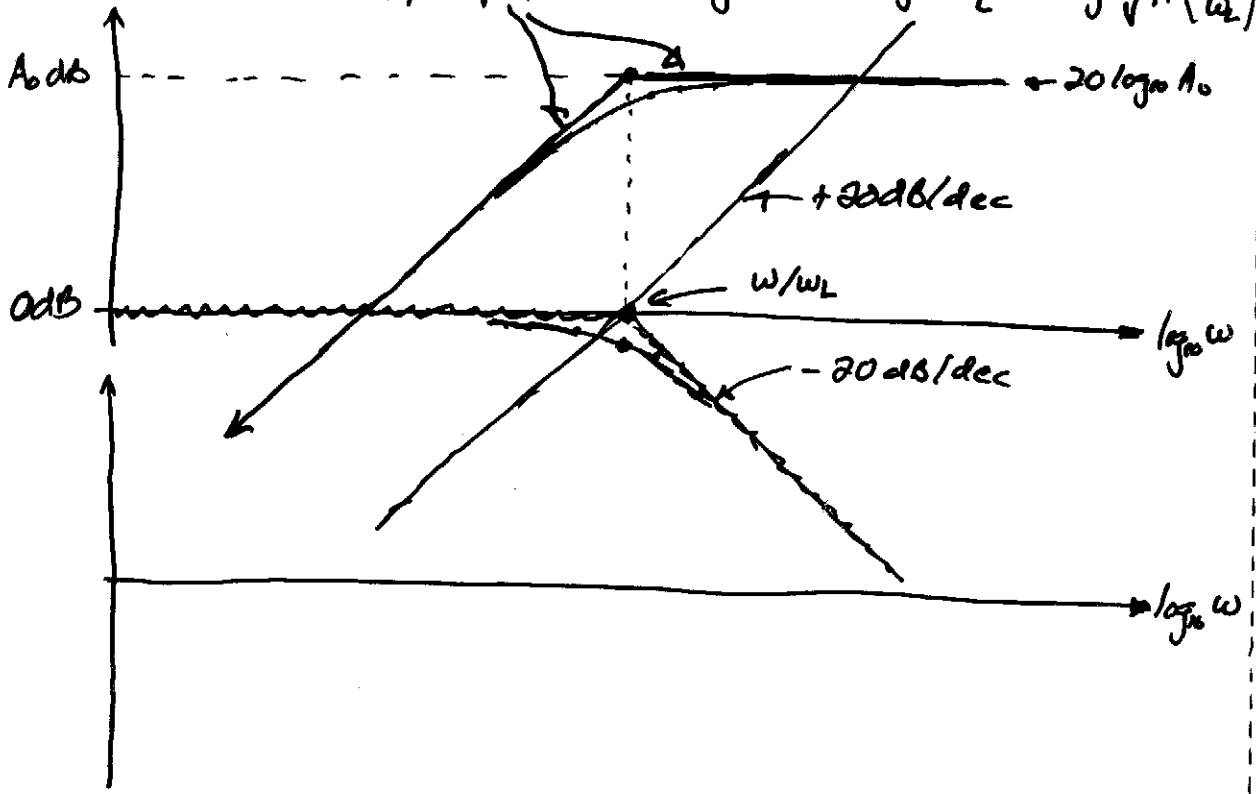


First-order High-Pass Amplifier



$$A_V(s) = \frac{A_0 s}{s + \omega_L} = \frac{A_0 \frac{s}{\omega_L}}{\frac{s}{\omega_L} + 1} \rightarrow |A_V(j\omega)| = \frac{A_0 \frac{\omega}{\omega_L}}{\sqrt{1 + (\frac{\omega}{\omega_L})^2}}$$

$$|A_V(j\omega)| \text{ dB} = 20 \log_{10} A_0 + 20 \log_{10} \frac{\omega}{\omega_L} - 20 \log_{10} \sqrt{1 + (\frac{\omega}{\omega_L})^2}$$



Bandpass

$$A_v(s) = \frac{A_0 s \omega_H}{(s + \omega_L)(s + \omega_H)} = A_0 \left(\frac{s}{\omega_L} \right) \left(\frac{1}{\frac{s}{\omega_H} + 1} \right)$$

