## EXAMINATION NO. 1 - SOLUTIONS

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(\text { Average score }=78.4 / 100)
$$

## Problem 1-( 25 points)

From the voltage transfer function curve shown, numerically identify, $V_{O H}, V_{O L}, V_{I L}, V_{I H}$, and $V_{S}$. From these values, find the value of $N M_{H}$ and $N M_{L}$.


From the curve we get the following numerical values.
$V_{O H}=\underline{\underline{1.8 \mathrm{~V}}}, V_{O L}=\underline{\underline{0.13 \mathrm{~V}}}, V_{I L}=\underline{\underline{0.6 \mathrm{~V}}}, V_{I H}=\underline{\underline{1.08 \mathrm{~V}}}$, and $V_{S}=\underline{\underline{0.92 \mathrm{~V}}}$
The noise margins are,

$$
N M_{H}=V_{O H}-V_{I H}=1.80-1.08=\underline{\underline{0.72 \mathrm{~V}}}
$$

and

$$
N M_{H}=V_{I L}-V_{O L}=0.60-0.13=\underline{\underline{0.47 \mathrm{~V}}}
$$

## Problem 2-( 25 points)

Solve for the dc value of the drain current, $I_{D S}$, for the NMOS transistor shown assuming $0.18 \mu \mathrm{~m}$ CMOS technology. The $W$ and $L$ for this transistor are given in Problem 3.

## Solution

Check for saturation.

$$
\begin{aligned}
& V_{D S}(\mathrm{sat})=\frac{\left(V_{G S}-V_{T}\right) E_{c} L}{\left(V_{G S}-V_{T}\right)+E_{c} L}=\frac{(1-0.5)(1.2)}{(1-0.5)+1.2}=0.353 \mathrm{~V} \quad \text { S04E1P2 } \mathrm{S} \\
& V_{D S}=1.5 \mathrm{~V} \Rightarrow \text { NMOS in saturation } \\
& \left.\therefore I_{D S}=W v_{s a t} C_{o x} \frac{\left(V_{G S}-V_{T}\right)^{2}}{\left(V_{G S}-V_{T}\right)+E_{c} L}\right) \\
& W=0.6 \times 10^{-4} \mathrm{~cm} \\
& v_{s a t}= \\
& C_{o x}=\frac{\mu_{e} E_{c}}{2}=\frac{270\left(\mathrm{~cm}^{2} / \mathrm{V} \cdot \mathrm{~s}\right) 6 \times 10^{4}(\mathrm{~V} / \mathrm{cm})}{2}=8.1 \times 10^{6} \mathrm{~cm} / \mathrm{sec} \\
& t_{o x}
\end{aligned}=\frac{4 \cdot 8.85 \times 10^{-14}(\mathrm{~F} / \mathrm{cm})}{35 \times 10^{-8} \mathrm{~cm}}=1.01 \times 10^{-6} \mathrm{~F} / \mathrm{cm}^{2} .
$$



## Problem 3-(25 points)

Given the layout for the NMOS transistor of Problem 2, find the value of $C_{g s}, C_{g d}, C_{g b}, C_{d b}$, and $C_{s b}$ assuming that the junction depth of the source-drain diffusions is $x_{j}=50 \mathrm{~nm}, m=0.5$ and the lateral diffusion is 10 nm .


## Solution

From Problem 2, we know that the NMOS transistor is in saturation. To make the calculations, we will need $C_{g}$ and $C_{o v}$. They are calculated as follows,

