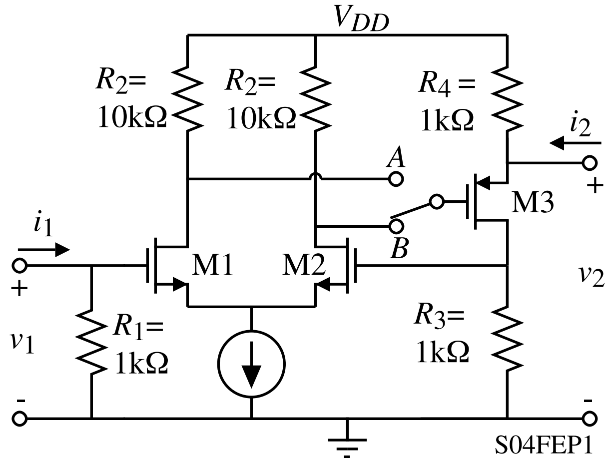
FINAL EXAMINATION - SOLUTIONS

(Average score = 78/100)

Problem 1 - (20 points - This problem must be attempted)

The simplified schematic of a feedback amplifier is shown. Assume that all transistors are matched and g_m = 1mA/V and $r_{ds} = \infty$. (a.) Where should the switch be connected for negative feedback? (b.) Use the method of feedback analysis to find v_2/v_1, $R_{in} = v_1/i_1$, and $R_{out} = v_2/i_2$.



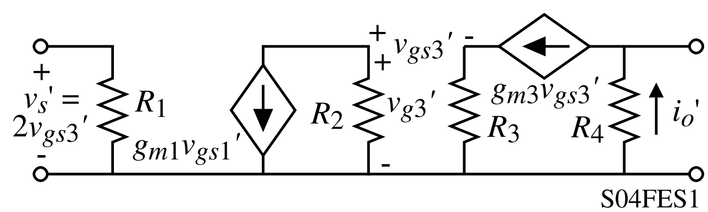
Solution

(a.) A quick check of the ac voltage changes around the loop show that the switch should be connected to A.

(b.) This feedback circuit is series-series. The units of A are A/V and the units of β are V/A.

$$F = z_{12f} = \left.\frac{v_{1f}}{i_{2f}}\right|_{i_{1f}=0} = R_3 = 1\text{k}\Omega$$

The circuit for calculating the small-signal open-loop gain is,



$$A = \frac{i_o'}{v_s'} = \left(\frac{i_o'}{v_{gs3}'}\right)\left(\frac{v_{gs3}'}{v_{g3}'}\right)\left(\frac{v_{g3}'}{v_{gs1}'}\right)\left(\frac{v_{gs1}'}{v_s'}\right) = (-g_{m3})\left(\frac{1}{1+g_{m3}R_4}\right)(-g_{m1}R_2)\left(\frac{1}{2}\right)$$

$$A = \frac{i_o'}{v_s'} = (1\text{mS})(0.5)(5) = 2.5\text{mS} \rightarrow A_F = \frac{i_o}{v_s} = \frac{A}{1+AF} = \frac{2.5\text{mS}}{1+2.5\cdot 1} = 0.714 \text{ mS}$$

$$\frac{v_2}{v_1} = \frac{v_2}{v_s} = \left(\frac{v_2}{i_o}\right)\left(\frac{i_o}{v_s}\right) = -R_4(0.714\text{mS}) = -0.714 \text{ V/V}$$

$$\boxed{\frac{v_2}{v_s} = -0.714 \text{ V/V}}$$

R_1 is not influenced by feedback so $\boxed{\frac{v_1}{i_1} = R_1 = 1\text{k}\Omega}$

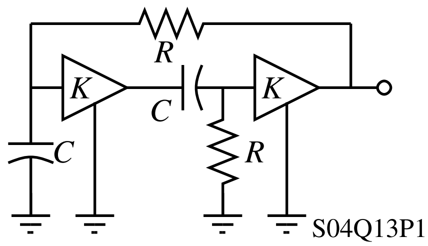
$$R_o = R_4 + (1/g_{m3}) = 1\text{k}\Omega + 1\text{k}\Omega = 2\text{k}\Omega \rightarrow \quad R_{oF} = 2\text{k}\Omega(1+2.5) = 7\text{k}\Omega$$

$$R_{out} = \frac{v_2}{i_2} = (R_{oF} - R_4) || R_4 = 7\text{k}\Omega || 1\text{k}\Omega = 875\Omega$$

$$\boxed{\frac{v_2}{i_2} = 875\Omega}$$

Problem 2 - (20 points - This problem must be attempted)

This problem deals with finding the open-loop gain and it application. The following questions are independent of each other.



