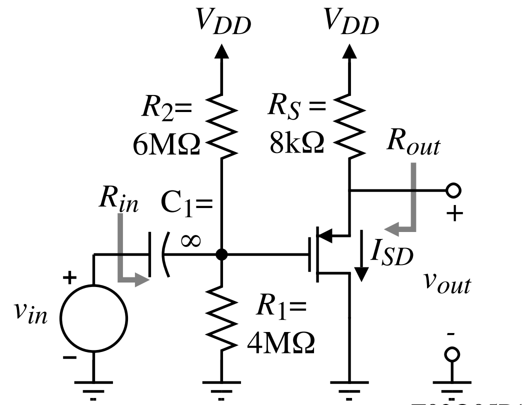
Homework Assignment No. 6 - Solution

1.) A PMOS common-drain amplifier is shown. Assume the parameters of the transistor are $k_F = 0.5\text{mA/V}^2$, $V_{TP} = -1\text{V}$, and $\lambda = 0$. (a.) If $I_{SD} = 0.5\text{mA}$, find the small signal model parameter values for g_m and r_o. (b.) Find an algebraic expression for the small signal input resistance, R_{in}, the output resistance, R_{out}, and the voltage gain, v_{out}/v_{in}. (c.) Numerically evaluate the small signal input resistance, R_{in}, the output resistance, R_{out}, and the voltage gain, v_{out}/v_{in}.

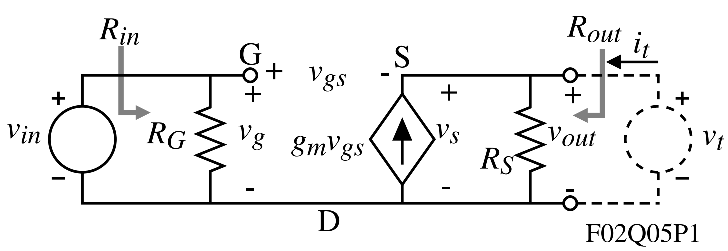


F02Q05P1

Solution

(a.) $g_m = \sqrt{2I_{SD}k_P} = \sqrt{2\cdot0.5\cdot0.5}\text{ mS} = \underline{\underline{0.707\text{mS}}}$ and $r_o = \underline{\underline{\infty}}$

(b.) First we need a small signal model.



F02Q05P1

Obviously, $\boxed{R_{in} = R_G = R_1 || R_2}$. For R_{out} we apply the voltage source, v_t, and set v_{in} = 0 and solve for v_t/i_t which equivalent to R_{out}.

$$\therefore\ i_t = G_S\,v_t - g_m v_{gs} = G_S\,v_t - g_m(v_g - v_s) = G_S\,v_t - g_m(0 - v_s)$$

$$= G_S\,v_t + g_m\,v_s = G_S\,v_t + g_m\,v_t = (G_S + g_m)v_t$$

$$\therefore \qquad R_{out} = \frac{v_t}{i_t} = \frac{1}{G_S + g_m} = \frac{R_S}{1 + g_m R_S} \quad \rightarrow \quad \boxed{R_{out} = \frac{R_S}{1 + g_m R_S}}$$

The output voltage can be expressed as,

$$v_{out} = g_m R_S v_{gs} = g_m R_S(v_g - v_s) = g_m R_S(v_{in} - v_{out})$$

$$\therefore \qquad v_{out}(1 + g_m R_S) = g_m R_S\,v_{in} \quad \rightarrow \quad \boxed{\frac{v_{out}}{v_{in}} = \frac{g_m R_S}{1 + g_m R_S}}$$

(c.) $R_{in} = R_G = R_1 || R_2 = \underline{\underline{2.4\text{M}\Omega}}$, $R_{out} = \frac{8\text{k}\Omega}{1+0.707\cdot8} = \underline{\underline{1.2\text{k}\Omega}}$ and $\frac{v_{out}}{v_{in}} = \frac{0.707\cdot8}{1+0.707\cdot8} = \underline{\underline{0.85\text{V/V}}}$

2.) Problem 14.16 (no equivelent problem in first ed.) of the text.

