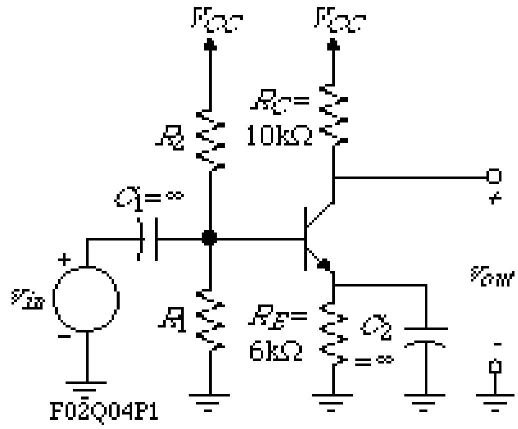
Homework Assignment No. 5 - Solution

1.) An NPN BJT common-emitter inverting amplifier is shown. Assume the parameters of the transistor are β_F = 100, V_T = 25mV, and V_A = 100V. (a.) If I_C = 0.5mA and V_{CE} = 3V, find the small signal model parameter values for g_m, r_π, and r_o. (b.) Find an algebraic expression for the small signal voltage gain, v_{out}/v_{in}. (c.) Numerically evaluate the small signal voltage gain, v_{out}/v_{in}.

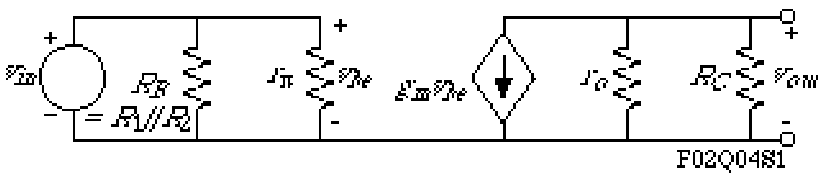


Solution

(a.) $$g_m = \frac{I_C}{V_T} = \frac{0.5\text{mA}}{25\text{mV}} = \underline{\underline{20\text{mS}}}$$

$$r_\pi = \beta_F \frac{V_T}{I_C} = \frac{100}{20\text{mS}} = \underline{\underline{5\text{k}\Omega}} \qquad r_o = \frac{V_A + V_{CE}}{I_C} = \frac{102}{0.5\text{mA}} = \underline{\underline{204\text{k}\Omega}}$$

(b.) To find the small signal voltage gain, we must first develop a small signal model. This model is given below:



$$\boxed{\frac{v_{out}}{v_{in}} = \frac{v_{out}}{v_{be}} = -g_m(r_o||R_C)}$$

(c.) The numerical value of this gain is

$$\frac{v_{out}}{v_{in}} = \text{-20mS(204k}\Omega||\text{10k}\Omega\text{) = -20mS(9.53k}\Omega\text{) = } \underline{\underline{\text{-190.65 V/V}}}$$

2.) Problem 13.102 (13.91) of the text.

$$g_m = \sqrt{2\left(500\frac{\mu A}{V^2}\right)(100\mu A)(1+0.02(5))} = 332\mu S \quad | \quad r_o = \frac{50+5V}{100\mu A} = 550k\Omega$$

$$A_v = -\left(\frac{6.8M\Omega}{6.8M\Omega + 0.1M\Omega}\right)(332\mu S)\left(550k\Omega \| 50k\Omega \| 120k\Omega\right) = -10.9$$

3.) Problem 13.111 (13.100) of the text. [A_v = -4.60 V/V]

$$g_m = \frac{2}{3}\sqrt{1mA(1mA)[1+0.015(9)]} = 710\mu S \quad | \quad r_o = \frac{\frac{1}{0.015}+9V}{1mA} = 75.7k\Omega$$

