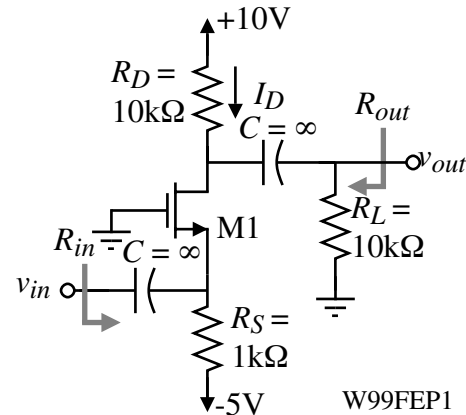


**Homework Assignment No. 7 - Solutions**

1.) If  $K = 0.1\text{mA/V}^2$  and  $V_t = 1\text{V}$  for the n-channel MOSFET shown, find the dc value of  $I_D$ ,  $V_{GS}$ , and  $V_{DS}$ . If the dc value of  $I_D = 1\text{mA}$  (not necessarily the answer above) find the value of the small-signal input resistance,  $R_{in}$ , voltage gain,  $v_{out}/v_{in}$ , and output resistance,  $R_{out}$ .

**Solution**

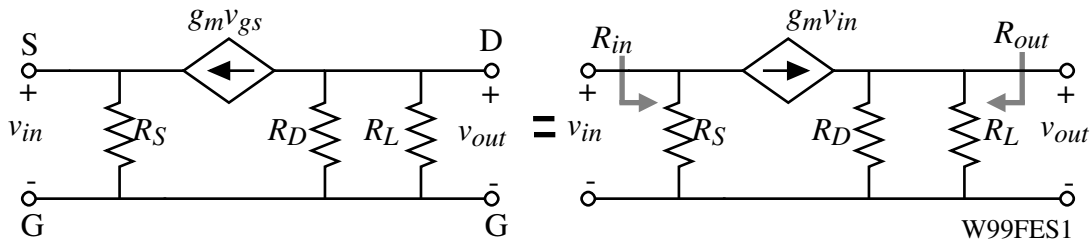
(a.) The key equations are  $I_D = K(V_{GS} - V_t)^2$  and  $5 = V_{GS} + I_D R_S$ . Substituting the first into the second gives,  $5 = V_{GS} + 0.1(V_{GS} - 1)^2 \rightarrow 5 = V_{GS} + 0.1V_{GS}^2 - 0.2V_{GS} + 0.1$

$$\text{or } V_{GS}^2 + 8V_{GS} - 49 = 0 \rightarrow V_{GS} = -4 \pm \frac{1}{2} \sqrt{64 + 4 \cdot 49} = -4 \pm 8.0623 = 4.0623\text{V}$$

$$I_D = 0.1\text{mA}(4.0623 - 1)^2 = 0.9377\text{mA} \text{ and } V_{DS} = 15\text{V} - I_D(11\text{k}\Omega) = 4.68\text{V}$$

$$\therefore \boxed{V_{GS} = 4.0623\text{V}, I_D = 0.9377\text{mA} \text{ and } V_{DS} = 4.68\text{V}}$$

(b.) The small signal model for this amplifier is shown below.  $r_{ds}$  has been neglected since  $\lambda$  was not given. The small signal parameter  $g_m$  is:  $g_m = 2\sqrt{KI_D} = 0.632\text{mA/V}$



The voltage gain is found as,

$$v_{out} = +g_m(R_D \parallel R_L)v_{in} \rightarrow \frac{v_{out}}{v_{in}} = g_m(R_D \parallel R_L) = 0.632 \cdot 5 = 3.16\text{V/V}$$

$$\therefore \boxed{\frac{v_{out}}{v_{in}} = 3.16\text{V/V}}$$

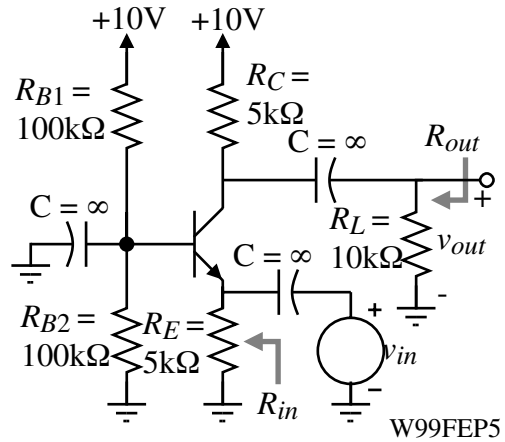
The output resistance is simply  $R_D \parallel R_S$  and is  $\boxed{R_{out} = 5\text{k}\Omega}$