14.16

$V_{EQ} = 12\frac{62k\Omega}{20k\Omega + 62k\Omega} = 9.07V$ | $R_{EQ} = 20k\Omega \| 62k\Omega = 15.1k\Omega$

$I_B = \frac{(12 - 0.7 - 9.07)V}{15.1k\Omega + (75+1)3.9k\Omega} = 7.16\mu A$ | $I_C = 537\ \mu A$ | $V_{EC} = 12 - 3900I_E - 8200I_C = 5.47\ V$

Active region is correct. | $r_\pi = \frac{75(0.025V)}{537\mu A} = 3.49k\Omega$ | V_A not specified, choose $r_o = \infty$

$R_{in} = 15.1k\Omega \| 3.49k\Omega = 2.83\ k\Omega$ | $R_{out} = r_o \| 8.2k\Omega = 8.2\ k\Omega$ | $g_m = 40I_C = 21.5\ mS$

$R_L = r_o \| 8.2k\Omega \| 100k\Omega = 8.2k\Omega \| 100k\Omega = 7.58k\Omega$

$$A_v = -g_m R_L\left(\frac{R_{in}}{R_I + R_{in}}\right) = -(21.5mS)(7.58k\Omega)\left(\frac{2.83k\Omega}{1k\Omega + 2.83k\Omega}\right) = -120$$

$$A_i = \frac{R_B}{R_B + r_\pi}(-\beta_o)\frac{R_{out}}{R_{out} + R_3} = \frac{15.1k\Omega}{15.1k\Omega + 3.49k\Omega}(-75)\frac{8.2k\Omega}{8.2k\Omega + 100k\Omega} = -4.62$$

$v_{be} = v_i\frac{R_{in}}{R_I + R_{in}} = v_i\frac{2.83k\Omega}{1k\Omega + 2.83k\Omega} = 0.739v_i$ | $v_i = \frac{5.00mV}{0.739} = 6.76\ mV$

$A_v \cong -10V_{CC} = -10(12) = -120.$ | The voltage gain is identical to the rule-of-thumb estimate.

3.) Problem 14.19 (14.15) of the text.

14.19

$V_{GS} = -(11k\Omega)I_D = -(11k\Omega)(20mA)\left(1 - \frac{V_{GS}}{-4}\right)^2 \rightarrow V_{GS} = -3.50V,\ I_D = -\frac{V_{GS}}{11k\Omega} = 318\ \mu A$

$V_{DS} = 20 - I_D(11k\Omega + 39k\Omega) = 4.10V$ | Active region operation is correct.

$g_m = \frac{2}{|-4|}\sqrt{20mA(318\mu A)} = 1.26mS$ | Assume $\lambda = 0$, $r_o = \infty$. | $R_L = 39k\Omega \| 500k\Omega = 36.2k\Omega$

$R_{in} = R_G = 1.00\ M\Omega$ | $R_{out} = 39k\Omega$

$$A_v = -\frac{g_m R_L}{1 + g_m R_S}\left(\frac{R_{in}}{R_I + R_{in}}\right) = -\frac{1.26mS(36.2k\Omega)}{1 + 1.26mS(11k\Omega)}\left(\frac{1M\Omega}{500\Omega + 1M\Omega}\right) = -3.07$$

$$A_i = -R_G\left(\frac{g_m}{1 + g_m R_S}\right)\left(\frac{R_D}{R_D + R_3}\right) = -10^6\left[\frac{1.26mS}{1 + 1.26mS(11k\Omega)}\right]\left(\frac{39k\Omega}{39k\Omega + 500k\Omega}\right) = -6.14$$

$v_{gs} = \frac{R_{in}}{R_I + R_{in}}v_i = 1.00v_i$ | $V_{GS} - V_P = -3.5 - (-4) = 0.500V$

$v_{gs} \le 0.2(V_{GS} - V_P)(1 + g_m R_S)$ | $v_i \le 0.2(0.5)[1 + 1.26mS(11k\Omega)] = 1.49\ V$

4.) Problem 14.27 (14.21) of the text.

