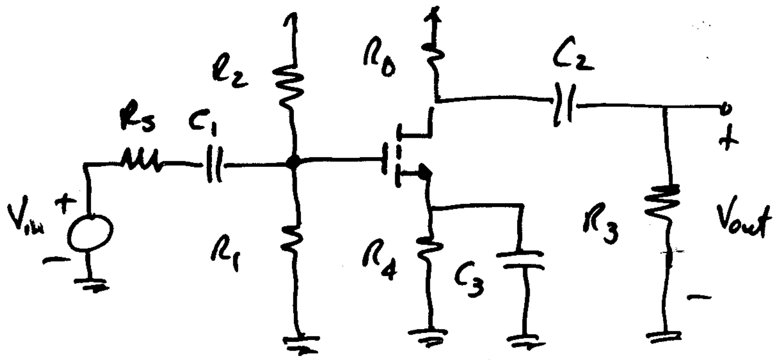
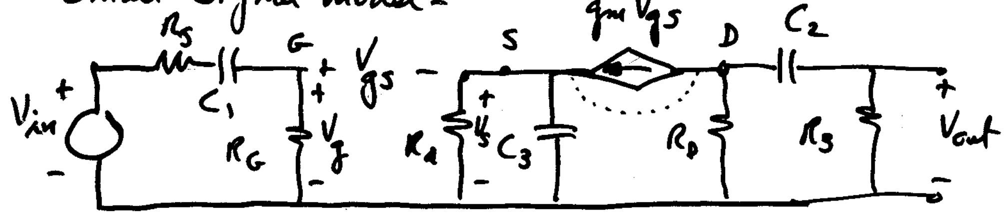
Warm-up Comments

MOSFET Example –



Small-signal model –



$R_G = R_1 || R_2$

1.) Direct analysis

$$\frac{V_{out}(s)}{V_{in}(s)} = \left(\frac{V_{out}}{V_{gs}}\right)\left(\frac{V_{gs}}{V_g}\right)\left(\frac{V_g}{V_{in}}\right)$$

$\frac{V_{gs}}{V_g}$:

$$V_{gs} = V_g - V_s = V_g - g_m V_{gs} Z_s$$

$$V_{gs}(1 + g_m Z_s) = V_g \rightarrow \frac{V_{gs}}{V_g} = \frac{1}{1+g_m Z_s}$$

$$\frac{V_{gs}}{V_g} = \frac{1}{1+g_m\left(\frac{R_4 \, 1/sC_3}{R_4 + \frac{1}{sC_3}}\right)} = \frac{1}{1+g_m\left(\frac{R_4}{sC_3R_4+1}\right)} = \frac{sC_3R_4+1}{sC_3R_4+1+g_mR_4}$$

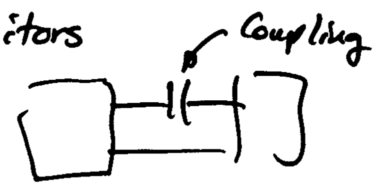
$$= \left(\frac{1}{1+g_mR_4}\right)\left[\frac{sC_3R_4+1}{\frac{sC_3R_4}{1+g_mR_4}+1}\right]$$

$$\text{zero} = -\frac{1}{R_4C_3} = z_2$$

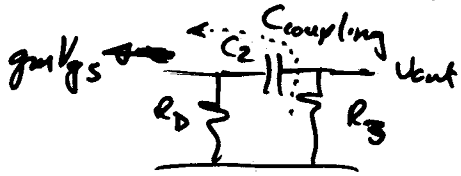
$$\text{Pole} = -\frac{1+g_mR_4}{C_3R_4} = P_2$$

$$\frac{V_{out}(s)}{V_{in}(s)} = MBG \times F_L(s) = MBG\left(\frac{s}{s+P_1}\right)\left(\frac{s+z_2}{s+P_2}\right)\left(\frac{s}{s+P_3}\right)$$