(a.) Find the loop gain of the feedback circuit shown, $T(s)$, if the amplifier is an ideal voltage amplifier with a gain of K.

(b.) If $T(s) = \dfrac{sKRC}{s^2R^2C^2 + 2RCs + 1}$, find f_{osc} and the value of K necessary for oscillation.

(c.) If $T(s)$ has the following properties: $T(0) = 10$ and two poles at $s = -100$, what is the phase margin of this feedback circuit?

Solution

(a.) Opening the loop gives,

$$T(s) = \frac{V_{out}}{V_{in}} = \left(\frac{KR}{R+(1/sC)}\right)\left(\frac{K(1/sC)}{R+(1/sC)}\right)$$

$$= \frac{K^2RCs}{(sRC+1)(sRC+1)} = \frac{K^2RCs}{s^2R^2C^2+s2RC+1}$$

$$\therefore \quad \boxed{T(s) = \frac{K^2RCs}{s^2R^2C^2+s2RC+1}}$$

(b.) $T(s) = \dfrac{sKRC}{s^2R^2C^2 + 2RCs + 1} \quad \rightarrow \quad T(j\omega) = \dfrac{j\omega KRC}{-\omega^2R^2C^2 + j\omega 2RC + 1}$

or $\quad T(j\omega) = \dfrac{j\omega KRC}{1-\omega^2R^2C^2 + j\omega 2RC} = 1+j0 \Rightarrow \quad \boxed{f_{osc} = \dfrac{1}{2\pi RC}}$ and $\boxed{K = 2}$

(c.) We could plot a Bode plot and estimate the phase margin or do the following:

$$|T(j\omega)| = \frac{10}{1+\left(\frac{\omega}{100}\right)^2}$$

Find the unity-gain frequency from, $\dfrac{10}{1+\left(\frac{\omega_c}{100}\right)^2} = 1 \quad \rightarrow \quad \omega_c{}^2 = 100^2\,(10–1) = 300^2$

or $\omega_c = 300$ rads/sec. The phase margin can be expressed as,

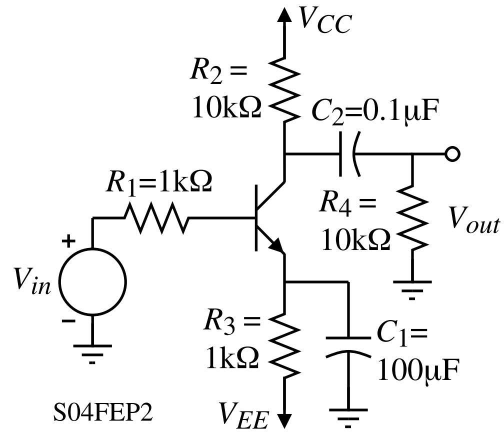
$PM = 180° - 2\tan^{-1}\left(\frac{300}{100}\right) = 180° - 2(71.56°) = \underline{\underline{36.87°}}$

Problem 3 - (20 points - This problem is optional)

A BJT amplifier is shown. Assume that the BJT has the small signal parameters of $\beta_F = 100$, $r_\pi = 1\text{k}\Omega$, and $V_A = \infty$.

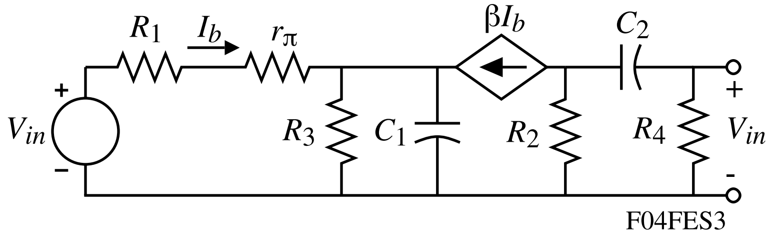
a.) Find the midband voltage gain of this amplifier, V_{out}/V_{in}.

b.) Find the value of the lower -3dB frequency, f_L, in Hz, using any method that is appropriate.



