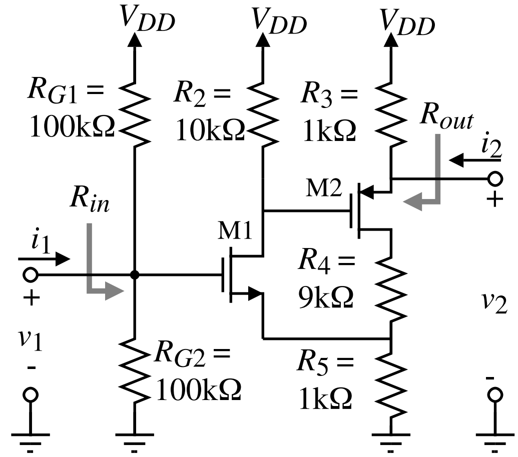
Homework Assignment No. 13 - Solutions

Problem 1

Use the method of feedback analysis to find v_2/v_1, $R_{in} = v_1/i_1$, and $R_{out} = v_2/i_2$. Assume that all transistors are matched and that $g_{m1} = g_{m2} = 1\text{mS}$. Neglect r_{ds}.

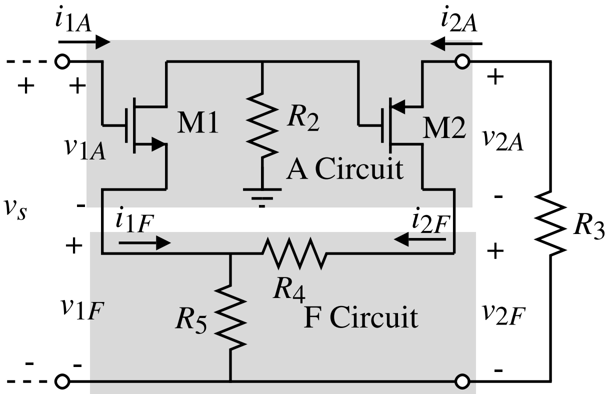


F02H12P4

Solution

The topology is series-series.

The circuit is redrawn below in order to help identify the various terms for the analysis:



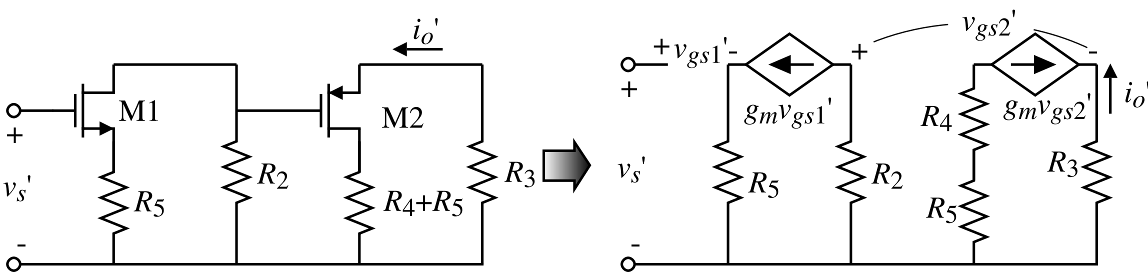
F02H12S4A

$$z_{11F} = \frac{v_{1F}}{i_{1F}}\bigg|_{i_{2F}=0} = R_5 = 1\text{k}\Omega$$

$$z_{22F} = \frac{v_{2F}}{i_{2F}}\bigg|_{i_{1F}=0} = R_4 + R_5 = 10\text{k}\Omega$$

$$z_{12F} = \beta = \frac{v_{1F}}{i_{2F}}\bigg|_{i_{1F}=0} = R_5 = 1\text{k}\Omega$$

Calculation of the A circuit:



F02H12S4B

$$A = \frac{i_o'}{v_s'} = \left(\frac{i_o'}{v_{gs2}'}\right)\left(\frac{v_{gs2}'}{v_{gs1}'}\right)\left(\frac{v_{gs2}'}{v_s'}\right) = (-g_m)\left(\frac{-g_m R_2}{1+g_m R_3}\right)\left(\frac{1}{1+g_m R_5}\right) = (10^{-3})(-5)(-0.5) = 2.5\text{ mS}$$

$$\therefore \frac{i_o}{v_s} = \frac{A}{1+A\beta} = \frac{2.5\text{mS}}{1+2.5} = 0.714\text{mS}$$

Since, $z_{11A} = \infty$, then $R_{in} = 50\text{k}\Omega \| \infty = \underline{50\text{k}\Omega}$

$$\frac{v_2}{v_1} = \frac{-i_o R_3}{v_s} = \frac{-i_o}{v_s} R_3 = -0.714\text{mS}(1\text{k}\Omega) = \underline{-0.714\text{ V/V}}$$

