QUIZ NO. 13 - SOLUTION

(Average Score = 6.4/10 of those taking this quiz.)

The open-loop transfer function of a negative feedback system is given as,

$$T(s) = \frac{A_o}{\left(\frac{s}{1000}+1\right)\left(\frac{s}{10,000}+1\right)}$$

What value of A_o gives a phase margin of 45°? The relationship $\tan^{-1}(a) + \tan^{-1}(b) = \tan^{-1}[(a+b)/(1-ab)]$ may be of use in solving this problem.

Solution

The phase margin can be expressed as,

$$\mathrm{PM} = 180° - \tan^{-1}\left(\frac{\omega}{1000}\right) - \tan^{-1}\left(\frac{\omega}{10,000}\right) = 180° - \tan^{-1}\left(\frac{\frac{\omega}{1000}+\frac{\omega}{10,000}}{1-\left(\frac{\omega}{1000}\right)\left(\frac{\omega}{10,000}\right)}\right)$$

If the phase margin is to be 45°, then

$$\tan^{-1}\left(\frac{\frac{\omega}{1000}+\frac{\omega}{10,000}}{1-\left(\frac{\omega}{1000}\right)\left(\frac{\omega}{10,000}\right)}\right) = 135° \quad \rightarrow \quad \left(\frac{\frac{\omega}{1000}+\frac{\omega}{10,000}}{1-\left(\frac{\omega}{1000}\right)\left(\frac{\omega}{10,000}\right)}\right) = \tan(135°) = -1$$

$$\therefore \quad \frac{\omega}{1000}+\frac{\omega}{10,000} = -1 + \left(\frac{\omega}{1000}\right)\left(\frac{\omega}{10,000}\right) \quad \rightarrow \quad \omega_{45°}^2 - 1.1\text{x}10^4\omega_{45°} - 10^7 = 0$$

$$\omega_{45°} = 5.5\text{x}10^3 \pm 46.34\text{x}10^3 = 11,844 \text{ radians/sec.}$$

At $\omega_{45°}$, $|T(j\omega_{45°})| = 1$.

$$\therefore \quad A_o^2 = \left[1+\left(\frac{\omega_{45°}}{1000}\right)^2\right]\left[1+\left(\frac{\omega_{45°}}{10,000}\right)^2\right] = (141.3)(1.402) = 198.2 \quad \rightarrow \quad A_o = \underline{\underline{14.1}}$$

This problem can also be solved graphically as shown below. $A_o = 23\text{dB} \rightarrow A_o = 14.1$