Solution

The small-signal model for this problem is:



(a.) If the capacitors are large, or the frequency large, the midband gain is,

$$\frac{V_{out}}{V_{in}} = \left(\frac{V_{out}}{I_b}\right)\left(\frac{I_b}{V_{in}}\right)$$

$$\frac{V_{out}}{V_{in}} = [-\beta(R_2||R_4)]\left(\frac{1}{R_1 + r_\pi}\right) = (-500)(0.5) = \underline{\underline{250 \text{ V/V}}}$$

(b.) Since the capacitors are independent, let us choose the short-circuit time constant approach.

$$R_{C1} = R_3||\left(\frac{R_1 + r_\pi}{1+\beta}\right) = 1||\left(\frac{2}{101}\right) \text{ k}\Omega = 19.42\Omega$$

$$\therefore \quad p_1 = \frac{-1}{C_1 R_{C1}} = \frac{-1}{10^{-4} \cdot 19.42} = -515 \text{ rads/sec.}$$

$$R_{C2} = R_2 + R_2 = 20\text{k}\Omega$$

$$\therefore \quad p_2 = \frac{-1}{C_2 R_{C2}} = \frac{-1}{10^{-7} \cdot 20\text{k}\Omega} = -500 \text{ rads/sec.}$$

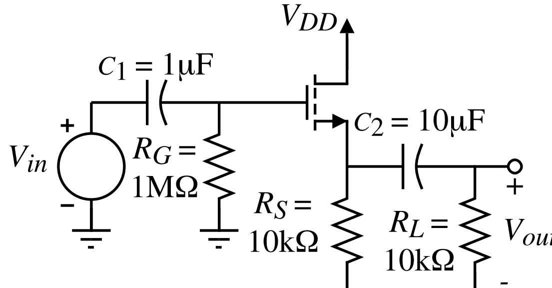
There is also a zero due to R_3 and C_1 given as $-1/(R_3C_1) = -10$ rads/sec.

$$\therefore \quad \omega_L \approx \sqrt{515^2 + 500^2 - 2(10^2)} = 717.65 \text{ rads/sec. or } \quad f_{-3\text{dB}} = \underline{\underline{114.2 \text{ Hz}}}$$

Problem 4 – (20 points - This problem is optional)

a.) If the g_m of the MOSFET is 0.1mA/V, find the midband gain and the location of all zeros and poles of the circuit shown.

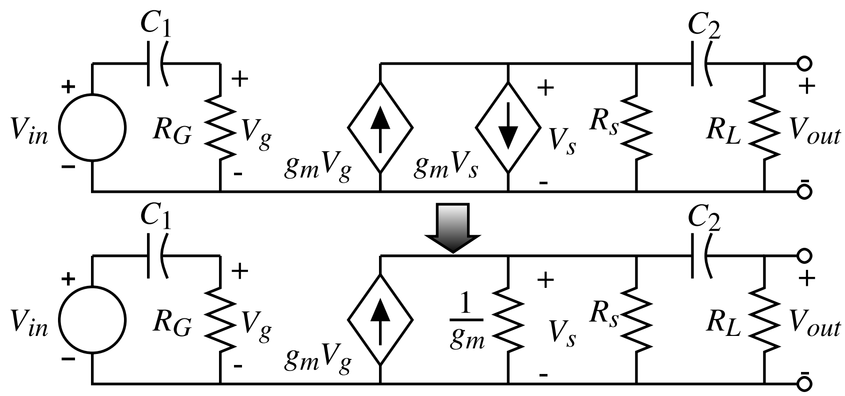
b.) If the amplifier above has two zeros at the origin and a pole at –1 rads/sec and –4 rads/sec., what is the lower –3dB frequency in Hz?