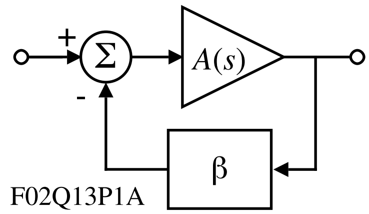
$$R_o = (z_{22T} + R_3)(1+A\beta) = \left(\frac{1}{g_m} + R_3\right)(1+A\beta) = 2\text{k}\Omega(3.5) = 7\text{k}\Omega$$

However, the output resistance specified by the problem, R_{out}, is found as

$$R_{out} = (R_o - R_3) \| R_3 = 6\text{k}\Omega \| 1\text{k}\Omega = \underline{857\Omega}$$

<u>Problem 2</u> - The amplifier in the feedback circuit shown has a transfer function of

$$A(s) = \frac{100}{\frac{s}{10^5} + 1}$$



F02Q13P1A

What value of β will increase the upper –3db frequency by a factor of 10 for the closed loop gain? What is the closed loop, low frequency gain?

<u>Solution</u>

$$A_F = \frac{A}{1+A\beta} = \frac{1}{\frac{1}{A}+\beta} = \frac{1}{\frac{s/10^5+1}{100}+\beta} = \frac{100}{\frac{s}{10^5}+1+100\beta} = \left(\frac{100}{1+100\beta}\right)\frac{1}{\frac{s}{10^5(1+100\beta)}+1}$$

$$\therefore\ 10^5(1+100\beta) = 10^6 \quad\rightarrow\quad 1+100\beta = 10 \quad\rightarrow\quad \underline{\underline{\beta = 9/100 = 0.09}}$$

The closed-loop, low frequency gain is,

$$A_F(0) = \frac{100}{1+100\beta} = \frac{100}{1+9} = 10 \quad\rightarrow\quad \underline{\underline{A_F(0) = 10}}$$

<u>3.) Problem 18.40 of the text.</u>

$$T = \frac{v_o}{v_x} = g_{m2}\left(r_{o2}\|r_{o4}\right)\frac{(\beta_o+1)R}{\left(r_{o2}\|r_{o4}\right)+r_{\pi 3}+(\beta_o+1)R} \quad | \quad g_{m1} = 40\left(10^{-4}\right) = 4.00mS$$

$$r_{o2} = \frac{50+1.4}{10^{-4}} = 514k\Omega \quad | \quad r_{o4} = \frac{50+11.3}{10^{-4}} = 613k\Omega \quad | \quad r_{\pi 3} = \frac{100(0.025)}{(12V/10k\Omega)} = 2.08k\Omega$$

$$T = \left(4x10^{-3}\right)(280k\Omega)\frac{(101)10k\Omega}{280k\Omega+2.08k\Omega+101(10k\Omega)} = 876 \quad (58.9\ dB)$$

<u>4.) Problem 18.59 of the text.</u>

$$(a)\ \ A(s) = \frac{\frac{2x10^{14}\pi^2}{\left(2\pi x10^3\right)\left(2\pi x10^5\right)}}{\left(1+\frac{s}{2\pi x10^3}\right)\left(1+\frac{s}{2\pi x10^5}\right)} = \frac{5x10^5}{\left(1+\frac{s}{2\pi x10^3}\right)\left(1+\frac{s}{2\pi x10^5}\right)}$$

$A(s)$ represents a low - pass amplifier with two widely - spaced poles

Open – loop: $A_o = 5x10^5 = 114dB \quad | \quad f_L = 0 \quad | \quad f_H \cong f_1 = 1000\ Hz$

$(b)A$ common mistake would be the following:

Closed – loop: $f_H = 1000Hz\left[1+5x10^5(0.01)\right] = 5MHz$

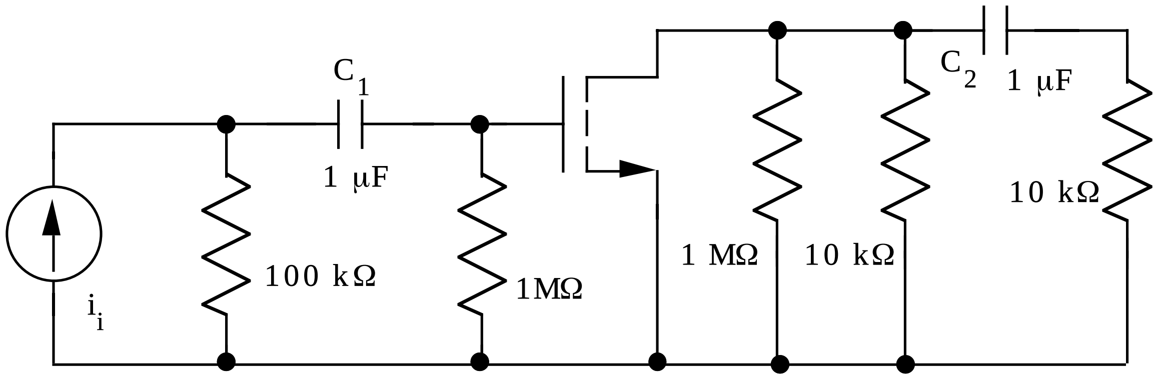
Oops! - This exceeds $f_2 = 100\ kHz$! This is a two - pole low - pass amplifier.

$$A_v(s) = \frac{\frac{2x10^{14}\pi^2}{\left(s+2\pi x10^3\right)\left(s+2\pi x10^5\right)}}{1+\frac{2x10^{14}\pi^2}{\left(s+2\pi x10^3\right)\left(s+2\pi x10^5\right)}(0.01)} = \frac{2x10^{14}\pi^2}{s^2+1.01\left(2\pi x10^5\right)s+2x10^{12}\pi^2}$$

Using dominant - root factorization: $f_1 = 101\ kHz, \quad f_2 = 4.95\ MHz$

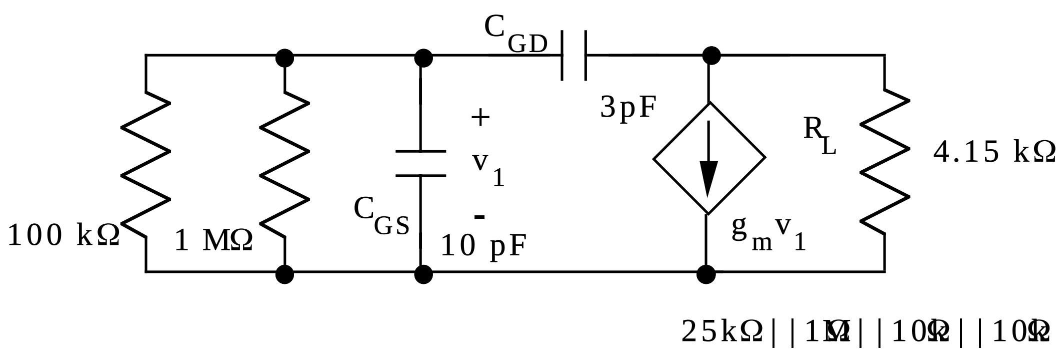
So the closed - loop values are $f_H = 101\ kHz$ *and* $f_L = 0$.

5.) Problem 18.62 (18.30) of the text.



$$\omega_1 = \frac{1}{10^{-6}(100k\Omega + 1M\Omega)} = 0.909\frac{rad}{s} \quad | \quad \omega_2 = \frac{1}{10^{-6}(10k\Omega + 25k\Omega \| 10k\Omega \| 1M\Omega)} = 58.5\frac{rad}{s}$$

Separate, widely-spaced, poles $\rightarrow f_L^A = f_2 = \frac{58.5}{2\pi} = 9.31\ Hz$



$$\omega_H^A = \frac{1}{r_{\pi o}C_T} = \frac{1}{(100k\Omega \| 1M\Omega)\left[10pF + 3pF\left(1 + 2mS(4.15k\Omega) + \frac{4.15k\Omega}{100k\Omega \| 1M\Omega}\right)\right]}$$

$$f_H^A = \frac{1}{2\pi}\frac{1}{(90.9k\Omega)(38.0pF)} = 46.1\ kHz$$

$$v_{gs} = i_s(100k\Omega \| 1M\Omega) = (90.9k\Omega)i_s \quad | \quad v_o = -(2x10^{-3})v_{gs}(25k\Omega \| 10k\Omega \| 10k\Omega \| 1M\Omega)$$

$$A = \frac{v_o}{i_s} = -(2mS)(4.15k\Omega)(90.9k\Omega) = -7.55x10^5\Omega \quad | \quad y_{12}^F = -10^{-5}S$$

$$1 + A\beta = 1 + (-7.55x10^5\Omega)(-10^{-6}S) = 1.76$$

$$f_L = \frac{9.31}{1.76} = 5.29\ Hz \qquad f_H = 46.1kHz(1.76) = 81.0\ kHz$$