Homework Assignment No. 3 - Solution

1.) The differential amplifier below uses an ideal op amp. Find the values of R_1, R_2, R_3 and R_4 if the single-ended input resistances, R_{in1} and R_{in2} are to be 100kΩ and the output voltage is to be $v_{out} = 10(v_1 - v_2)$.

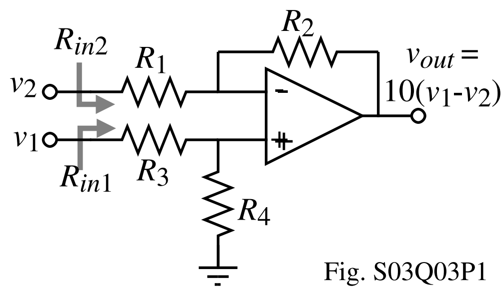


Fig. S03Q03P1

Solution

The first step is to find v_{out} as a function of v_1 and v_2 and to find R_{in1} and R_{in2}.

The output voltage can be found by using superposition applied to the inputs v_1 and v_2. The result is,

$$v_{out} = \left(\frac{v_{out}}{v_1}\right)\Big|_{v_2=0} + \left(\frac{v_{out}}{v_2}\right)\Big|_{v_1=0} = \left(\frac{R_1+R_2}{R_1}\right)\left(\frac{R_4}{R_3+R_4}\right)v_1 - \left(\frac{R_2}{R_1}\right)v_2$$

$R_{in1} = R_3 + R_4$ (remember to set v_2 to zero in this calculation – only one excitation at a time)

$R_{in2} = R_1$ (remember to set v_1 to zero in this calculation – only one excitation at a time)

From the input resistance results, we can write that,

$R_3 + R_4 = 100\text{k}\Omega$ and $\underline{\underline{R_1 = 100\text{k}\Omega}}$

Substituting these values in the voltage gain expression gives,

$$v_{out} = \left(\frac{R_1+R_2}{100\text{k}\Omega}\right)\left(\frac{R_4}{100\text{k}\Omega}\right)v_1 - \left(\frac{R_2}{100\text{k}\Omega}\right)v_2 = 10(v_1 - v_2)$$

This gives us $\underline{\underline{R_2 = 1\text{M}\Omega.}}$ Substituting this back into the voltage gain expression gives,

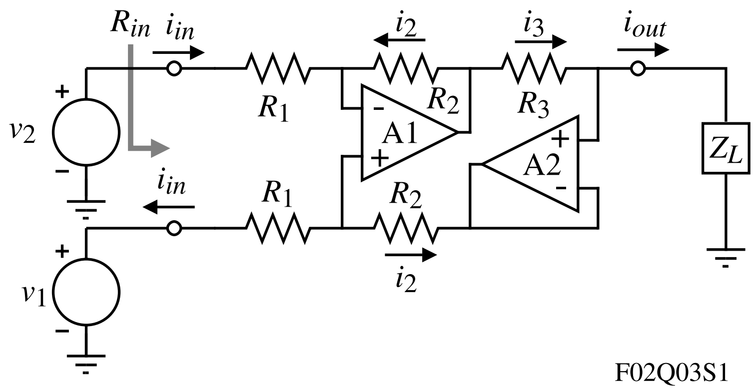
$$v_{out} = \left(\frac{1100\text{k}\Omega}{100\text{k}\Omega}\right)\left(\frac{R_4}{100\text{k}\Omega}\right)v_1 - 10\,v_2 = 10(v_1 - v_2) \rightarrow R_4 = \frac{1000\text{k}\Omega}{11} = \underline{\underline{90.9\text{k}\Omega}}$$

Since the sum of R_3 and R_4 must equal 100kΩ, we get

$R_3 = 100\text{k}\Omega - 90.9\text{k}\Omega = \underline{\underline{9.1\text{k}\Omega}}$

Substituting these values back into the top three equations satisfies the requirements.

2.) Assume that the op amps are ideal and find i_{out} as a function of the inputs, v_1 and v_2. Find the input resistance defined as $R_{in} = (v_2 - v_1)/i_{in}$.