14.27

$V_{EQ} = 18\frac{51k\Omega}{51k\Omega + 100k\Omega} = 6.08V$ | $R_{EQ} = 51k\Omega \| 100k\Omega = 33.8k\Omega$

$I_B = \frac{(6.08 - 0.7 + 18)V}{33.8k\Omega + (126)(4.7k\Omega)} = 37.3\mu A$ | $I_C = 4.67\ mA$ | $V_{CE} = 36 - 2000I_C - 4700I_E = 4.54$ V

Active region is correct. | $r_\pi = \frac{125(0.025V)}{4.67mA} = 669\Omega$ | $r_o = \frac{(50 + 4.54)V}{4.67mA} = 11.7k\Omega$

$R_B = R_1 \| R_2 = 51k\Omega \| 100k\Omega = 33.8k\Omega$ | $R_L = R_3 \| R_E \| r_o = 24k\Omega \| 4.7k\Omega \| 11.7k\Omega = 2.94k\Omega$

$R_{in} = R_B \| [r_\pi + (\beta_o + 1)R_L] = 33.8k\Omega \| [669\Omega + (126)2.94k\Omega] = 31.0\ k\Omega$

$$A_v = +\frac{(\beta_o + 1)R_L}{r_\pi + (\beta_o + 1)R_L}\left(\frac{R_{in}}{R_I + R_{in}}\right) = \frac{126(2.94k\Omega)}{0.669k\Omega + 126(2.94k\Omega)}\left(\frac{31.0k\Omega}{500\Omega + 31.0\ k\Omega}\right) = 0.982$$

$$v_{be} = v_i\left(\frac{R_{in}}{R_I + R_{in}}\right)\left(\frac{r_\pi}{r_\pi + (\beta_o + 1)R_L}\right) = \left(\frac{31.0k\Omega}{500\Omega + 31.0\ k\Omega}\right)\left[\frac{0.669k\Omega}{0.669k\Omega + 126(2.94k\Omega)}\right] = 1.77x10^3 v_i$$

$v_i \le \frac{0.005V}{1.77x10^{-3}} = 2.82\ V$ | $R_{out} = R_E \| \frac{(R_B \| R_I) + r_\pi}{\beta_o + 1} = 4.7k\Omega \| \frac{(33.8k\Omega \| 500\Omega) + 669\Omega}{126} = 9.22\ \Omega$

5.) Problem 14.28 (14.23) of the text

14.28

$V_{GS} = 5V$ | $I_D = \frac{4x10^{-4}}{2}(5 - 1)^2 = 3.2mA$ | $V_{DS} = 5 - (-5) = 10V$ - Pinchoff region operation is correct. | $g_m = \sqrt{2(4x10^{-4})(3.2mA)[1 + 0.02(10)]} = 1.75mS$

$r_o = \frac{\frac{1}{0.02} + 10}{3.2}\frac{V}{mA} = 18.8k\Omega$ – Cannot *neglect*! | $R_L = 18.8k\Omega \| 100k\Omega = 15.8k\Omega$

$R_{in} = R_G = 1\ M\Omega$ | $R_{out} = \frac{1}{g_m} \| r_o = 555\ \Omega$

$$A_v = +\frac{R_{in}}{R_I + R_{in}}\left(\frac{g_m R_L}{1 + g_m R_L}\right) = +\frac{1M\Omega}{10k\Omega + 1M\Omega}\left(\frac{1.75mS(15.8k\Omega)}{1 + 1.75mS(15.8k\Omega)}\right) = 0.956$$

$$\mathbf{v}_{gs} = \mathbf{v}_i\left(\frac{R_{in}}{R_I + R_{in}}\right)\left(\frac{1}{1 + g_m R_L}\right) = v_i\left(\frac{10^6\Omega}{10^4\Omega + 10^6\Omega}\right)\left[\frac{1}{1 + 1.75mS(15.8k\Omega)}\right] = 0.0346\mathbf{v}_i$$

$v_i \le \frac{0.2(5 - 1)}{0.0346} = 23.2\ V$ But, v_{DS} must exceed $v_{GS} - V_{TN} \cong V_{GS} - V_{TN} = 4V$ for pinchoff.

$V_{DS} = 10 - v_o = 10 - 0.956v_i \ge 4 \rightarrow v_i \le 6.28\ V$ – Limited by the Q-point voltages