## Homework No. 3 - Solutions

P3.1. The general approach for the first two parameters is to figure out which variables should remain constant, so that when you have two currents, you can divide them, and every variable but the ones you want to calculate remain. In this case, since the long-channel transistor is in saturation for all values of $V_{G S}$ and $V_{D S}$, only one equation needs to be considered:

$$
I_{D S}=\frac{1}{2} \mu_{N} C_{O X} \frac{W}{L}\left(V_{G S}-V_{T}\right)^{2}\left(1+\lambda V_{D S}\right)
$$

For the last two parameters, now that you have enough values, you can just choose one set of numbers to compute their final values.
a. The threshold voltage, $V_{T 0}$, can be found by choosing two sets of numbers with the same $V_{D S}$ 's but with different $V_{G S}$ 's. In this case, the first two values in the table can be used.

$$
\begin{gathered}
I_{D S 1}=\frac{1}{2} \frac{W}{L} \mu_{N} C_{O X}\left(V_{G S 1}-V_{T}\right)^{2}\left(1+\lambda V_{D S 1}\right) \\
I_{D S 2}=\frac{1}{2} \frac{W}{L} \mu_{N} C_{O X}\left(V_{G S 2}-V_{T}\right)^{2}\left(1+\lambda V_{D S 2}\right) \\
\frac{I_{D S 1}}{I_{D S 2}}=\frac{\left(1.2-V_{T 0}\right)^{2}}{\left(0.8-V_{T 0}\right)^{2}}=\left(\frac{1.2-V_{T 0}}{0.8-V_{T 0}}\right)^{2}=\frac{1000}{280} \\
\therefore V_{T 0}=0.35 \mathrm{~V}
\end{gathered}
$$

b. The channel modulation parameter, $\lambda$, can be found by choosing two sets of numbers with the same $V_{G S}$ 's but with different $V_{D S}$ 's. In this case, the second and third values in the table can be used.

$$
\frac{I_{D S 2}}{I_{D S 2}}=\frac{(1+1.2 \lambda)}{(1+0.8 \lambda)}=\frac{250}{247} \quad \therefore \lambda=0.04 \mathrm{~V}^{-1}
$$

c. The electron mobility, $\mu_{n}$, can now be calculated by looking at any of the first three sets of numbers, but first, let's calculate $C_{O X}$.

$$
\begin{aligned}
& t_{O X}=22 \mathscr{A} \times \frac{1 \not \mathrm{X}}{10^{10} \AA} \times \frac{10^{6} \mu \mathrm{~m}}{1 \mathrm{M}}=2.2 \square 0^{-3} \mu \mathrm{~m} \\
& C_{O X}=\frac{0.0351}{2.2 \square 0^{-3}}=1.6 \times 10^{-6} \mathrm{~F} / \mathrm{cm}^{2}
\end{aligned}
$$

Now calculate the mobility by using the first set of numbers.

$$
\begin{aligned}
& I_{D S 1}=\frac{1}{2} \frac{W}{L} \mu_{N} C_{O X}\left(V_{G S 1}-V_{T}\right)^{2}\left(1+\lambda V_{D S 1}\right)=\frac{1}{2} \frac{W}{L} \mu_{N} C_{O X}\left(1.2-V_{T 0}\right)^{2}(1+1.2 \lambda) \\
& \mu_{N}=\frac{2 I_{D S 1}}{C_{O X} \frac{W}{L}\left(V_{G S 1}-V_{T 0}\right)^{2}\left(1+\lambda V_{D S 1}\right)}=\frac{2(1000 \mu A)}{\left(1.6 \times 10^{-6}\right)(4.75)(1.2-0.35)^{2}(1+(0.04)(1.2))}=348 \frac{\mathrm{~cm}^{2}}{\mathrm{~V}-\mathrm{s}}
\end{aligned}
$$

P3.1-Continued
d. The body effect coefficient gamma, $\gamma$, can be calculated by using the last set of numbers since it is the only one that has a $V_{S B}$ greater than 0 V .

$$
\begin{aligned}
& I_{D S 4}=\frac{1}{2} \frac{W}{L} \mu_{N} C_{O X}\left(V_{G S 4}-V_{T}\right)^{2}\left(1+\lambda V_{D S 1}\right) \\
&\left(V_{G S 4}-V_{T}\right)^{2}=\frac{2 I_{D S 4}}{\mu_{N} \frac{W}{L} C_{O X}\left(1+\lambda V_{D S 1}\right)} \\
& V_{G S 4}-V_{T}=\sqrt{\frac{2 I_{D S 4}}{(W / L) \mu_{N} C_{O X}\left(1+\lambda V_{D S 1}\right)}} \\
& V_{T}= V_{G S 4}-\sqrt{\frac{2 I_{D S 4}}{(W / L) \mu_{N} C_{O X}\left(1+\lambda V_{D S 1}\right)}}=1.2-\sqrt{\frac{2(741 \mu A)}{(W / L)(348)\left(1.6 \times 10^{-6}\right)(1+(0.04)(1.2))}}=0.468 \mathrm{~V} \\
& V_{T}=V_{T 0}+\gamma\left(\sqrt{V_{S B}+2\left|\phi_{F}\right|}-\sqrt{2\left|\phi_{F}\right|}\right) \\
& V_{T}-V_{T 0}=\gamma\left(\sqrt{V_{S B}+2\left|\phi_{F}\right|}-\sqrt{2\left|\phi_{F}\right|}\right) \\
& \gamma=\frac{V_{T}-V_{T 0}}{\left(\sqrt{V_{S B}+2\left|\phi_{F}\right|}-\sqrt{2\left|\phi_{F}\right|}\right)}=\frac{0.468-0.35}{(\sqrt{0.4+0.88}-\sqrt{0.88})}=0.6 \mathrm{~V}^{1 / 2}
\end{aligned}
$$

Problems P3.4, P3.6, and P3.7 are SPICE problems and no answers are given.
P3.8 First, let's look at the various parameters and identify how they affect $V_{T}$.

- $\quad L$ - Shorter lengths result in a lower threshold voltage due to DIBL.
- $\quad W$ - Narrow width can increase the threshold voltage.
- $\quad V_{S B}$ - Larger source-bulk voltages (in magnitude) result in a higher threshold voltage.
- $\quad V_{D S}$-Larger drain-source voltages (in magnitude) result in a lower threshold voltage due to DIBL.
The transistor with the lowest threshold voltage has the shortest channel, larger width, smallest source-bulk voltage and largest drain-source voltage. This would be the first transistor listed.

The transistor with the highest threshold voltage has the longest channel, smallest width,largest source-bulk voltage and smallest drain-source voltage. This would be the last transistor listed.

