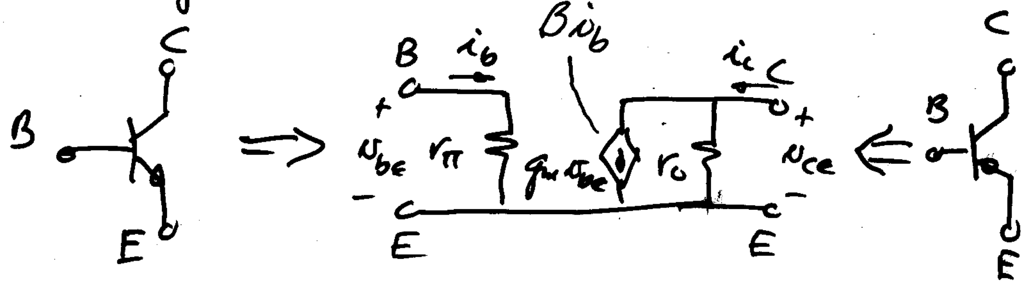
Small-signal analysis of transistor amplifiers

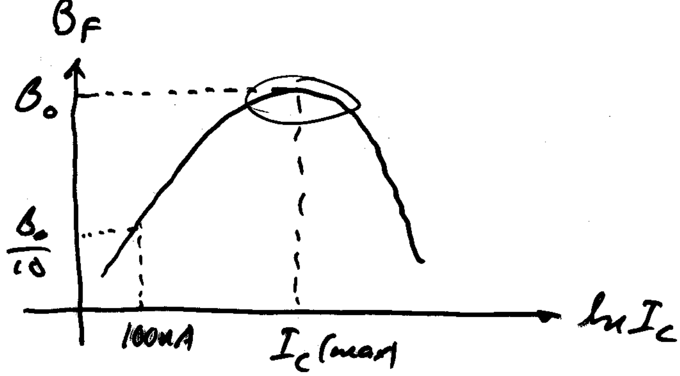
Small-signal -



Small-signal model parameter – (r_π, g_m, β, r_o)

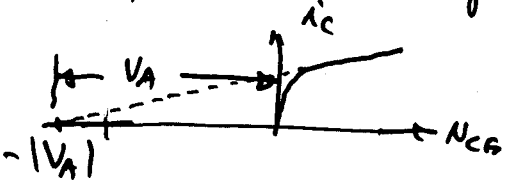
$(\beta_F = g_m r_\pi)$ $\beta_o = g_m r_\pi$

- $\beta_F = f(I_c)$?



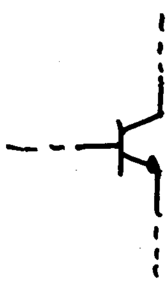
- $g_m = \frac{I_C}{V_T} = \frac{q I_C}{kT}$, $g_\pi = \frac{I_C}{\beta_o V_T} = \frac{g_m}{\beta_o} \rightarrow r_\pi = \frac{\beta_o}{g_m}$

$\frac{1}{r_o} = g_o = \frac{I_C}{V_A + V_{CE}} \simeq \frac{I_C}{|V_A|}$



- A s.s. model is linear $\Rightarrow v_{be} \ll V_T \simeq 26\,mV$

Example

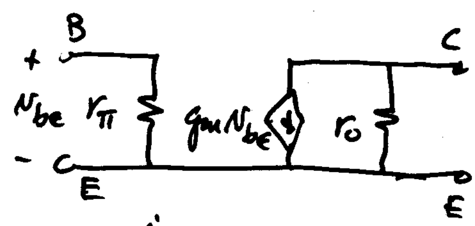


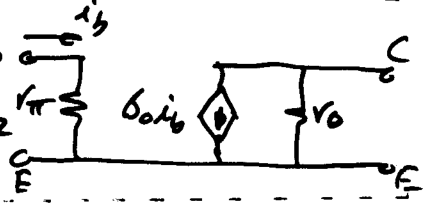
$I_c = 1mA$
$\beta_o = 100$
$V_A = 100V$

