Diode Small-Signal Model – Cont'd

$$i_D = i_d + I_D = I_S\left[\exp\left(\frac{v_D}{V_T}\right) - 1\right] = I_S\left[\exp\left(\frac{v_d + V_D}{V_T}\right) - 1\right]$$

$$i_d + I_D = I_S\left[\exp\left(\frac{v_d}{V_T}\right)\exp\left(\frac{V_D}{V_T}\right) - 1\right]$$

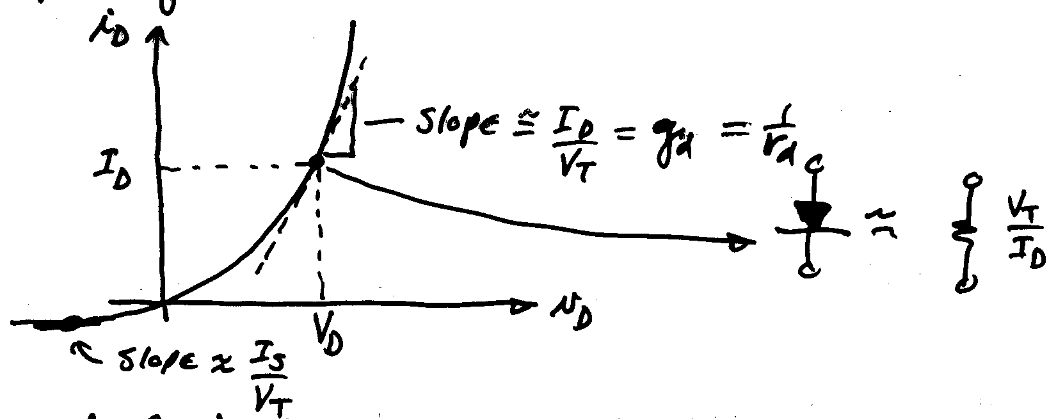
If $v_d < V_T$, then $\exp\left(\frac{v_d}{V_T}\right) \simeq 1 + \frac{v_d}{V_T} + \frac{1}{2}\left(\frac{v_d}{V_T}\right)^2 + \cdots$

$$\therefore \quad i_d + I_D \simeq I_S\left[\exp\left(\frac{V_D}{V_T}\right)\right]\left[1 + \frac{v_d}{V_T}\right] - I_S$$

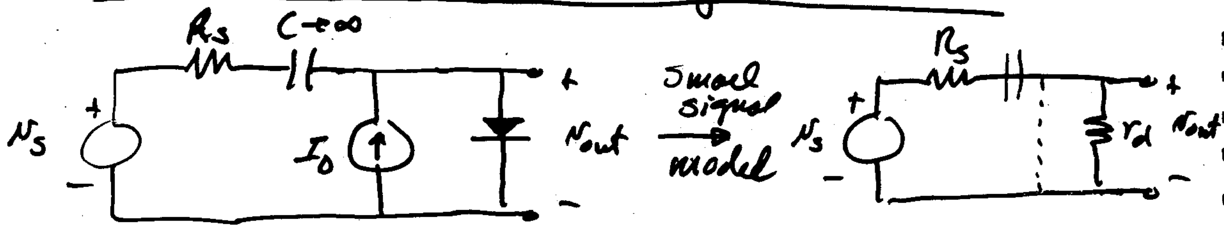
$$\simeq \underbrace{I_S\left[\exp\left(\frac{V_D}{V_T}\right) - 1\right]}_{I_D} + \underbrace{\left[I_S \exp\left(\frac{V_D}{V_T}\right)\right]}_{I_D + I_S}\left(\frac{v_d}{V_T}\right)$$

$$\therefore \quad i_d \simeq \left(\frac{I_D + I_S}{V_T}\right) v_d \simeq \left(\frac{I_D}{V_T}\right) v_d \quad \text{if } V_D > 0$$

Graphically –



Current-Controlled Small-Signal Attenuator



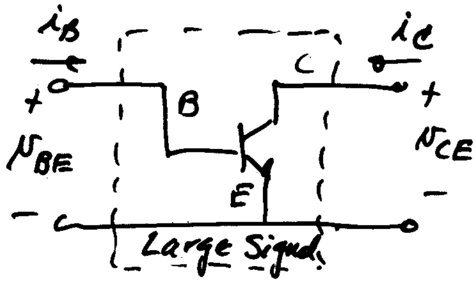
$$\frac{v_{out}}{v_S} = \frac{r_d}{r_d + R_S} = \frac{1}{1 + \frac{R_S}{r_d}} = \frac{1}{1 + \frac{R_S I_D}{V_T}}$$

SMALL-SIGNAL TRANSISTOR MODELS

Approach:

1.) $i_A = i_a + I_A \rightarrow i_a = i_A - I_A = \Delta i_A = d i_A$

2.) CE BJT

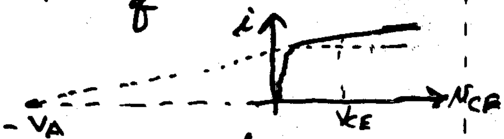


Forward active (BE is fwd. biased and CE is reverse biased)

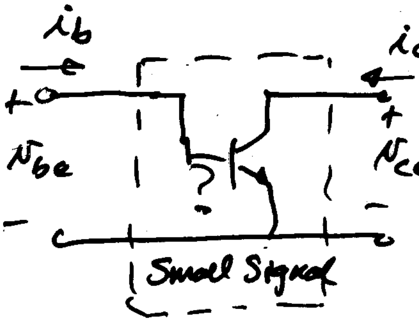
$$i_C = I_S \left[\exp\left(\frac{v_{BE}}{V_T}\right)\right]\left[1 + \frac{v_{CE}}{V_A}\right]$$

$$i_B = \frac{I_S}{\beta_F} \exp\left(\frac{v_{BE}}{V_T}\right)$$

$$V_T = \frac{kT}{q} \simeq 26\,mV$$



"Linearize"



$$i_c = k_1 v_{be} + k_2 v_{ce}$$

$$\rightarrow \quad i_b = k_3 v_{be}$$

What are k_1, k_2 & k_3?

Define:

$$k_1 = \left.\frac{i_c}{v_{be}}\right|_{v_{ce}=0} = \left.\frac{\partial i_C}{\partial v_{BE}}\right|_{Q \Rightarrow v_{ce}=0} = \frac{I_C}{V_T} = g_m$$

$$k_2 = \left.\frac{i_c}{v_{ce}}\right|_{v_{be}=0} = \left.\frac{\partial i_C}{\partial v_{CE}}\right|_{Q} = \frac{I_C}{V_A + V_{CE}} \simeq \frac{I_C}{V_A} = g_o$$

$$k_3 = \left.\frac{i_b}{v_{be}}\right|_{Q} = \left.\frac{\partial i_B}{\partial v_{BE}}\right|_{Q} = \frac{I_C}{\beta_F V_T} = g_\pi = \frac{1}{r_\pi}$$

$\therefore$ $i_c = g_m v_{be} + g_o v_{ce}$

$i_b = g_\pi v_{be}$