Homework Assignment No. 4 - Solutions

Problem 1

13.45

\[ r_D = \frac{1}{20k\Omega + r_D} \quad \Rightarrow \quad r_D = 2.22k\Omega \quad \text{and} \quad 40I_D = \frac{1}{2.22k\Omega} \quad \Rightarrow \quad I_D = 11.3 \, \mu A \quad \text{and} \quad v_i = 10(5mV) = 50 \, mV \]

Problem 2

13.52

\[ r_o = \frac{V_A + V_{CE}}{I_C} \quad \text{solving for} \quad V_A = I_C r_o - V_{CE} \]

Using the values from row 1: \( V_A = 0.002(40000) - 10 = 70 \, V \)

Using the values from the second row: \( \beta_o = g_m r_o = 0.12(500) = 60 \) and \( \beta_f = \beta_o = 60. \)

Row 1: \( g_m = 40I_C = 40(0.002) = 0.08 \, S \quad \| \quad r_o = \frac{\beta_o}{g_m} = \frac{60}{0.08} = 750 \, \Omega \)

\[ \mu_f = g_m r_o = 0.08(40000) = 3200 \]

Row 2: \( I_C = \frac{g_m}{40} = 0.12 \quad 3 \, mA \quad \| \quad r_o = \frac{V_A + V_{CE}}{I_C} = \frac{80}{0.003} = 26.7 \, k\Omega \)

\[ \mu_f = g_m r_o = 0.12(26700) = 3200 \]

Row 3: \( g_m = \frac{\beta_o}{r_o} = \frac{60}{4.8 \times 10^3} = 1.25 \times 10^{-6} \, S \quad \| \quad I_C = \frac{g_m}{40} = \frac{1.25 \times 10^{-4}}{40} = 3.13 \, \mu A \)

\[ r_o = \frac{V_A + V_{CE}}{I_C} = \frac{80}{3.13 \times 10^6} = 25.6 \, M\Omega \quad \| \quad \mu_f = g_m r_o = 1.25 \times 10^{-3}(25.6 \times 10^6) = 3200 \]

Problem 3

13.57

For the hybrid pi model: \( y_{11} = \frac{1}{r_e} \)

For the T - model: \( i_x = \frac{v_x}{r_e} = \alpha_o \frac{v_x}{r_e} = \frac{1 - \alpha_o}{r_e} v_x \)

\[ y_{11} = i_x = \frac{1 - \alpha_o}{r_e} = \frac{1 - \beta_o}{r_e} \quad \Rightarrow \quad r_e = (\beta_o + 1) r_e \]

\[ r_e = \frac{\beta_o}{(\beta_o + 1)} = \frac{g_m}{g_m (\beta_o + 1)} = \frac{\alpha_o}{g_m} \frac{\alpha_o}{g_m} \frac{\alpha_o}{g_m} = \frac{V_T}{I_C} = \frac{I_E}{I_E} \]
Problem 4

13.64

(a) \( V_{EO} = -9 + \frac{20k\Omega}{62k\Omega + 20k\Omega} \times 18 = -4.61V \) \( R_{EO} = 20k\Omega \| 62k\Omega = 15.1k\Omega \)

\( I_B = \frac{-4.61 - 0.7 - (-9)}{15.1k\Omega + 136(3.9k\Omega)} = 6.76\mu A \) \( I_C = 135I_B = 913\mu A \)

\( V_{CE} = 9 - 13000I_C - 3900I_E - (-9) = 2.54V \)

\( g_m = 40I_C = 0.0365S \) \( r_\pi = \frac{135}{g_m} = 3.70k\Omega \) \( r_o = \infty \)

\( A_v = -\left( \frac{2.97k\Omega}{1k\Omega + 2.97k\Omega} \right)(0.0365)(11.5k\Omega) = -314 \)

(b) For \( V_{CC} = 18V \), the answers are the same: \( I_C = 913\mu A \) \( V_{EC} = 2.54V \) \( A_v = -314 \)

5.) An NPN BJT common-emitter inverting amplifier is shown. Assume the parameters of the transistor are \( \beta_F = 100 \), \( V_T = 25mV \), and \( V_A = 100V \). (a.) If \( I_C = 0.5mA \) and \( V_{CE} = 3V \), find the small signal model parameter values for \( g_m \), \( r_\pi \), and \( r_o \). (b.) Find an algebraic expression for the small signal voltage gain, \( v_{out}/v_{in} \). (c.) Numerically evaluate the small signal voltage gain, \( v_{out}/v_{in} \).

Solution

(a.) \( g_m = \frac{I_C}{V_T} = \frac{0.5mA}{25mV} = 20mS \)

\( r_\pi = \beta_F I_C = \frac{100}{20mS} = 5k\Omega \)

\( r_o = \frac{V_A + V_{CE}}{I_C} = \frac{102}{0.5mA} = 204k\Omega \)

(b.) To find the small signal voltage gain, we must first develop a small signal model. This model is given below:

(c.) The numerical value of this gain is

\[ \frac{v_{out}}{v_{in}} = -20mS(204k\Omega \| 10k\Omega) = -20mS(9.53k\Omega) = -190.65 \text{V/V} \]