Homework No. 4 – Solutions

Problem 1 – P4.2
Problem should refer to Figure P4.1

a. Resistive load

\[ V_{OH} = V_{DD} \]

\[ V_{OL} = \frac{V_{DD}}{1 + kR_L \left( V_{DD} - V_T \right)} = \frac{V_{DD}}{1 + \mu n C_{ox} \frac{W}{L} R_L \left( V_{DD} - V_T \right)} \]

\[ = \frac{1.2}{1 + (260)(10^{-6})(10 \times 10^3)(1.2 - 0.4)} = 0.055\text{V} \]

b. Saturated-enhancement load

\[ V_{OH} = V_{DD} - V_T = V_{DD} - \left( V_{T0} + \gamma \left( \sqrt{V_{SB}} + 2 \left| \phi_f \right| - \sqrt{2} \left| \phi_f \right| \right) \right) \]

\[ = V_{DD} - V_{T0} - \gamma \sqrt{V_{OH}} + 2 \left| \phi_f \right| + \gamma \sqrt{2} \left| \phi_f \right| \]

\[ = 1.2 - 0.4 - 0.2 \sqrt{V_{OH}} + 0.88 + 0.2 \sqrt{0.88} \]

\[ = 0.988 - 0.2 \sqrt{V_{OH}} + 0.88 \]

Iterate to produce:

\[ V_{OH} = 0.733\text{V} \]

To compute \( V_{OL} \) we can ignore body effect and equate currents:

\[ \therefore \frac{W_I}{L_I} \left( \frac{\mu n}{1 + \frac{V_{OL}}{E_{CN} L_I}} \right) \left( (V_{DD} - V_{TI}) V_{OL} - \frac{V_{OL}^2}{2} \right) = \frac{W_I V_{sat} (V_{DD} - V_{OL} - V_{TI})^2}{(V_{DD} - V_{OL} - V_{TI}) + E_{CN} L_I} \]

\[ \therefore 1.2 \left( \frac{270 \text{cm}^2}{0.1} \right) \left[ (1.2 - 0.4) V_{OL} - \frac{V_{OL}^2}{2} \right] = \frac{(0.2 \mu m)(8 \times 10^6 \text{cm/s})(1.2 - V_{OL} - 0.4)^2}{(1.2 - V_{OL} - 0.4) + 0.6} \]

Solve for \( V_{OL} \approx 0.03\text{V} \)

c. Linear-enhancement load
\[ V_{OH} = V_{GG} - V_T = V_{DD} - \left( V_{T0} + \gamma \left( \sqrt{V_{SB} + 2|\phi_f|} - \sqrt{2|\phi_f|} \right) \right) \]
\[ = V_{GG} - V_{T0} - \gamma \sqrt{V_{OH}} + 2|\phi_f| + \gamma \sqrt{2|\phi_f|} \]
\[ = 1.6 - 0.4 - 0.2\sqrt{V_{OH} + 0.88} + 0.2\sqrt{0.88} \]
\[ = 1.388 - 0.2\sqrt{V_{OH} + 0.88} \]

Iterate to produce:

\[ V_{OH} = 1.11V \]

This tells us that \( V_{GG} \) should have been above 1.6V (closer to 1.7 V).

To compute \( V_{OL} \) we can ignore body effect and equate currents. Note that the load is saturated even though we call it a linear-enhancement load. The driver is also saturated due to the device sizes used.

\[ \frac{W_n I_{sat} (V_{DD} - V_{Tn})^2}{(V_{DD} - V_{Tn}) + E_{CN} L_t} = \frac{W_p I_{sat} (V_{GG} - V_{OL} - V_{TL})^2}{(V_{GG} - V_{OL} - V_{TL}) + E_{CN} L_p} \]
\[ \therefore \frac{(0.1\mu m)(8 \times 10^6 \text{cm/s})(1.2 - 0.4)^2}{(1.2 - 0.4) + 0.6} = \frac{(0.1\mu m)(8 \times 10^6 \text{cm/s})(1.6 - V_{OL} - 0.4)^2}{(1.6 - V_{OL} - 0.4) + 0.6} \]

Solve for \( V_{OL} \approx 0.69V \)

d. CMOS

\[ V_{OH} = V_{DD} \quad V_{OL} = 0V \]

