**Homework No. 4 – Solutions**

Problem 1 – P4.2

Problem should refer to Figure P4.1

a. Resistive load

$$V_{OH} = V_{DD}$$

$$V_{OL} = \frac{V_{DD}}{1 + kR_L\left(V_{DD} - V_T\right)} = \frac{V_{DD}}{1 + \mu_N C_{OX} \frac{W}{L} R_L\left(V_{DD} - V_T\right)}$$

$$= \frac{1.2}{1 + (260)\left(10^{-6}\right)\left(\frac{1}{0.1}\right)\left(10 \times 10^3\right)\left(1.2 - 0.4\right)} = 0.055\text{V}$$

b. Saturated-enhancement load

$$V_{OH} = V_{DD} - V_T = V_{DD} - \left(V_{T0} + \gamma\left(\sqrt{V_{SB} + 2\left|\phi_f\right|} - \sqrt{2\left|\phi_f\right|}\right)\right)$$

$$= V_{DD} - V_{T0} - \gamma\sqrt{V_{OH} + 2\left|\phi_f\right|} + \gamma\sqrt{2\left|\phi_f\right|}$$

$$= 1.2 - 0.4 - 0.2\sqrt{V_{OH} + 0.88} + 0.2\sqrt{0.88}$$

$$= 0.988 - 0.2\sqrt{V_{OH} + 0.88}$$

Iterate to produce:

$$V_{OH} = 0.733\text{V}$$

To compute $V_{OL}$ we can ignore body effect and equate currents:

$$\therefore \frac{W_I}{L_I}\frac{\mu_N}{\left(1 + \frac{V_{OL}}{E_{CN}L_I}\right)}\left[\left(V_{DD} - V_{TI}\right)V_{OL} - \frac{V_{OL}^2}{2}\right] = \frac{W_L v_{sat}(V_{DD} - V_{OL} - V_{TL})^2}{(V_{DD} - V_{OL} - V_{TL}) + E_{CN}L_L}$$

$$\therefore \frac{1.2}{0.1}\frac{(270\,cm^2/V-s)}{\left(1 + \frac{V_{OL}}{0.6}\right)}\left[\left(1.2 - 0.4\right)V_{OL} - \frac{V_{OL}^2}{2}\right] = \frac{(0.2\mu m)(8 \times 10^6\,cm/s)(1.2 - V_{OL} - 0.4)^2}{(1.2 - V_{OL} - 0.4) + 0.6}$$

Solve for $V_{OL} \approx 0.03V$

c. Linear-enhancement load

$$V_{OH} = V_{GG} - V_T = V_{DD} - \left(V_{T0} + \gamma\left(\sqrt{V_{SB} + 2|\phi_f|} - \sqrt{2|\phi_f|}\right)\right)$$
$$= V_{GG} - V_{T0} - \gamma\sqrt{V_{OH} + 2|\phi_f|} + \gamma\sqrt{2|\phi_f|}$$
$$= 1.6 - 0.4 - 0.2\sqrt{V_{OH} + 0.88} + 0.2\sqrt{0.88}$$
$$= 1.388 - 0.2\sqrt{V_{OH} + 0.88}$$

Iterate to produce:

$$V_{OH} = 1.11\text{V}$$

This tells us that $V_{GG}$ should have been above 1.6V (closer to 1.7 V).

To compute $V_{OL}$ we can ignore body effect and equate currents. Note that the load is saturated even though we call it a linear-enhancement load. The driver is also saturated due to the device sizes used.

$$\therefore \frac{W_I v_{sat}(V_{DD} - V_{TI})^2}{(V_{DD} - V_{TI}) + E_{CN}L_I} = \frac{W_L v_{sat}(V_{GG} - V_{OL} - V_{TL})^2}{(V_{GG} - V_{OL} - V_{TL}) + E_{CN}L_L}$$

$$\therefore \frac{(0.1\mu m)(8\times 10^6 cm/s)(1.2-0.4)^2}{(1.2-0.4)+0.6} = \frac{(0.1\mu m)(8\times 10^6 cm/s)(1.6-V_{OL}-0.4)^2}{(1.6-V_{OL}-0.4)+0.6}$$

Solve for $V_{OL} \approx 0.69\text{V}$

d. CMOS

$V_{OH} = V_{DD}$ $\qquad V_{OL} = 0\text{V}$

Problem 2 – P4.3

For this problem, you are required to use the formulae:

$$V_{IL} = \frac{2V_{out} - V_{DD} - |V_{TP}| + (k_N/k_P)(V_{TN})}{1 + (k_N/k_P)} \qquad V_{IH} = \frac{2V_{out} + V_{TN} + (k_P/k_N)(V_{DD} - |V_{TP}|)}{1 + (k_P/k_N)}$$