The input resistance is found as,

$$i_{in} = \frac{v_{in}}{R_S} + g_m v_{in} \rightarrow R_{in} = \frac{v_{in}}{i_{in}} = \frac{1}{\frac{1}{R_S} + g_m} = \frac{1}{\frac{1}{5\text{k}\Omega} + 0.632} = 613\Omega$$

$$\therefore \boxed{R_{in} = 613\Omega}$$

2.) (a.) If  $\beta = 100$  and  $V_T = 25\text{mV}$  of the NPN transistor shown, solve for the dc value of collector current. (b.) If this collector current is 1mA (so if you found  $I_C$  incorrectly, you will not be penalized on the rest of the problem) find the small signal values of the input resistance,  $R_{in}$ , output resistance,  $R_{out}$ , and voltage gain,  $v_{out}/v_{in}$ .



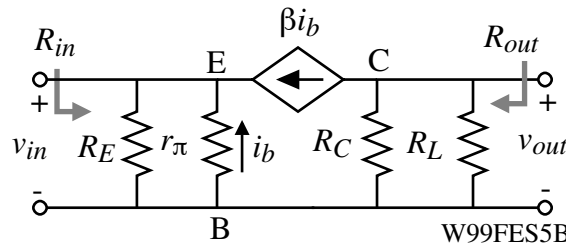
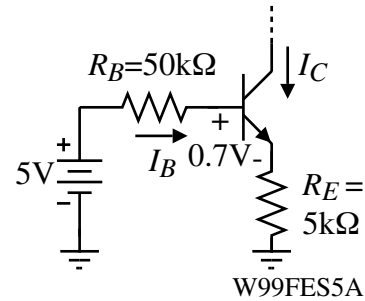
Solution

(a.) The circuit used to calculate the collector current is shown. The base current is

$$I_B = \frac{5 - 0.7}{50\text{k}\Omega + 101 \cdot 5\text{k}\Omega} = 7.75\mu\text{A}$$

$$\therefore I_C = 100 \cdot I_B = 0.775\text{mA}$$

(b.) The small signal parameters are  $g_m = \frac{1\text{mA}}{25\text{mV}} = 0.04\text{mS}$  and  $r_\pi = 101 \cdot 25\Omega = 2.525\text{k}\Omega$ . The small-signal model for the above circuit is given as



$$i_{in} = \frac{v_{in}}{R_E} - (1+\beta)i_b = \frac{v_{in}}{R_E} + \frac{(1+\beta)v_{in}}{r_\pi} \rightarrow R_{in} = \frac{v_{in}}{i_{in}} = R_E \parallel \frac{1}{g_m} = 24.87\Omega$$

$$v_{out} = -\beta i_b (R_C \parallel R_L) = -\beta (R_C \parallel R_L) \frac{-v_{in}}{r_\pi} \rightarrow \frac{v_{out}}{v_{in}} = \frac{100 \cdot 3.33}{2.525} = 132\text{V/V}$$

Finally,  $R_{out} = R_C \parallel R_L = 3.33\text{k}\Omega$

**14.26**

$$I_B = \frac{(5 - 0.7)V}{1M\Omega + (100 + 1)430k\Omega} = 96.8nA \quad | \quad I_C = 9.68\mu A \quad | \quad V_{CE} = 10 - 430000I_E = 5.80V$$

$$\text{Active region is correct.} \quad | \quad r_\pi = \frac{100(0.025V)}{9.68\mu A} = 258k\Omega \quad | \quad r_o = \frac{(60 + 5.80)V}{9.68\mu A} = 6.80M\Omega - \text{neglected}$$

In the ac model,  $R_1$  appears in parallel with  $r_\pi$ . The circuit appears to be using a transistor with  $r_\pi' = 500k\Omega \parallel r_\pi = 170k\Omega$  and  $\beta_o' = g_m r_\pi' = 40(9.68\mu A)170k\Omega = 65.8$

$$R_L = 500k\Omega \parallel 430k\Omega \parallel 500k\Omega = 158k\Omega \quad | \quad R_{in} = r_\pi' + (\beta_o' + 1)R_L = 170k\Omega + 66.8(158k\Omega) = 10.7M\Omega$$

$$A_v = \frac{(\beta_o' + 1)R_L}{r_\pi' + (\beta_o' + 1)R_L} \left( \frac{R_{in}}{R_I + R_{in}} \right) = \frac{66.8(158k\Omega)}{170k\Omega + 66.8(158k\Omega)} \left( \frac{10.7M\Omega}{500\Omega + 10.7M\Omega} \right) = +0.984$$

