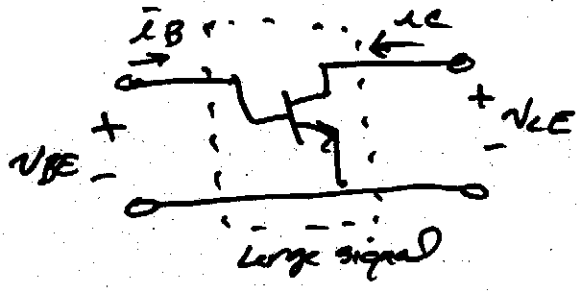
Small Signal Transistor Models

1.) Small Signal large signal relationships

$$i_A = i_a + I_A \rightarrow i_a = i_A - I_A = \Delta i_A = d i_A$$

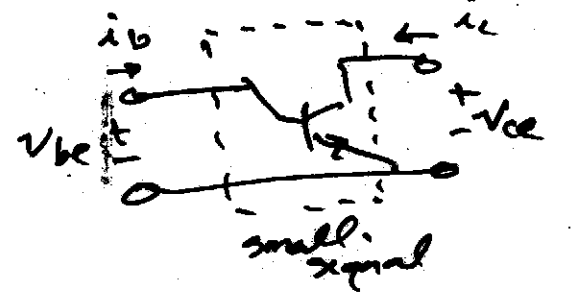
2) CE BJT



Fwd active region

$$i_c = I_S\left[\exp\left(\frac{v_{BE}}{V_T}\right)\right]\left[1+\frac{v_{CE}}{V_A}\right]$$

$$i_B = \frac{I_S}{\beta_F}\exp\left(\frac{v_{BE}}{V_T}\right)$$



$$i_c = K_1 v_{be} + K_2 v_{ce}$$

$$i_b = K_3 v_{be}$$

what are K_1, K_2, K_3?

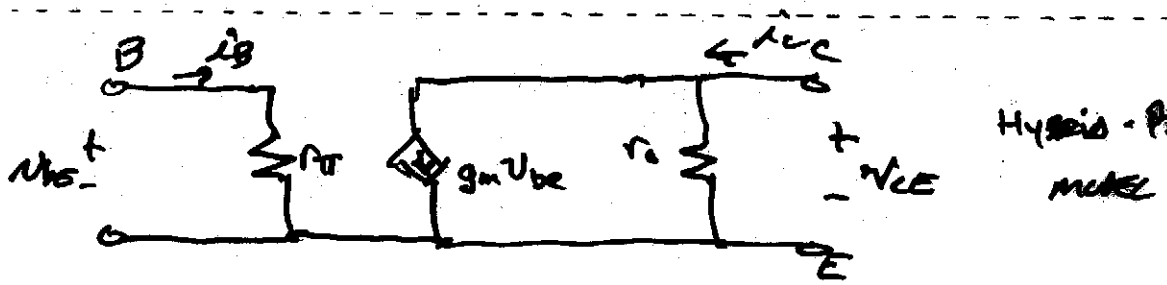
$$K_1 = \left.\frac{i_c}{v_{be}}\right|_{v_{ce}=0} = \left.\frac{\partial i_c}{\partial v_{BE}}\right|_Q = \boxed{\frac{I_C}{V_T} = g_m}$$

$$K_2 = \left.\frac{i_c}{v_{ce}}\right|_{v_{be}=} = \left.\frac{\partial i_c}{\partial v_{ce}}\right|_Q = \boxed{\frac{I_C}{V_A + V_{CE}} \approx \frac{I_C}{V_A} = g_o}$$

$$\boxed{\frac{V_A}{I_C} = r_o}$$

$$K_3 = \left.\frac{i_b}{v_{be}}\right|_{v_{CE}} = \frac{\partial i_B}{\partial v_{BE}}$$

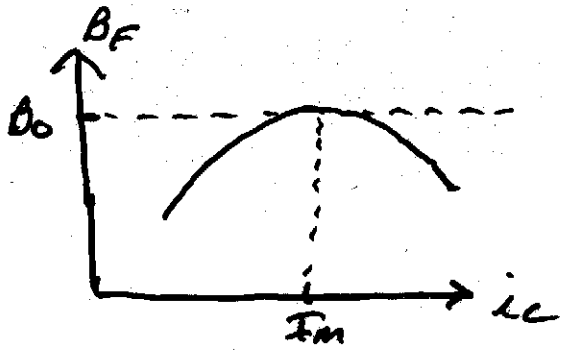
$$= \boxed{\frac{I_C}{\beta_o V_T} = g_\pi} \Rightarrow \boxed{r_\pi = \frac{\beta_o V_T}{I_C}}$$