We already know that $V_{OH}$=1.2 V and $V_{OL}$=0 V. For $V_S$ use:

$$W_N = 4\lambda, \quad W_P = 16\lambda:$$

$$X = \sqrt{\frac{\dfrac{W_N}{E_{CN}L_N}}{\dfrac{W_P}{E_{CP}L_P}}} = \sqrt{\frac{W_N E_{CP}}{W_P E_{CN}}} = \sqrt{\frac{(4)(24)}{(16)(6)}} = 1$$

$$V_S = \frac{0.8 + (0.4)1}{1+1} = 0.6V$$

Next $V_{IL}$ and $V_{IH}$ are estimated as follows:

$$V_{IL} = \frac{2V_{out} - V_{DD} - |V_{TP}| + (k_N / k_P)(V_{TN})}{1 + (k_N / k_P)} = \frac{2V_{out} - 1.2 - |-0.4| + (1)(0.4)}{1+(1)} = \frac{2V_{out} - 1.2}{2} = 0.55V$$

$$V_{IH} = \frac{2V_{out} + V_{TN} + (k_P / k_N)(V_{DD} - |V_{TP}|)}{1 + (k_P / k_N)} = \frac{2V_{out} + 0.4 + (1)(1.2 - 0.4)}{1+(1)} = \frac{2V_{out} + 1.2}{2} = 0.65V$$

Therefore

$$NM_L = 0.55 - 0 = 0.55V$$
$$NM_H = 1.2 - 0.65V = 0.55V$$

When we cut the size of the PMOS device in half, the VTC shifts to the left. So $V_{IL}$, $V_S$, and $V_{IH}$ will all shift to the left. The recalculation of the switching threshold produces $V_S$=0.566V.

We can compute $V_{IL}$ to be roughly 0.533V and $V_{IH}$ to be roughly 0.667V.

Therefore

$$NM_L = 0.533 - 0 = 0.533V$$
$$NM_H = 1.2 - 0.667V = 0.533V$$

<u>Problem 3 – P4.9</u>

Resistive Load inverter:

$$\frac{V_{DD} - V_{OL}}{R_L} = \frac{W_N}{L_N} \frac{\mu_n C_{ox}}{\left(1 + \dfrac{V_{OL}}{E_C L}\right)} [2(V_{OH} - V_T)V_{OL} - V_{OL}^2]$$

$$\frac{1.2-0.1}{10k}=\frac{W_N}{0.1}\frac{(270)(1.6\times10^{-6})}{\left(1+\frac{0.1}{0.6}\right)}[2(1.2-0.4)0.1-0.1^2]$$

$$\therefore W_N=0.2\mu m$$

Saturated Enhancement Load inverter (ignoring body-effect):

$$\frac{W_I}{L_I}\frac{\mu_N C_{ox}}{\left(1+\frac{V_{out}}{E_{CN}L_I}\right)}\left[\left(V_{in}-V_{TI}\right)V_{out}-\frac{V_{out}{}^2}{2}\right]=\frac{W_L v_{sat}C_{ox}(V_{DD}-V_{out}-V_{TL})^2}{(V_{DD}-V_{out}-V_{TL})+E_{CN}L_L}$$

$$\frac{W_I}{0.1}\frac{(270)(1.6\times10^{-6})}{\left(1+\frac{0.1}{0.6}\right)}[2(1.2-0.4)0.1-0.1^2]=\frac{0.1(10^{-4})(8)(1.6)(1.2-0.1-0.4)^2}{(1.2-0.1-0.4)+0.6}$$

$$\therefore W_N=0.1\mu m$$

Linear Enhancement Load inverter (ignoring body-effect):

$$\frac{W_I}{L_I}\frac{\mu_N C_{ox}}{\left(1+\frac{V_{out}}{E_{CN}L_I}\right)}\left[\left(V_{in}-V_{TI}\right)V_{out}-\frac{V_{out}{}^2}{2}\right]=\frac{W_L v_{sat}C_{ox}(V_{DD}-V_{out}-V_{TL})^2}{(V_{DD}-V_{out}-V_{TL})+E_{CN}L_L}$$

$$\frac{W_I}{0.1}\frac{(270)(1.6\times10^{-6})}{\left(1+\frac{0.1}{0.6}\right)}[2(1.2-0.4)0.1-0.1^2]=\frac{0.1(10^{-4})(8)(1.6)(1.6-0.1-0.4)^2}{(1.6-0.1-0.4)+0.6}$$

$$\therefore W_N=0.6\mu m$$

The linear enhancement load inverter requires the largest pull-down device since it has the strongest pull up device. The resistive load inverter is next and the saturated enhancement load requires the smallest pull-down device.