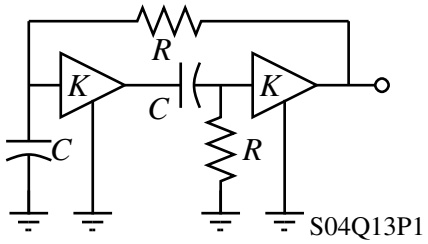
QUIZ NO. 13 - SOLUTION

(Average score = 6.8/10 of those taking the quiz)

This quiz deals with finding the open-loop gain and it application. The following questions are independent of each other.

(a.) Find the loop gain of the feedback circuit shown, $T(s)$, if the amplifier is an ideal voltage amplifier with a gain of K.

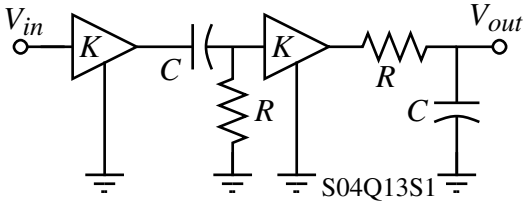


(b.) If $T(s) = \dfrac{sKRC}{s^2R^2C^2 + 2RCs + 1}$, find f_{osc} and the value of K necessary for oscillation.

(c.) If $T(s)$ has the following properties: $T(0) = 10$ and two poles at $s = -100$, what is the phase margin of this feedback circuit?

Solution

(a.) Opening the loop gives,



$$T(s) = \frac{V_{out}}{V_{in}} = \left(\frac{KR}{R+(1/sC)}\right)\left(\frac{K(1/sC)}{R+(1/sC)}\right)$$

$$= \frac{K^2RCs}{(sRC+1)(sRC+1)} = \frac{K^2RCs}{s^2R^2C^2+s2RC+1}$$

$$\therefore \quad \boxed{T(s) = \frac{K^2RCs}{s^2R^2C^2+s2RC+1}}$$

(b.) $T(s) = \dfrac{sKRC}{s^2R^2C^2 + 2RCs + 1} \quad \rightarrow \quad T(j\omega) = \dfrac{j\omega KRC}{-\omega^2R^2C^2 + j\omega 2RC + 1}$

or $T(j\omega) = \dfrac{j\omega KRC}{1-\omega^2R^2C^2 + j\omega 2RC} = 1+j0 \quad \Rightarrow \quad \boxed{f_{osc} = \dfrac{1}{2\pi RC}}$ and $\boxed{K = 2}$

(c.) We could plot a Bode plot and estimate the phase margin or do the following:

$$|T(j\omega)| = \frac{10}{1+\left(\dfrac{\omega}{100}\right)^2}$$

Find the unity-gain frequency from, $\dfrac{10}{1+\left(\dfrac{\omega_c}{100}\right)^2} = 1 \quad \rightarrow \; \omega_c^2 = 100^2\,(10-1) = 300^2$

or $\omega_c = 300$ rads/sec. The phase margin can be expressed as,

$$PM = 180° - 2\tan^{-1}\left(\frac{300}{100}\right) = 180° - 2(71.56°) = \underline{36.87°}$$