Hybrid-π model

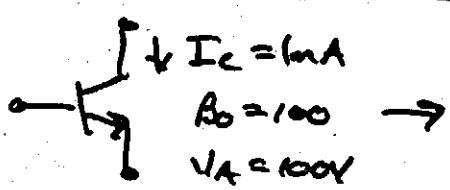
$i_c = g_m v_{be} + g_o v_{ce}$ $\quad i_b = g_\pi v_{be}$

- β_F is not constant with I_C



- $g_m = \frac{I_C}{V_T}$ and $g_\pi \beta_o = \frac{I_C}{V_T} \Rightarrow g_m = g_\pi \beta_o = \frac{\beta_o}{r_\pi}$
- Linearity: requires that $v_{be} \ll 2 V_T$
- PNP s.s. model is identical to NPN.

Example:

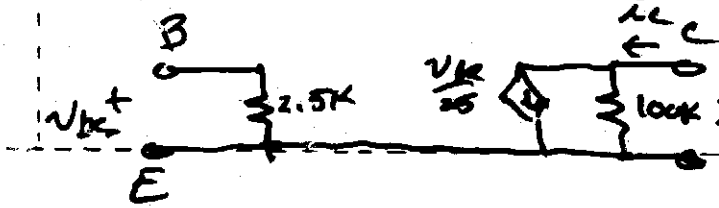


$g_m = \frac{1E-3}{25E-3} = 40mS$

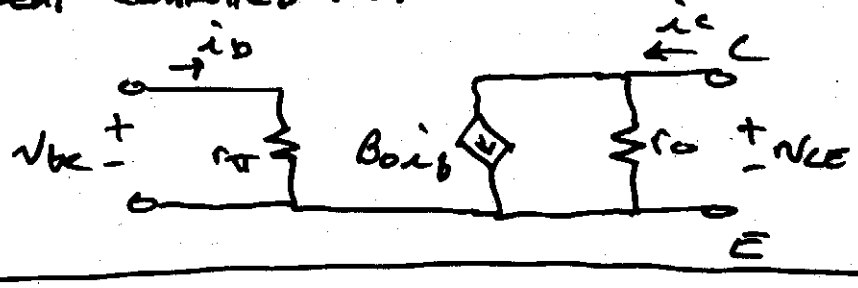
$g_\pi = \frac{1E-3}{(25E-3)\,100} = 0.4mS$

$\hookrightarrow r_\pi = 2.5k$

$g_o = \frac{1}{100} mS \rightarrow r_o = 100k\Omega$



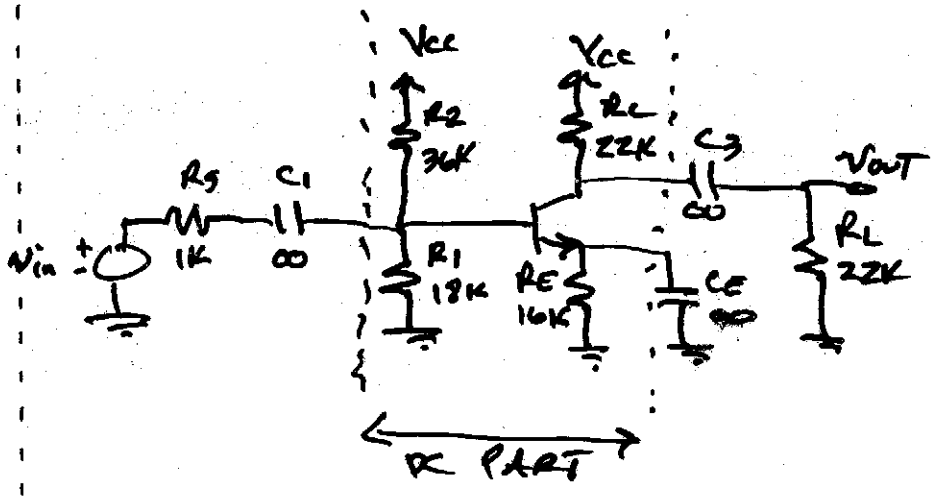
5.) CURRENT CONTROLLED BJT S.S. MODEL



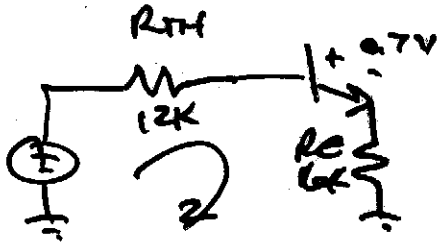
Example: CE BJT Amplifier

Given: $\beta_F = \beta_o = 100$
$V_A = \infty$
$V_{cc} = 10V$
$V_{BE} = 0.7V$

FIND: A_v, R_{IN}, R_{OUT}, Q POINT (V_C, V_B, V_E, I_C, I_E, I_B)