Problem 2 – P4.3

For this problem, you are required to use the formulae:

\[ V_{IL} = \frac{2V_{out} - V_{DD} - |V_{TP}| + (k_N / k_P)(V_{TN})}{1 + (k_N / k_P)} \]
\[ V_{BH} = \frac{2V_{out} + V_{TN} + (k_P / k_N)(V_{DD} - |V_{TP}|)}{1 + (k_P / k_N)} \]

We already know that \( V_{OH} = 1.2V \) and \( V_{OL} = 0V \). For \( V_S \) use:

\[ \text{null} \]
Next $V_{IL}$ and $V_{IH}$ are estimated as follows:

\[ V_{IL} = \frac{2V_{out} - V_{DD} - |V_{TP}| + (k_N / k_P)(V_{TN})}{1 + (k_N / k_P)} = \frac{2V_{out} - 1.2 - |0.4| + (1)(0.4)}{1 + (1)} = \frac{2V_{out} - 1.2}{2} = 0.55V \]

\[ V_{IH} = \frac{2V_{out} + V_{TN} + (k_P / k_N)(V_{DD} - |V_{TP}|)}{1 + (k_P / k_N)} = \frac{2V_{out} + 0.4 + (1)(1.2 - 0.4)}{1 + (1)} = \frac{2V_{out} + 1.2}{2} = 0.65V \]

Therefore

\[ NM_L = 0.55 - 0 = 0.55V \]
\[ NM_H = 1.2 - 0.65V = 0.55V \]

When we cut the size of the PMOS device in half, the VTC shifts to the left. So $V_{IL}$, $V_S$, and $V_{IH}$ will all shift to the left. The recalculation of the switching threshold produces $V_S = 0.566V$.

We can compute $V_{IL}$ to be roughly 0.533V and $V_{IH}$ to be roughly 0.667V.

Therefore

\[ NM_L = 0.533 - 0 = 0.533V \]
\[ NM_H = 1.2 - 0.667V = 0.533V \]

Problem 3 – P4.9

Resistive Load inverter:

\[ \frac{V_{DD} - V_{OL}}{R_L} = \frac{W_N}{L_N} \frac{\mu_n C_{ox}}{2(V_{OH} - V_T) V_{OL} - V_{OL}^2} \left( 1 + \frac{V_{OL}}{E_c L} \right) \]
\[
\frac{1.2 - 0.1}{10k} = \frac{W_N (270)(1.6 \times 10^{-6})}{0.1 \left(1 + \frac{0.1}{0.6}\right)} [2(1.2 - 0.4)0.1 - 0.1^2]
\]
\[
\therefore W_N = 0.2 \mu m
\]

Saturated Enhancement Load inverter (ignoring body-effect):

\[
\frac{W_L}{L_I} \frac{\mu_N C_{ox}}{1 + \frac{V_{out}}{E_{CN}L_I}} \left[ (V_{in} - V_{TI}) V_{out} - \frac{V_{out}^2}{2} \right] = \frac{W_L V_{out} C_{ox} (V_{DD} - V_{out} - V_{TL})^2}{(V_{DD} - V_{out} - V_{TL}) + E_{CN}L_I}
\]
\[
W_L (270)(1.6 \times 10^{-6}) \left[2(1.2 - 0.4)0.1 - 0.1^2\right] = \frac{0.1(10^{-4})(8)(1.6)(1.2 - 0.1 - 0.4)^2}{(1.2 - 0.1 - 0.4) + 0.6}
\]
\[
\therefore W_N = 0.1 \mu m
\]

Linear Enhancement Load inverter (ignoring body-effect):

\[
\frac{W_L}{L_I} \frac{\mu_N C_{ox}}{1 + \frac{V_{out}}{E_{CN}L_I}} \left[ (V_{in} - V_{TI}) V_{out} - \frac{V_{out}^2}{2} \right] = \frac{W_L V_{out} C_{ox} (V_{DD} - V_{out} - V_{TL})^2}{(V_{DD} - V_{out} - V_{TL}) + E_{CN}L_I}
\]
\[
W_L (270)(1.6 \times 10^{-6}) \left[2(1.2 - 0.4)0.1 - 0.1^2\right] = \frac{0.1(10^{-4})(8)(1.6)(1.6 - 0.1 - 0.4)^2}{(1.6 - 0.1 - 0.4) + 0.6}
\]
\[
\therefore W_N = 0.6 \mu m
\]

The linear enhancement load inverter requires the largest pull-down device since it has the strongest pull up device. The resistive load inverter is next and the saturated enhancement load requires the smallest pull-down device.