## Homework No. 5 - Solutions

Problem 1-P4.10
We will illustrate the process and estimate the solutions for this problem.
We already know that $\mathrm{V}_{\mathrm{OH}}=1.2 \mathrm{~V}$ and $\mathrm{V}_{\mathrm{OL}}=0 \mathrm{~V}$. For $\mathrm{V}_{\mathrm{S}}$ use:

$$
\begin{aligned}
W_{N} & =0.4 u \mathrm{~m}, \quad W_{P}=0.8 u m: \\
X & =\sqrt{\frac{\frac{W_{N}}{\frac{E_{C N} L_{N}}{W_{P}}}}{E_{C P} L_{P}}}=\sqrt{\frac{W_{N} E_{C P}}{W_{P} E_{C N}}}=\sqrt{\frac{(0.4)(24)}{(0.8)(6)}}=1.41 \\
V_{S} & =\frac{0.8+(0.4) 1.41}{1+1.41}=0.566
\end{aligned}
$$

Next $V_{\text {IL }}$ and $V_{I H}$ are estimated as follows:

$$
\begin{gathered}
V_{I L}=\frac{2 V_{\text {out }}-V_{D D}-\left|V_{T P}\right|+\left(k_{N} / k_{P}\right)\left(V_{T N}\right)}{1+\left(k_{N} / k_{P}\right)} \\
V_{I L}=\frac{2 V_{\text {out }}-1.2-|-0.4|+(2)(0.4)}{1+(2)}=\frac{2 V_{\text {out }}-0.6}{3}
\end{gathered}
$$

We can compute $\mathrm{V}_{\text {IL }}$ to be roughly 0.533 V .

$$
\begin{gathered}
V_{I H}=\frac{2 V_{\text {out }}+V_{T N}+\left(k_{P} / k_{N}\right)\left(V_{D D}-\left|V_{T P}\right|\right)}{1+\left(k_{P} / k_{N}\right)} \\
V_{I H}=\frac{2 V_{\text {out }}+0.4+(2)(1.2-0.4)}{1+(2)}=\frac{2 V_{\text {out }}+2}{3}
\end{gathered}
$$

We can compute $\mathrm{V}_{\mathrm{IH}}$ to be roughly 0.667 V .
When we double the size of the PMOS device, the VTC shifts to the right. So $\mathrm{V}_{\mathrm{IL}}$, $\mathrm{V}_{\mathrm{S}}$, and $\mathrm{V}_{\mathrm{IH}}$ will all shift to the right. The recalculation of the switching threshold produces $\mathrm{V}_{\mathrm{S}}=0.6 \mathrm{~V}$.

We can compute $\mathrm{V}_{\mathrm{IL}}$ to be roughly 0.55 V and $\mathrm{V}_{\mathrm{IH}}$ to be roughly 0.65 V .

## Problem 2-P5.1

For each problem, restate each Boolean equation into a form such that it can be translated into the p and n -complex of a CMOS gate.
a. Out $=A B C+B D=\overline{\overline{A B C+B D}}=\overline{(\bar{A}+\bar{B}+\bar{C})(\bar{B}+\bar{D})}$
b. Out $=A B+\bar{A} C+B C=\overline{\overline{A B+\bar{A} C+B C}}=\overline{(\bar{A}+\bar{B})(A+\bar{C})(\bar{B}+\bar{C})}$
c. Out $=\overline{A+B+C D}+A=\bar{A} \bar{B}(\bar{C}+\bar{D})+A=\overline{\overline{\overline{A+B+C D}+A}}=\overline{(A+B+C D) \bar{A}}$


## Problem 3 - P5.8

The solution is shown below. Notice that there is no relevance with the lengths and widths of the transistors when it comes to $V_{O H}$, although they the do matter when calculating $V_{O L}$.

$$
\begin{aligned}
V_{\text {out }} & =V_{G G}-V_{T} \\
V_{G G} & =V_{\text {out }}+V_{T 0}+\gamma\left(\sqrt{V_{\text {out }}+2\left|\phi_{F}\right|}-\sqrt{2\left|\phi_{F}\right|}\right) \\
& =1.8+0.5+0.3(\sqrt{1.8+0.88}-\sqrt{0.88})=2.51 \mathrm{~V}
\end{aligned}
$$

## Problem 4 - P5.9

For $t_{P L H}$, we need to size the pull-up PMOS appropriately.

$$
\begin{aligned}
t_{P L H} & =0.7 R C=0.7 R_{\text {eqq }} \frac{L}{W} C_{L O A D} \\
W_{p} & =0.7 R_{S Q} \frac{L}{t_{P L H}} C_{L O A D}=0.7(30 \mathrm{k} \Omega) \frac{(2 \lambda)}{\left(50 \times 10^{-12}\right)}\left(100 \times 10^{-15}\right)=84 \lambda
\end{aligned}
$$

For $V_{O L}$ :

$$
\begin{aligned}
I_{P}(\text { sat }) & =\frac{W_{P} v_{\text {sat }} C_{O X}\left(V_{G S}-V_{T}\right)^{2}}{V_{G S}-V_{T}+E_{C P} L}=\frac{\left(4.2 \times 10^{-4}\right)\left(8 \times 10^{6}\right)\left(1.6 \times 10^{-6}\right)(1.2-0.4)^{2}}{1.2-0.4+(24)(0.1)}=1.08 \mathrm{~mA} \\
I_{P}(\text { sat }) & =\frac{W_{N} \mu_{N} C_{O X}\left(V_{O L}-V_{T N}-\frac{V_{O L}}{2}\right) V_{O L}}{L_{N}\left(1+\frac{V_{O L}}{E_{C N} L}\right)}=\frac{W_{N}(270)\left(1.6 \times 10^{-6}\right)\left(1.2-0.4-\frac{0.1}{2}\right) 0.1}{L_{N}\left(1+\frac{0.1}{0.6}\right)} \\
\frac{W_{N}}{L_{N}} & =38.5 \quad W_{N}=77 \lambda \quad W_{3}=3 \times 77 \lambda=232 \lambda \quad(3 \text { stack }) \quad W_{2}=155 \lambda \quad(2 \text { stack })
\end{aligned}
$$