Types of Low Frequency Capacitors



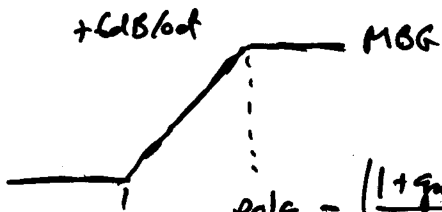
1.) Coupling $\longrightarrow \dfrac{s}{s+p}$

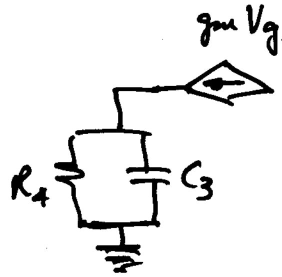


$$V_{out} = -g_m V_{gs}\left(\frac{R_D}{R_D+R_3+\frac{1}{sC_3}}\right)R_3$$

$$\frac{V_{out}}{V_{gs}} = \frac{-g_m R_D R_3}{R_D+R_3+\frac{1}{sC_3}} = \frac{-g_m R_D R_3 \; s}{s(R_D+R_3)+\frac{1}{C_3}} = \left(\frac{-g_m R_D R_3}{R_D+R_3}\right)\left(\frac{s}{s+\frac{1}{C_3(R_D+R_3)}}\right)$$

2.) Bypass $\longrightarrow \dfrac{s+z}{s+p}$





$$\text{pole} = \left|\frac{1+g_m R_4}{R_4 C_3}\right|$$

$$\text{zero} = \frac{1}{R_4 C_3}$$

Back to the MOSFET Prob.

$$\boxed{\omega_L = \omega_{3dB}\text{ (lower)}}$$

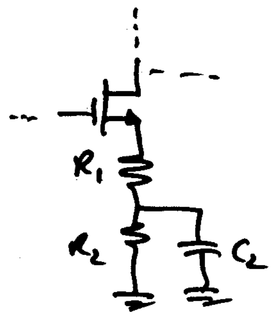
1.) Direct analysis
   a.) Bode plot
   b.) Dominant
   c.) $\sqrt{\sum p_i^2 - 2\sum z_i^2}$

2.) Short-circuit time constant
   a.) If the C's are independent then the $\dfrac{1}{R_{sci}C_i} = |p_i|$
   b.) $\omega_L \cong \sum_i \dfrac{1}{R_{sci}C_i}$

Comment: One approach good for quizzes is to use the S.C. time constant approach (when caps are indep.) to find poles and zeros.

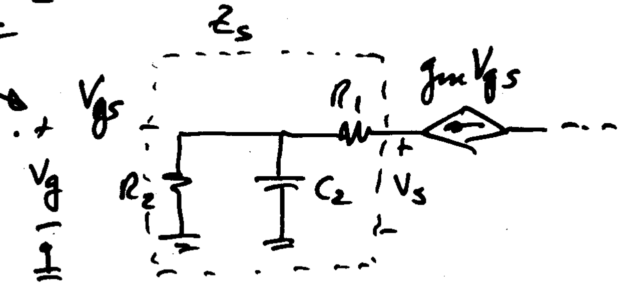
Bypass cap.: $z = \dfrac{-1}{R_i C_i}$ $\quad p = \dfrac{-1}{R_{eqi} C_i}$

R_i = bypassed resistance $\quad R_{eqi}$ = Resistance seen by C_i



$$z = \frac{-1}{R_2 C_2}$$

$$p = -\frac{1}{C_2\left[R_2 \,\|\, \left(R_1 + \frac{1}{g_m}\right)\right]}$$



$$\frac{V_{gs}}{V_g} = \frac{1}{1 + g_m Z_s} = \frac{1}{1 + g_m\left[R_1 + \frac{R_2}{sR_2C_2 + 1}\right]}$$

$$= \frac{1}{1 + g_m R_1 + \frac{g_m R_2}{sR_2C_2 + 1}} = \frac{sR_2C_2 + 1}{(1 + g_m R_1)sR_2C_2 + (1 + g_m R_1) + g_m R_2}$$

$$= \left(\frac{1}{1 + g_m R_1}\right)\left(\frac{sR_2C_2 + 1}{sR_2C_2 + 1 + \frac{g_m R_2}{1 + g_m R_1}}\right)$$