$$R_{out} = R_E \parallel R_2 \parallel \frac{R_I + r_\pi'}{\beta_o' + 1} = 430k\Omega \parallel 500k\Omega \parallel \frac{500\Omega + 170k\Omega}{66.8} = 2.52k\Omega$$

$$v_{be} = v_i \frac{r_\pi'}{R_I + r_\pi' + (\beta_o' + 1)R_L} = v_i \frac{170k\Omega}{500\Omega + 170k\Omega + 66.8(158k\Omega)} = 1.58 \times 10^{-2} v_i$$

$$v_i \leq \frac{0.005V}{1.58 \times 10^{-2}} = 0.315V$$

**14.38**

$$V_{GS} = -12 + (33k\Omega)I_D \quad | \quad V_{GS} = -12 + \frac{(3.3 \times 10^4)(2 \times 10^{-4})}{2} (V_{GS} + 1)^2$$

$$V_{GS} = -2.68V \quad \& \quad I_D = \frac{(2 \times 10^{-4})}{2} (V_{GS} + 1)^2 = 282\mu A$$

$$V_{DS} = -[24 - I_D(33k\Omega + 24k\Omega)] = -7.93V \quad - \text{Active region operation is correct.}$$

$$g_m = \sqrt{2(2 \times 10^{-4})(2.82 \times 10^{-4})} = 3.36 \times 10^{-4} S \quad | \quad R_I \parallel R_S = 0.5k\Omega \parallel 33k\Omega = 493\Omega$$

$$\text{Assume } \lambda = 0, r_o = \infty \quad | \quad R_L = R_D \parallel R_3 = 24k\Omega \parallel 100k\Omega = 19.4k\Omega$$

$$A_v = \frac{g_m R_L}{1 + g_m(R_I \parallel R_S)} \left( \frac{R_S}{R_I + R_S} \right) = \frac{0.336mS(19.4k\Omega)}{1 + 0.336mS(493\Omega)} \left( \frac{33k\Omega}{500\Omega + 33k\Omega} \right) = 5.51$$

$$A_i = 1 - \frac{R_S}{R_S + \frac{1}{g_m}} \left( \frac{R_D}{R_D + R_3} \right) = \frac{33k\Omega}{33k\Omega + 2.98k\Omega} \left( \frac{24k\Omega}{24k\Omega + 100k\Omega} \right) = 0.178$$

$$R_{in} = R_S \parallel \frac{1}{g_m} = 2.73k\Omega \quad | \quad R_{out} = R_D = 24k\Omega$$

$$|v_{gs}| = v_i \frac{R_{IN}}{R_I + R_{IN}} \leq 0.2|V_{GS} + 1| \quad | \quad v_i \frac{2.73k\Omega}{0.5k\Omega + 2.73k\Omega} \leq 0.2(1.68) \rightarrow v_s \leq 0.398V$$

**14.39**

$$I_B = \frac{(9 - 0.7)V}{100k\Omega + (50 + 1)82k\Omega} = 1.94\mu A \quad | \quad I_C = 96.9 \mu A$$

$$V_{CE} = 18 - 82000I_E - 39000I_C = 6.12 V \quad | \quad \text{Active region operation is correct.}$$

$$g_m = 40I_C = 3.88mS \quad | \quad r_\pi = \frac{\beta_o}{g_m} = 12.9k\Omega \quad | \quad r_o = \frac{(50 + 6.12)V}{96.9\mu A} = 579k\Omega \quad - \text{neglected}$$

$$R_I \parallel R_E = 0.5k\Omega \parallel 82k\Omega = 497\Omega \quad | \quad R_L = R_C \parallel R_3 = 39k\Omega \parallel 100k\Omega = 28.1k\Omega$$

$$A_v = \frac{g_m R_L}{1 + g_m (R_I \parallel R_E)} \left( \frac{R_E}{R_I + R_E} \right) = \frac{3.88mS(28.1k\Omega)}{1 + 3.88mS(497\Omega)} \left( \frac{82k\Omega}{500\Omega + 82k\Omega} \right) = 37.0$$

$$R_{in} = 82k\Omega \parallel \frac{r_\pi}{\beta_o + 1} = 252\Omega \quad | \quad A_i = A_v \frac{R_I + R_{in}}{R_3} = 37.0 \frac{500\Omega + 252\Omega}{100k\Omega} = 0.278$$

$$R_{out} = R_C = 39.0 k\Omega \quad | \quad v_{eb} = v_i \frac{R_{in}}{R_I + R_{in}} \leq 5.00mV \quad | \quad 0.335v_i \leq 5.00mV \quad | \quad v_i \leq 14.9 mV$$