Assume that Q1 and Q2 and the resistors $R_C$ and $R_E$ of the differential amplifier shown are matched. If $\beta_F = 100$, $V_t = 25\text{mV}$, and $V_A = \infty$, find (a.) Find the numerical value of $v_{C1}/v_{id}$ where $v_{id} = v_1 - v_2$. (Hint: assume node $x$ is at ac ground.) (b.) Find the numerical value of the differential input resistance defined as, 

$$R_{id} = \frac{v_{id}}{i_{in}}$$

when $v_1 = 0.5v_{id}$ and $v_2 = -0.5v_{id}$ (c.) Find the numerical value of $v_{C1}/v_{cm}$ where $v_{cm} = v_1 = v_2$.

**Solution**

a.) Simplifying the circuit for differential mode analysis gives the model shown.

\[
g_m = \frac{I_{C1}}{V_t} = \frac{0.5\text{mA}}{25\text{mV}} = 20\text{mS}
\]

\[
r_\pi = \frac{\beta}{g_m} = \frac{100}{20\text{mS}} = 5k\Omega
\]

$$R_{i1} = r_{\pi} + (1+\beta)R_E = 5k\Omega + (101)1k\Omega = 106k\Omega$$

\[
\frac{v_{C1}}{v_{id}} = \frac{1}{2} \frac{v_{C1}}{i_{b1}} = \frac{1}{2} \left( \frac{i_{b1}}{v_{id}/2} \right) = \frac{1}{2} \left( \frac{v_{C1}}{i_{b1}} \right) \left( \frac{1}{R_{i1}} \right) = \frac{-\beta R_C}{2R_{i1}} = \frac{-100 \cdot 10k\Omega}{2 \cdot 106k\Omega} = 4.717 \text{ V/V}
\]

b.) $R_{id} = \frac{v_{id}}{i_{in}} = \frac{2}{2i_{b1}} = \frac{v_{id}/2}{i_{b1}} = 2R_{id} = 2(106k\Omega) = 212k\Omega$

c.) The common mode gain is zero because the resistance of the $I_{EE}$ current source ($R_{EE}$) is infinite. This is illustrated in the circuit shown for the common mode analysis.