DC



$$V_{TH} = V_{cc}\left(\frac{R_1}{R_1+R_2}\right) = 3.33V$$

$$3.33 - I_B R_{TH} - 0.7 - (1+\beta) I_B R_E = 0$$

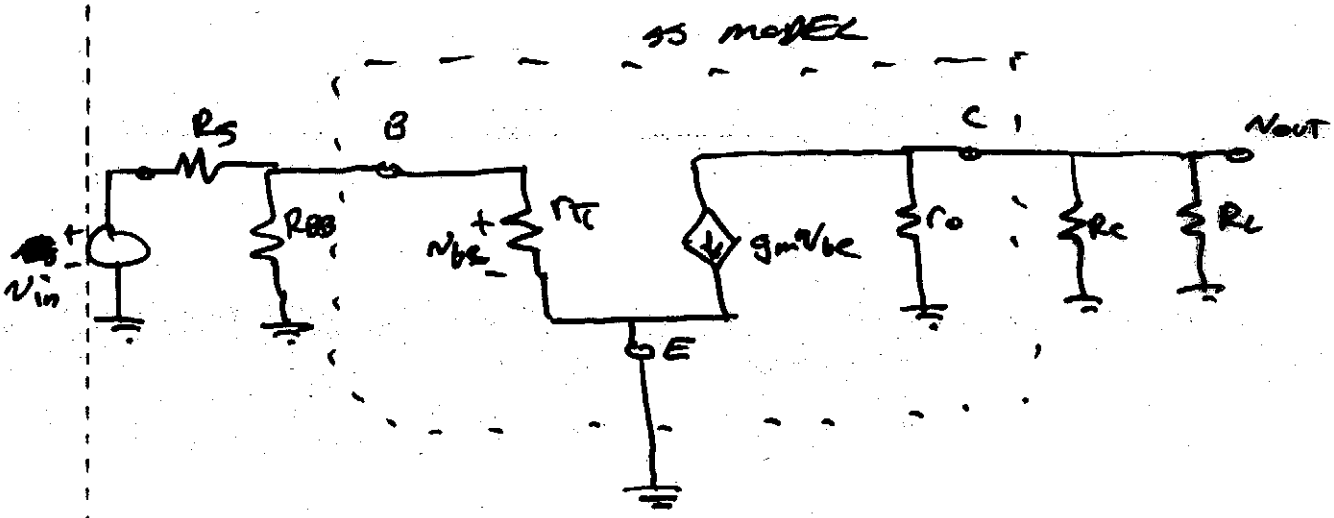
$$\Rightarrow I_B = 1.62mA,\ I_C = 162mA,\ I_E = 164mA$$

$$\Rightarrow V_C = 6.44V,\ V_E = 2.62V,\ V_B = 3.32V$$

AC $g_m = \frac{I_C}{V_T} = 6.48mS$, $r_o = \frac{V_A}{I_C} = \infty$, $r_\pi = \frac{\beta_o V_T}{I_C} = \frac{\beta_o}{g_m}$

$= 15.4K\Omega$

$g_m = 6.48mS$
$r_o = \infty$
$r_\pi = 15.4K$



$$A_v = \frac{V_{out}}{V_{in}} = \left(\frac{V_{be}}{V_{in}}\right)\left(\frac{V_{out}}{V_{be}}\right) = \left(\frac{R_{BB} \| r_\pi}{R_{BB} \| r_\pi + R_S}\right)\left(-g_m [R_L \| R_c \| r_o]\right)$$

$$= (0.871)(-71.28)$$

$$\boxed{A_v = -62.1}$$

$\underline{R_{in}} = R_S + R_{BB} \| r_\pi$ $\quad$ $\underline{R_{out}} = R_L \| R_c \| r_o$

$= 7.74K\Omega$ $\quad$ $= 11K\Omega$