S04FEP4

Solution

1.) Small-signal model:



F02Q09S1

$$\frac{V_{out}}{V_{in}} = \left(\frac{g_m\,(1/g_m)||R_s}{(1/g_m)||R_s + R_L + \frac{1}{sC_2}} \text{ x } R_L\right)\left(\frac{R_G}{R_G + \frac{1}{sC_2}}\right) = \left(\frac{5\text{k}}{15\text{k} + \frac{1}{sC_2}}\right)\left(\frac{1\text{M}}{1\text{M} + \frac{1}{sC_1}}\right)$$

$$= \left(\frac{1}{3}\right)\left(\frac{s}{s + \frac{1}{15\text{k}\ C_2}}\right)\left(\frac{s}{s + \frac{1}{1\text{M}\ C_1}}\right) = \left(\frac{1}{3}\right)\left(\frac{s}{s + 6.67}\right)\left(\frac{s}{s + 1}\right)$$

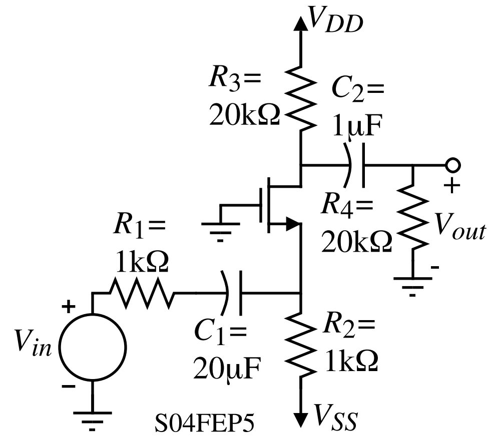
∴ MGB = 0.333, two zeros at 0 rads/sec. and poles at –1 rad/sec and –6.67 rads/sec.

2.) $\omega_L \approx \sqrt{p_1{}^2 + p_2{}^2 - 2(z_1{}^2 + z_2{}^2)} = \sqrt{1^2 + 4^2 - 2(0)} = \sqrt{17} = 4.123$ rads/sec.

∴ $f_L = \frac{4.123}{6.28} = 0.656$ Hz

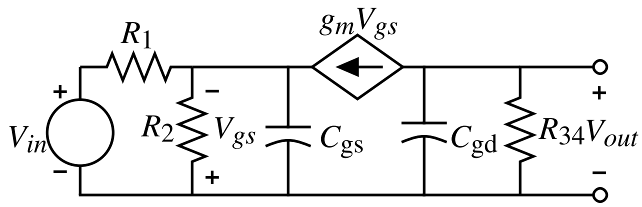
Problem 5 - (20 points - This problem is optional)

The FET in the amplifier shown has g_m = 1mA/V, $r_d = \infty$, C_{gd} = 0.5pF, and C_{gs} = 10pF. (a.) Find the midband gain, V_{out}/V_{in}. (b.) Find the upper -3dB frequency, f_H, in Hz. (Note: You cannot use the Miller's theorem on this problem because there is no bridging capacitor.)



Solution

The small signal model for the high frequency range is shown where $R_{34} = R_3 || R_4$ = 10kΩ.



Because the capacitors are independent, probably the best way to work this problem is by superposition (open-circuit time constants). Therefore,

C_{gs}:

$$R_{Cgs} = R_1||R_2||(1/g_m) = 1\text{K}||1\text{K}||1\text{K} = 333\Omega \rightarrow \omega_{Cgs} = \frac{1}{C_{gs}\cdot 333\Omega} = 300 \text{ Mrads/sec.}$$

C_{gd}:

$$R_{Cgd} = R_{34} = 10\text{k}\Omega \rightarrow \omega_{Cgd} = \frac{1}{C_{gd}\cdot 10\text{k}\Omega} = 200 \text{ Mrads/sec.}$$

$$\therefore \ \omega_H \approx \frac{1}{\sqrt{\left(\frac{1}{300\text{Mrads/sec}}\right)^2 + \left(\frac{1}{200\text{Mrads/sec}}\right)^2}} = 166 \text{ Mrads/sec.}$$