$g_m = \frac{I_c}{V_T} = \frac{1}{40} S$

$r_\pi = \frac{\beta_o}{g_m} = 4k\Omega$

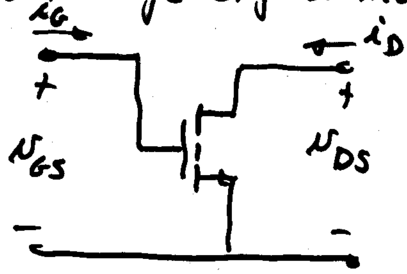
$r_o = \frac{100V}{1mA} = 100k\Omega$





MOSFET Small-Signal Models

1.) Large signal model



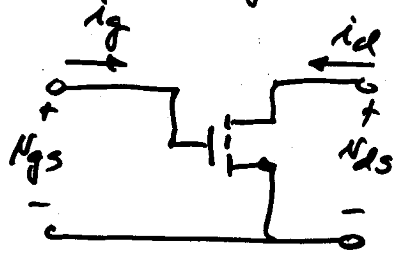
$\xrightarrow[\text{Model}]{\text{Mathematical}}$

Assumes saturation

$$i_D = \frac{K_n'}{2}(v_{GS} - V_{TN})^2(1+\lambda v_{DS})$$

$$i_G = 0$$

2.) Small-signal model



$\xrightarrow[\text{Model}]{\text{Mathematical}}$

$$i_d = k_1 v_{gs} + k_2 v_{ds}$$

$$i_g = 0$$

$$k_1 = \frac{i_d}{v_{gs}}\bigg|_{v_{ds}=0} = \frac{\Delta i_D}{\Delta v_{GS}}\bigg|_{V_{DS}} = \frac{\partial i_D}{\partial v_{GS}}\bigg|_Q = K_n'(V_{GS}-V_{TN})(1+\lambda V_{DS})$$

$$\approx K_n'(V_{GS}-V_{TN})$$

where $\lambda V_{DS} << 1$

$$k_1 = g_m = K_n'(V_{GS}-V_{TN})$$

$$I_D = \frac{K_n'}{2}(V_{GS}-V_{TN})^2 \rightarrow V_{GS}-V_{TN} = \sqrt{\frac{2I_D}{K_n'}}$$

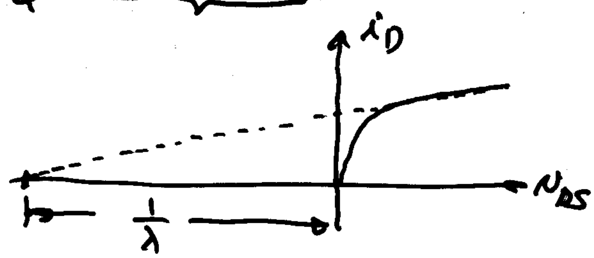
$$g_m = K_n'\sqrt{\frac{2I_D}{K_n'}} = \sqrt{2K_n' I_D}$$

$$k_2 = \frac{i_d}{v_{ds}}\bigg|_{v_{gs}=0} = \frac{\partial i_D}{\partial v_{DS}}\bigg|_Q = \underbrace{\frac{K_n'}{2}(V_{GS}-V_{TN})^2}\lambda \approx I_D\lambda$$

$$g_{ds} = g_o = k_2 = \frac{I_D\lambda}{1+\lambda V_{DS}} \approx I_D\lambda$$

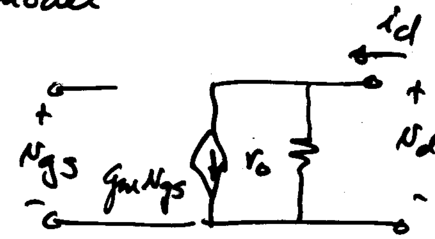
Amplification factor

$$\mu \equiv g_m r_o$$



3.) Schematic small-signal model

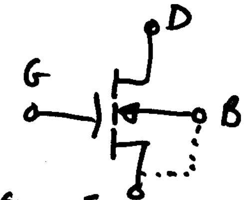
$$\left.\begin{array}{l} i_d = g_m v_{gs} + g_o v_{ds} \\ i_g = 0 \end{array}\right\}$$