F02Q03S1

Solution

From the circuit we can write the following equations based on an ideal op amp:

$$i_{out} = i_3, \qquad v_2 - v_1 = 2R_1 i_{in}, \qquad i_2R_2 + i_2R_2 = i_3R_3, \qquad i_{in} = -i_2$$

$$\therefore \qquad i_{out} = i_3 = \frac{2R_2 i_2}{R_3} = \frac{2R_2}{R_3}(-\,i_{in}) = \frac{2R_2}{R_3}\left(-\frac{v_2 - v_1}{2R_1}\right) = \frac{R_2}{R_1R_3}(v_1 - v_2)$$

$$\boxed{i_{out} = \frac{R_2}{R_1R_3}(v_1 - v_2)}$$

The input resistance, R_{in} is seen to be equal to $2R_1$. $\boxed{R_{in} = 2R_1}$

3.) Problem 11.38 (12.24) of the text

Applying op-amp assumption 1 to the circuit on the next page, the voltage at the top of R_2 is v_{O2}, and applying op-amp assumption 2,

$$\frac{\mathbf{v_S}}{R_1} = -\frac{\mathbf{v_{O2}}}{R_2} \quad \text{or} \quad \mathbf{v_{O2}} = -\mathbf{v_S}\frac{R_2}{R_1}$$

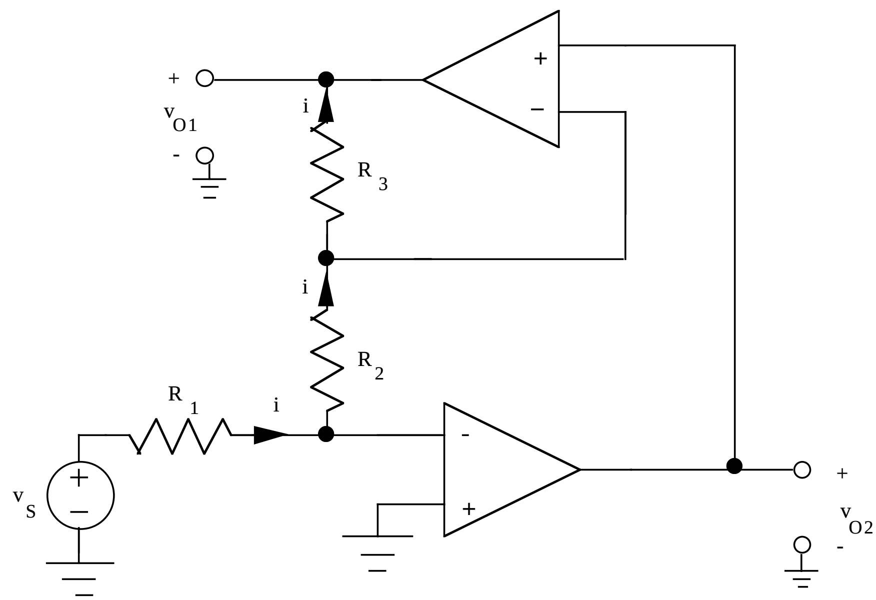
Since the op-amp input currents are zero, and

$$\mathbf{i} = \frac{\mathbf{v_S}}{R_1}, \qquad \mathbf{v_{O1}} = -\mathbf{i}R_2 - \mathbf{i}R_3 = -\left(\frac{R_2}{R_1} + \frac{R_3}{R_1}\right)\mathbf{v_S}$$

Alternatively, the voltage at the bottom of R_2 is zero, so

$$\mathbf{v_{O1}} = \left(1 + \frac{R_3}{R_2}\right)\mathbf{v_{O2}} = \left(1 + \frac{R_3}{R_2}\right)\left(-\frac{R_2}{R_1}\right)\mathbf{v_S} = -\left(\frac{R_2}{R_1} + \frac{R_3}{R_1}\right)\mathbf{v_S}$$

See next page for figure



4.) Problem 11.39 of the text.

$$V_O = -V_{REF}\left(\frac{R}{4R}+\frac{R}{8R}\right) = -3.2\left(\frac{1}{4}+\frac{1}{8}\right) = -1.2\ V$$

$$V_O = -V_{REF}\left(\frac{R}{2R}+\frac{R}{16R}\right) = -3.2\left(\frac{1}{2}+\frac{1}{16}\right) = -1.8\ V$$

0000	0.000 V	1000	-1.60 V
0001	-0.200 V	1001	-1.80 V
0010	-0.400 V	1010	-2.00 V
0011	-0.600 V	1011	-2.20 V
0100	-0.800 V	1100	-2.40 V
0101	-1.00 V	1101	-2.60 V
0110	-1.20 V	1110	-2.80 V
0111	-1.40 V	1111	-3.00 V

5.) Problem 11.98 (12.74) of the text.

$$\beta = \frac{2k\Omega}{2k\Omega + 40k\Omega} = \frac{1}{21} \quad | \quad A\beta = \frac{10^5}{21} = 4760 >> 1$$

$$\text{(a)}\ A_V = -\frac{R_2}{R_1} = -\frac{40k\Omega}{2k\Omega} = -20 \quad | \quad f_H = \beta f_T = \frac{3x10^6\,Hz}{21} = 143kHz$$

$$\text{(b)}\ A_V = (-20)^3 = -8000\ (78dB) \quad | \quad f_{H3} = 0.51 f_H = 72.9kHz$$