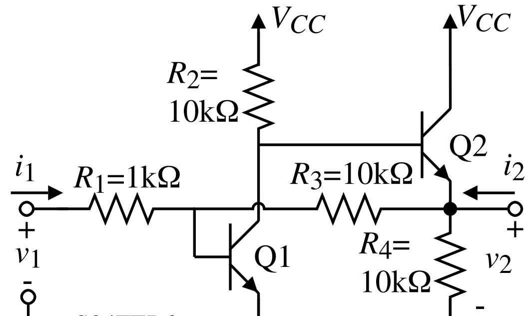
$$\boxed{f_L = 26.48\text{MHz}}$$

The midband gain is given as

$$\boxed{\text{MBG} = \left(\frac{R_2||\frac{1}{g_m}}{R_1 + R_2||\frac{1}{g_m}}\right)\left(\frac{-g_m R_3 R_4}{R_3+R_4}\right) = \left(\frac{-0.5}{1.5}\right)(-10) = 3.33\text{V/V}}$$

Problem 6 - (20 points - This problem is optional).

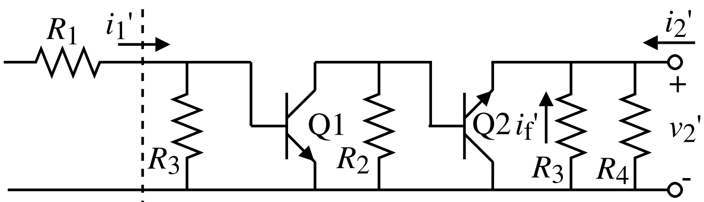
A feedback amplifier is shown. Use the methods of feedback analysis to find the numerical values of v_2/v_1, v_1/i_1, and v_2/i_2. Assume that all transistors are matched and that V_t= 25mV, β (of the BJT) = 100, $I_{C1} = I_{C2}$ = 100μA, and $r_o = \infty$.



S04FEP6

Solution

An open-loop version of the feedback amplifier is shown below.



F04FES6

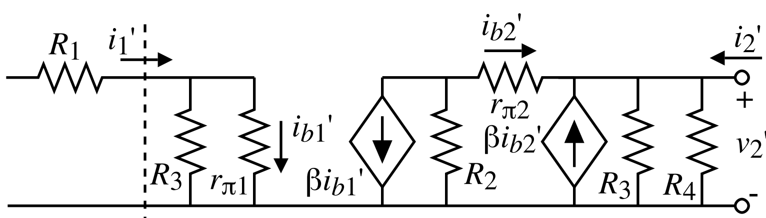
$$F = g_{12F} = \frac{i_F'}{v_2'}\Big|_{v_1'=0} = \frac{-1}{R_3} = \frac{-1}{10\text{k}\Omega}$$

Small signal BJT parameters are:

$$g_m = \frac{I_C}{25\text{mV}} = 4\text{mS}$$

and $r_\pi = \dfrac{1+\beta}{g_m} \approx 25\text{k}\Omega$

The open-loop ac schematic is given as,



F04FES6A

$$\frac{v_2'}{i_1'} = \left(\frac{v_2'}{i_{b2}'}\right)\left(\frac{i_{b2}'}{i_{b1}'}\right)\left(\frac{i_{b1}'}{i_1'}\right) = [(1+\beta)(R_3||R_4)]\left(\frac{-\beta R_2}{R_2+r_{\pi2}+(1+\beta)(R_3||R_4)}\right)\left(\frac{R_3}{R_3 + r_{\pi1}}\right)$$

$$= (101\cdot 10\text{K}||10\text{K})\left(\frac{-100\cdot 10\text{K}}{540\text{K}}\right)\left(\frac{-10\text{K}}{10\text{K}+25\text{K}}\right) = (505\text{K})(-1.852)(0.286) = -267.2\text{k}\Omega$$

$$R_T = \frac{v_2'}{i_1'} = -267.2\text{k}\Omega \quad \Rightarrow \quad \frac{v_2}{i_1} = \frac{R_T}{1+FR_T} = \frac{-267.2\text{K}\Omega}{1+26.72} = -9.64\text{k}\Omega$$

$$R_{in} = R_4||(r_{\pi1}) = 10\text{K}||25\text{K} = 7.14\text{k}\Omega, \ R_{inF} = \frac{R_{in}}{1+FR_T} = \frac{7.14\text{k}\Omega}{27.72} = 257\Omega$$