$$
\begin{aligned}
C_{g}= & C_{o x} \cdot W \cdot L=1.01 \times 10^{-6}(\mathrm{~F} / \mathrm{cm} 2) \cdot 0.6 \times 10^{-4}(\mathrm{~cm}) \cdot 0.2 \times 10^{-4}(\mathrm{~cm})=1.212 \times 10^{-15} \mathrm{~F} \\
& \left(C_{o x} \text { was calculated in Problem } 2\right) \\
C_{o l}= & C_{o x} \cdot L D=1.01 \times 10^{-6}(\mathrm{~F} / \mathrm{cm} 2) \cdot 10 \times 10^{-7}(\mathrm{~cm})=1.01 \times 10^{-12} \mathrm{~F} / \mathrm{cm} \\
\therefore C_{g s}= & C_{o l} \cdot W+0.667 C_{g}=\left(1.01 \times 10^{-12}\right)\left(0.6 \times 10^{-4}\right)+(0.667)(1.212)=(0.061+0.808) \mathrm{fF} \\
= & \underline{\underline{0.868 \mathrm{fF}}} \\
C_{g d}= & 0.061 \mathrm{fF} \approx \underline{=} \\
C_{g b}= & \underline{\underline{0}} \\
C_{J}= & \left.\frac{C_{j b}\left(A_{b}+A_{s w}\right)}{V_{j}}=\frac{C_{j 0}\left(A_{b}+A_{s w}\right)}{\phi_{B}}\right) \\
C_{b d}= & \frac{1.6 \mathrm{fF} / \mu \mathrm{m}^{2}[(0.3)(0.6)+(0.05)(0.6)]}{\left.\sqrt{\phi_{B}}\right)}=\underline{\underline{0.206 \mathrm{fF}}} \\
C_{b s}= & \frac{1.6 \mathrm{fF} / \mu \mathrm{m}^{2}[(0.3)(0.6)+(0.05)(0.6)]}{\sqrt{1+0}}=\underline{\underline{0.336 \mathrm{fF}}}
\end{aligned}
$$

## Problem 4-(25 points)

A voltage transfer curve of the circuit shown is given for $T=0^{\circ} \mathrm{C}, 27^{\circ} \mathrm{C}$, and $54^{\circ} \mathrm{C}$. The solid curve corresponds to $T=27^{\circ} \mathrm{C}$. Identify which of the two dashed curves corresponds to $T=0^{\circ} \mathrm{C}$ and $T=54^{\circ} \mathrm{C}$ ? Justify your reason (for full credit) using the relationships for the temperature dependence of the mobility and threshold voltage given in the text. Assume that the resistor, $R_{L}$, has no temperature dependence and the technology is $0.18 \mu \mathrm{~m}$ CMOS.


Assume the transistor is in saturation which might correspond for example to a value of $v_{I N}=$ 0.8 V . Therefore, assume that $v_{I N}$ is fixed at a constant value such as 0.8 V .

The output voltage can be found as

$$
\begin{aligned}
& v_{O U T}(T)=1.8-I_{D} R_{L}=1.8-30 \mathrm{k} \Omega\left[\frac{W \mu_{e}(T) E_{C} C_{o x}\left[V_{I N^{-}} V_{T}(T)\right]^{2}}{2\left[\left(V_{I N^{-}} V_{T}(T)\right)+E_{C} L\right]}\right] \\
& \approx 1.8-30 \mathrm{k} \Omega\left[\frac{W \mu_{e}(T) C_{o x}\left[V_{I N^{-}} V_{T}(T)\right]^{2}}{2 L}\right], \text { where } \frac{\mu_{e}\left(27^{\circ} \mathrm{C}\right) C_{o x} W}{2 L}=0.54 \mathrm{~mA} / \mathrm{V}^{2}
\end{aligned}
$$

assuming $0.18 \mu \mathrm{~m}$ CMOS technology. Assuming also that $V_{I N}=0.8 \mathrm{~V}$, $v_{\text {OUT }}(T)=1.8-16.2\left(\frac{T}{T_{o}}\right)^{-1.5}\left[0.8-0.5+0.002\left(T-T_{o}\right)\right]^{2}=1.8-16.2\left(\frac{T}{T_{o}}\right)^{-1.5}\left[0.3+0.002\left(T-T_{o}\right)\right]^{2}$
For $T=T_{o}=27^{\circ} \mathrm{C}$ we get, $v_{O U T}\left(27^{\circ} \mathrm{C}\right)=1.8-16.2(0.09)=1.8-1.45=0.342 \mathrm{~V}$
For $T=T_{o}=0^{\circ} \mathrm{C}$ we get, $v_{\text {OUT }}\left(0^{\circ} \mathrm{C}\right)=1.8-16.2(1.15)(0.3-0.054)^{2}=1.8-1.13=0.67 \mathrm{~V}$
For $T=T_{o}=54^{\circ} \mathrm{C}$ we get, $v_{\text {OUT }}\left(54^{\circ} \mathrm{C}\right)=1.8-16.2(0.78)(0.3+0.054)^{2}=1.8-1.58=0.22 \mathrm{~V}$
(Unfortunately I used the wrong mobility on the curves so that the values don't correspond but the trend is correct.)