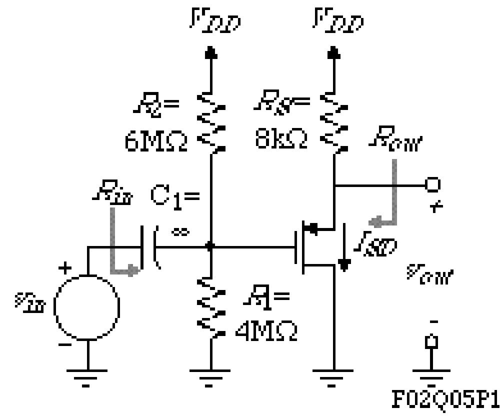
$$A_v = -\left(\frac{1M\Omega}{1M\Omega + 10k\Omega}\right)(710\mu S)\left(75.7k\Omega \| 7.5k\Omega \| 160k\Omega\right) = -4.60V/V$$

4.) Problem 13.118 (13.108) of the text.

$$R_{in} = R_G = 6.8M\Omega \quad | \quad R_{out} = 50k\Omega \| r_o$$

$$r_o = \frac{(50+5)V}{0.1mA} = 550k\Omega \quad | \quad R_{out} = 50k\Omega \| 550k\Omega = 45.8k\Omega$$

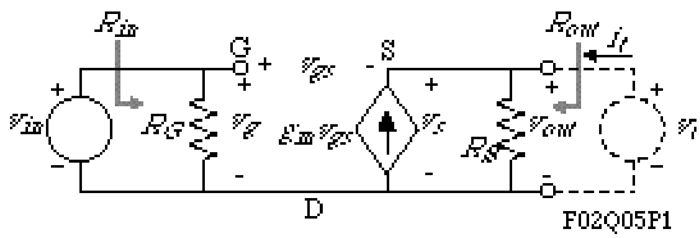
5.) A PMOS common-drain amplifier is shown. Assume the parameters of the transistor are k_F = 0.5mA/V^2, V_{TP} = -1V, and λ = 0. (a.) If I_{SD} = 0.5mA, find the small signal model parameter values for g_m and r_o. (b.) Find an algebraic expression for the small signal input resistance, R_{in}, the output resistance, R_{out}, and the voltage gain, v_{out}/v_{in}. (c.) Numerically evaluate the small signal input resistance, R_{in}, the output resistance, R_{out}, and the voltage gain, v_{out}/v_{in}.



Solution

(a.) $g_m = \sqrt{2I_{SD}k_P} = \sqrt{2\cdot0.5\cdot0.5}$ mS = $\underline{\underline{0.707\text{mS}}}$ and $r_o = \underline{\underline{\infty}}$

(b.) First we need a small signal model.



Obviously, $\boxed{R_{in} = R_G = R_1 \| R_2}$. For R_{out} we apply the voltage source, v_t, and set v_{in} = 0 and solve for v_t/i_t which equivalent to R_{out}.

$$i_t = G_S v_t - g_m v_{gs} = G_S v_t - g_m(v_g - v_s) = G_S v_t - g_m(0 - v_s)$$

$$= G_S v_t + g_m v_s = G_S v_t + g_m v_t = (G_S + g_m)v_t$$

$$R_{out} = \frac{v_t}{i_t} = \frac{1}{G_S + g_m} = \frac{R_S}{1 + g_m R_S} \quad \rightarrow \quad \boxed{R_{out} = \frac{R_S}{1 + g_m R_S}}$$

The output voltage can be expressed as,

$$v_{out} = g_m R_S v_{gs} = g_m R_S(v_g - v_s) = g_m R_S(v_{in} - v_{out})$$

$$v_{out}(1 + g_m R_S) = g_m R_S v_{in} \quad \rightarrow \quad \boxed{\frac{v_{out}}{v_{in}} = \frac{g_m R_S}{1 + g_m R_S}}$$

(c.) $R_{in} = R_G = R_1 \| R_2 = \underline{\underline{2.4M\Omega}}$, $R_{out} = \frac{8k\Omega}{1+0.707\cdot8} = \underline{\underline{1.2k\Omega}}$ and $\frac{v_{out}}{v_{in}} = \frac{0.707\cdot8}{1+0.707\cdot8} = \underline{\underline{0.85\text{V/V}}}$