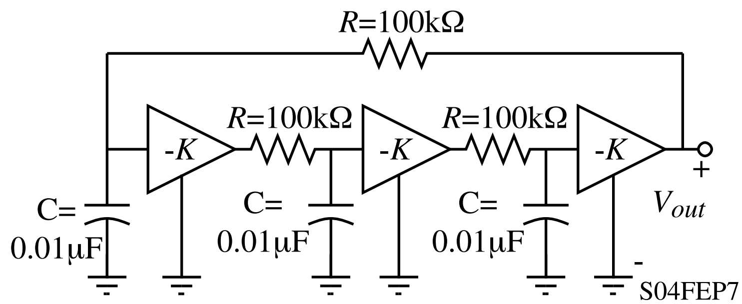
$$\therefore \ \frac{v_1}{i_1} = R_1 + R_{inF} = 1000 + 257 = \underline{\underline{1257\Omega}} \quad \frac{v_2}{v_1} = \frac{v_2}{i_1}\frac{i_1}{v_1} = \frac{-9.64\text{K}}{1257} = \underline{\underline{-7.67\text{V/V}}}$$

$$R_{out} = R_3||R_4||\left(\frac{r_{\pi2}+R_2}{1+\beta}\right) = (10||10||0.346)\text{k}\Omega = 324\Omega$$

$$\therefore \quad \frac{v_2}{i_2} = \frac{R_{out}}{1+FR_T} = \frac{324\Omega}{27.72} = \underline{\underline{11.7\Omega}}$$

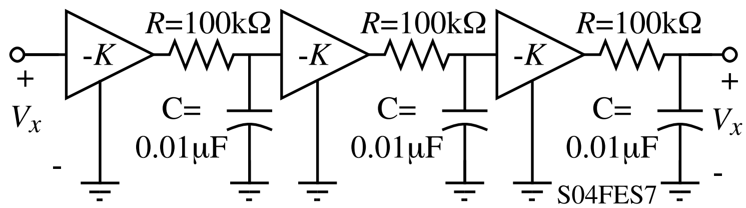
Problem 7 – (20 points, this problem is optional)

An *RC* oscillator is shown. Express the frequency of oscillation of this circuit in terms of the components and evaluate. What is the value of the voltage amplifier gain, *K*, necessary for oscillation? In words, how is the amplitude of oscillation determined?



Solution

Open the loop as follows,



$$T(s) = \frac{V_x}{V_x'} = \left(\frac{-K(1/sC)}{R + (1/sC)}\right)^3 = \left(\frac{-K}{sCR + 1}\right)^3 = \frac{-K^3}{(sRC+1)^3}$$

$$= \frac{-K^3}{s^3R^3C^3 + 3s^2R^2C^2 + 3sRC + 1}$$

Now,

$$T(j\omega) = \frac{-K^3}{(1 - 3\omega^2R^2C^2) + j\omega[3RC - \omega^2R^3C^3]} = 1 + j0$$

$$\therefore \quad 3RC = \omega_{osc}{}^2R^3C^3 \quad \rightarrow \quad \boxed{\omega_{osc} = \frac{\sqrt{3}}{RC}}$$

and $$\frac{-K^3}{1 - 3\omega_{osc}{}^2R^2C^2} = \frac{-K^3}{1 - 9} = \frac{K^3}{8} = 1 \quad \rightarrow \quad K = 8^{1/3} = 2 \quad \boxed{K = 2}$$

Substituting the values gives $f_{osc} = \frac{\sqrt{3}}{2\pi\cdot 10^5 \cdot 10^{-8}} =$ <u>275.67 Hz (1,732 rads/sec)</u>

The amplitude of the oscillation is determined by the amplifiers *K* having a nonlinear transfer function as illustrated. The slope for small values is greater than 2 while the slope for large values is less than 2. The amplitude of the oscillator will stabilize where the effective gain is exactly 2.