Homework Assignment No. 4 - Solutions

Problem 1

13.45

$$\frac{r_D}{20k\Omega + r_D} = \frac{1}{10} \rightarrow r_D = 2.22k\Omega \;\mid\; 40I_D = \frac{1}{2.22k\Omega} \rightarrow I_D = 11.3\ \mu A \;\mid\; v_i = 10(5mV) = 50\ mV$$

Problem 2

13.52

$r_o = \dfrac{V_A + V_{CE}}{I_C}$; solving for V_A: $V_A = I_C r_o - V_{CE}$

Using the values from row 1: $V_A = 0.002(40000) - 10 = 70\ V$

Using the values from the second row: $\beta_o = g_m r_\pi = 0.12(500) = 60\ and\ \beta_F = \beta_o = 60.$

Row 1: $g_m = 40I_C = 40(0.002) = 0.08\ S \;\mid\; r_\pi = \dfrac{\beta_o}{g_m} = \dfrac{60}{0.08} = 750\ \Omega$

$\mu_f = g_m r_o = 0.08(40000) = 3200$

Row 2: $I_C = \dfrac{g_m}{40} = \dfrac{0.12}{40} = 3\ mA \;\mid\; r_o = \dfrac{V_A + V_{CE}}{I_C} = \dfrac{80}{0.003} = 26.7\ k\Omega$

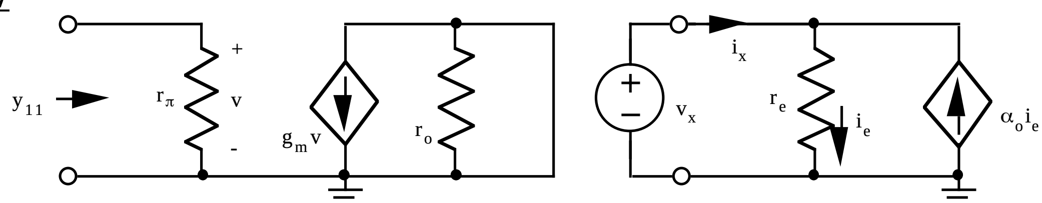
$\mu_f = g_m r_o = 0.12(26700) = 3200$

Row 3: $g_m = \dfrac{\beta_o}{r_\pi} = \dfrac{60}{4.8\ x\ 10^5} = 1.25\ x\ 10^{-4} S \;\mid\; I_C = \dfrac{g_m}{40} = \dfrac{1.25\ x\ 10^{-4}}{40} = 3.13\ \mu A$

$r_o = \dfrac{V_A + V_{CE}}{I_C} = \dfrac{80}{3.13\ x\ 10^{-6}} = 25.6\ M\Omega \;\mid\; \mu_f = g_m r_o = 1.25\ x\ 10^{-4}(25.6\ x\ 10^6) = 3200$

Problem 3

13.57



For the hybrid pi model: $y_{11} = \dfrac{1}{r_\pi}$

For the T-model: $i_x = \dfrac{v_x}{r_e} - \alpha_o \dfrac{v_x}{r_e} = \dfrac{1-\alpha_o}{r_e} v_x$

$$y_{11} = \frac{i_x}{v_x} = \frac{1-\alpha_o}{r_e} = \frac{1 - \dfrac{\beta_o}{\beta_o + 1}}{r_e} = \frac{1}{(\beta_o + 1) r_e} \rightarrow r_\pi = (\beta_o + 1) r_e$$

$$r_e = \frac{r_\pi}{(\beta_o + 1)} = \frac{\beta_o}{g_m(\beta_o + 1)} = \frac{\alpha_o}{g_m} = \frac{\alpha_o V_T}{I_C} = \frac{V_T}{I_E}$$

Problem 4

13.64

$$(a)\ V_{EQ} = -9 + \frac{20k\Omega}{62k\Omega + 20k\Omega}18 = -4.61V \ \mid \ R_{EQ} = 20k\Omega \| 62k\Omega = 15.1k\Omega$$

$$I_B = \frac{-4.61 - 0.7 - (-9)}{15.1k\Omega + 136(3.9k\Omega)} = 6.76\mu A \ \mid \ I_C = 135 I_B = 913\mu A$$

$$V_{CE} = 9 - 13000 I_C - 3900 I_E - (-9) = 2.54V$$

$$g_m = 40 I_C = 0.0365S \ \mid \ r_\pi = \frac{135}{g_m} = 3.70k\Omega \ \mid \ r_o = \infty$$

$$A_v = -\left(\frac{2.97k\Omega}{1k\Omega + 2.97k\Omega}\right)(0.0365)(11.5k\Omega) = -314$$

(b) For $V_{CC} = 18V$, the answers are the same: $I_C = 913\mu A \ \mid \ V_{EC} = 2.54V \ \mid \ A_v = -314$

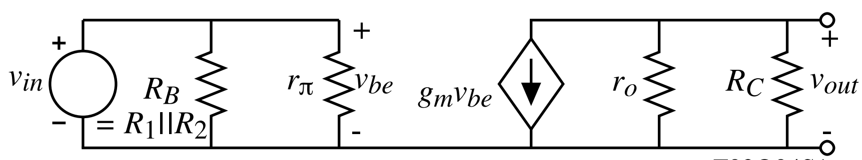
5.) An NPN BJT common-emitter inverting amplifier is shown. Assume the parameters of the transistor are $\beta_F = 100$, $V_T = 25$mV, and $V_A = 100$V. (a.) If $I_C = 0.5$mA and $V_{CE} = 3$V, find the small signal model parameter values for g_m, r_π, and r_o. (b.) Find an algebraic expression for the small signal voltage gain, v_{out}/v_{in}. (c.) Numerically evaluate the small signal voltage gain, v_{out}/v_{in}.

Solution

(a.) $g_m = \frac{I_C}{V_T} = \frac{0.5\text{mA}}{25\text{mV}} = \underline{\underline{20\text{mS}}}$

$r_\pi = \beta_F \frac{V_T}{I_C} = \frac{100}{20\text{mS}} = \underline{\underline{5\text{k}\Omega}}$ $r_o = \frac{V_A + V_{CE}}{I_C} = \frac{102}{0.5\text{mA}} = \underline{\underline{204\text{k}\Omega}}$

(b.) To find the small signal voltage gain, we must first develop a small signal model. This model is given below:



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$$\frac{v_{out}}{v_{in}} = \frac{v_{out}}{v_{be}} = -g_m(r_o || R_C)$$

(c.) The numerical value of this gain is

$$\frac{v_{out}}{v_{in}} = -20\text{mS}(204\text{k}\Omega || 10\text{k}\Omega) = -20\text{mS}(9.53\text{k}\Omega) = \underline{\underline{-190.65\ \text{V/V}}}$$