Good for both NMOS and PMOS

4.) Body effect

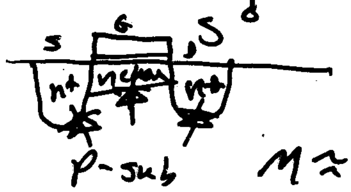
$$V_{TN} = V_{TO} + \gamma\sqrt{|v_{SB}| + 2\phi_F} - \gamma\sqrt{2\phi_F} = f(v_{SB})$$



$$i_d = g_m v_{gs} + g_o v_{ds} + k_s v_{sb}$$

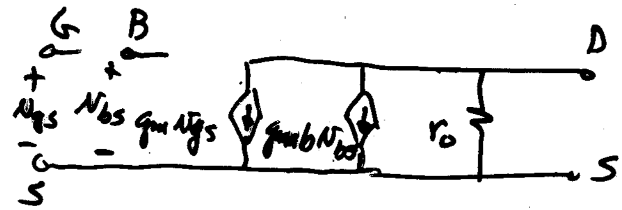
$$\left.\frac{\partial i_D}{\partial v_{SB}}\right|_Q = g_{mb} = \frac{\gamma\, k_n (V_{GS} - V_{TO})}{2\sqrt{V_{SB} + 2\phi_F}} = M\, g_m$$

g_m



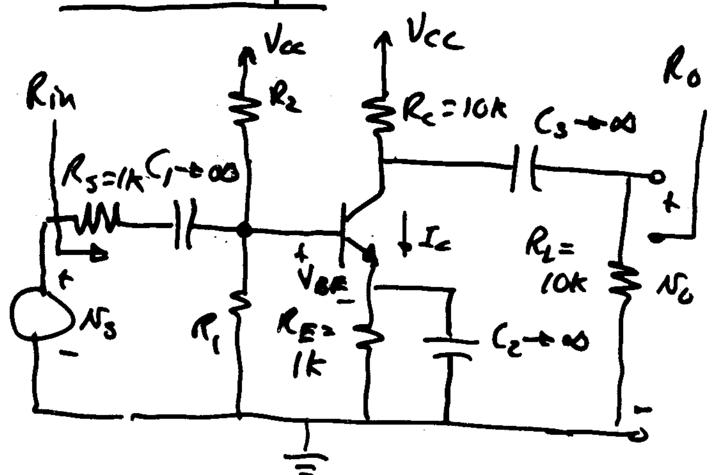
$M \approx \frac{1}{10}$

$\therefore$



$g_m v_{ab} \Rightarrow$ the arrow ALWAYS points to the 2nd subscript.

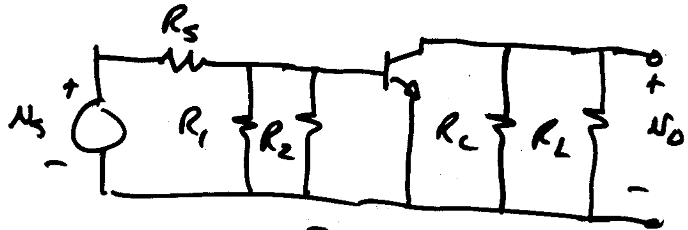
BJT Example



Find the small-signal input resistance, R_{in}, the output resistance, R_{out}, and $\frac{v_o}{v_s}$ where $I_c = 1mA$ $\beta_o = 100$ and $V_A = 100V$.

$R_1 \| R_2 = R_B = 20k\Omega$

SS eq. ckt. ($C \to \infty \to$ short $V_{cc} = 0$)



To be continued