$p = ?$ $\qquad -\left(1 + \frac{g_m R_2}{1 + g_m R_1}\right)\frac{1}{R_2 C_2}$

$$p = -\frac{1}{R_2 C_2}\left(\frac{1 + g_m R_1 + g_m R_2}{1 + g_m R_1}\right) = \frac{-1}{R_2 C_2}\left[\frac{\frac{1}{g_m} + R_1 + R_2}{\frac{1}{g_m} + R_1}\right]$$

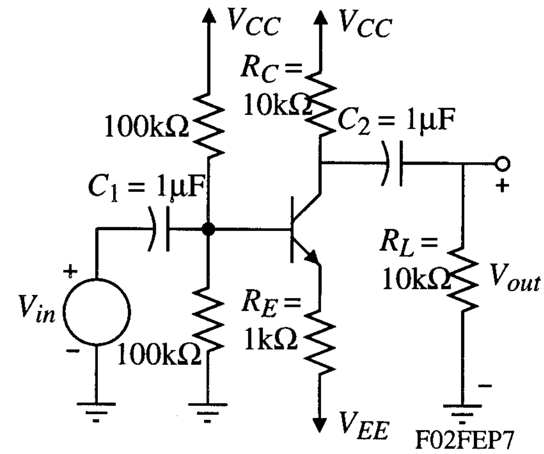
Low Frequency Example (Not covered in class)

A BJT amplifier is shown. Assume that the BJT has the small signal parameters of g_m = 50mA/V, $r_\pi = 2\text{k}\Omega$, and $r_o = \infty$.

a.) Find the midband voltage gain of this amplifier, V_{out}/V_{in}.

b.) Find the numerical value of all poles and zeros of the low frequency response.

c.) Find the value of the lower -3dB frequency, f_L, in Hz.



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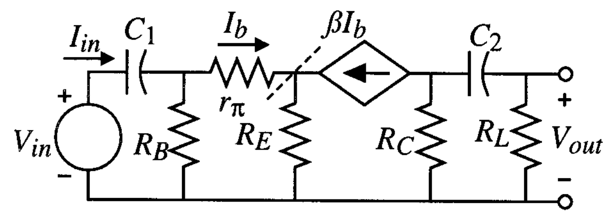
Solution

The low-frequency, small signal model for this problem is shown where R_B = 50kΩ.



The algebraic approach to this problem is:

$$\frac{V_{out}}{V_{in}} = \left(\frac{V_{out}}{I_b}\right)\left(\frac{I_b}{I_{in}}\right)\left(\frac{I_{in}}{V_{in}}\right) = \left(\frac{-\beta R_L R_C}{R_C+R_L+\frac{1}{sC_2}}\right)\left(\frac{R_B}{R_B+r_\pi+(1+\beta)R_E}\right)\left(\frac{1}{\frac{1}{sC_1}+R_B\|[r_\pi+(1+\beta)R_E]}\right)$$

$$= \left(\frac{-\beta R_L R_C}{(R_C+R_L)[r_\pi+(1+\beta)R_E]}\right)\left(\frac{s}{s+\frac{1}{C_2(R_C+R_L)}}\right)\left(\frac{s}{s+\frac{1}{C_1(R_B\|[r_\pi+(1+\beta)R_E])}}\right)$$

$$= \frac{-100\cdot 10\text{K}\cdot 10\text{K}}{20\text{K}\cdot 103\text{K}}\left(\frac{s}{s+50}\right)\left(\frac{s}{s+29.7}\right) = -4.854\left(\frac{s}{s+50}\right)\left(\frac{s}{s+29.7}\right)$$

The midband gain is $\boxed{MBG = 4.854\text{ V/V}}$

$\therefore \quad \omega_L \approx \sqrt{(29.7)^2 + (50)^2} = 58.2$ rads/sec. $\rightarrow$ $\boxed{f_L = 9.26\text{Hz}}$

The poles and zeros are,

$\boxed{\text{Two zeros at s = 0, a pole at s = -29.7 rads/sec. and a pole at s = -50 rads/sec.}}$