Homework No. 8 – Solutions

Problem 1 – P6.6

For optimum sizing given four inverters.

$$PE = \prod LE \times FO = (1)(1)(1)(1)(1200) = 1200$$

$$SE = \sqrt[N]{PE} = \sqrt[4]{1200} = 5.89$$

$$C_4 = \frac{LE \times C_{OUT}}{SE} = \frac{1(1200)}{5.89} = 203.89$$

$$C_3 = \frac{LE \times C_4}{SE} = \frac{1(203.89)}{5.89} = 34.64$$

$$C_2 = \frac{LE \times C_3}{SE} = \frac{1(34.64)}{5.89} = 5.89$$

$$C_1 = \frac{LE \times C_2}{SE} = \frac{1(5.89)}{5.89} = 1$$

$$D = \sum_1^N (LE \times FO + P) = \sum_1^4 (SE + P) = 4(5.89 + 0.5) = 25.5$$

For the number of devices for optimum delay, proceed as follows:

$$SE = \sqrt[N]{PE}$$

$$SE^N = PE$$

$$\log SE^N = \log PE$$

$$N \log SE = \log PE$$

$$guess \quad that \quad SE \approx 4 \ (FO4 \quad assumption)$$

$$N = \frac{\log PE}{\log SE} = \frac{\log 1200}{\log 4} = 5.11$$

Try $N = 5, 6, 7$ and see which gives the smallest delay:

$$N = 5: \quad SE = \sqrt[N]{PE} = \sqrt[5]{1200} = 4.12$$

$$D = \sum_1^N (LE \times FO + P) = \sum_1^5 (SE + P) = 5(4.12 + 0.5) = 23.1$$

$$N = 6: \quad SE = \sqrt[N]{PE} = \sqrt[6]{1200} = 3.26$$

$$D = \sum_1^N (LE \times FO + P) = \sum_1^6 (SE + P) = 6(3.26 + 0.5) = 22.6$$

$$N = 7: \quad SE = \sqrt[N]{PE} = \sqrt[7]{1200} = 2.75$$

$$D = \sum_1^N (LE \times FO + P) = \sum_1^7 (SE + P) = 7(2.75 + 0.5) = 22.8$$

Problem 1 – P6.6 - Continued

Therefore, N=6 is gives the smallest delay. Use this to compute capacitance size.

$$C_7 = \frac{LE \times C_{OUT}}{SE} = \frac{1(1200)}{3.26} = 368$$
$$C_6 = \frac{LE \times C_5}{SE} = 113$$
$$C_4 = \frac{LE \times C_5}{SE} = 34.6$$
$$C_3 = \frac{LE \times C_4}{SE} = 10.6$$
$$C_2 = \frac{LE \times C_3}{SE} = 3.26$$
$$C_1 = \frac{LE \times C_2}{SE} = 1$$

These capacitances can be converted to widths by assigning 2/3 of the capacitance to the PMOS device and 1/3 to the NMOS device and then dividing by 2fF/um.

Problem 2 – P6.9

a. $LE = \frac{5}{3}$ b. $LE = \frac{5}{3}$ c. $LE_R = \frac{8}{3}, LE_F = \frac{2}{3}$ d. $LE_R = \frac{4}{3}, LE_F = 2$

Problem 3 – P6.12

$$PE = \prod LE \times FO \times BE = \left(\frac{5}{3}\right)\left(\frac{4}{3}\right)\left(\frac{6}{3}\right)(4)(1000) = 17778$$
$$SE = \sqrt[N]{PE} = \sqrt[4]{17778} = 11.55$$
$$C_4 = \frac{LE \times C_{OUT} \times BE}{SE} = \frac{\left(\frac{6}{3}\right)(1000)(1)}{11.55} = 173.21$$
$$C_3 = \frac{LE \times C_4 \times BE}{SE} = \frac{\left(\frac{5}{3}\right)(173.21)(1)}{11.55} = 25$$
$$C_2 = \frac{LE \times C_3 \times BE}{SE} = \frac{\left(\frac{4}{3}\right)(25)(4)}{11.55} = 11.55$$
$$C_1 = \frac{LE \times C_2 \times BE}{SE} = \frac{(1)(11.55)(1)}{11.55} = 1$$
$$D = \sum_1^N (SE + P_N) = \sum_1^4 (SE + P_N) = 4(11.55) + 0.5 + 1 + 1.5 + 2 = 51.2$$

Problem 4 – P6.13

$$PE = \prod LE \times FO \times BE = (1)\left(\frac{4}{3}\right)\left(\frac{5}{3}\right)\left(\frac{7}{3}\right)(2)(2)(4)(8000) = 667303$$

$$SE = \sqrt[N]{PE} = \sqrt[5]{663703} = 14.6$$

$$C_5 = \frac{LE \times C_{OUT} \times BE}{SE} = \frac{\left(\frac{6}{3}\right)(8000)(1)}{14.6} = 1095.8$$

$$C_4 = \frac{LE \times C_5 \times BE}{SE} = \frac{\left(\frac{7}{3}\right)(1095)(1)}{14.6} = 175.1$$

$$PE = \prod LE \times FO \times BE = (1)\left(\frac{4}{3}\right)\left(\frac{5}{3}\right)(2)(4 \times 175.1 + 500) = 5335$$

$$SE = \sqrt[N]{PE} = \sqrt[3]{5335} = 17.47$$

$$C_3 = \frac{LE \times C_4 \times BE}{SE} = \frac{\left(\frac{5}{3}\right)(1200)(1)}{17.5} = 114.3$$

$$C_2 = \frac{LE \times C_3 \times BE}{SE} = \frac{\left(\frac{4}{3}\right)(114.3)(2)}{17.5} = 17.5$$

$$C_1 = \frac{LE \times C_2 \times BE}{SE} = \frac{(1)(17.5)(1)}{17.5} = 1$$

$$D = \sum_{1}^{N}\left(SE + P_N\right) = \sum_{1}^{5}\left(SE + P_N\right) = 3(17.5) + 2\left(14.6\right) + 0.5 + 1 + 1.5 + 2.25 + 2 = 88.9$$

To minimize the delay, a estimate of the number of needed stages can be performed :

$$SE = \sqrt[N]{PE}$$

$$\therefore N = \frac{\log PE}{\log SE} = \frac{\log 663704}{\log 4} = 9.6 \approx 10$$

The additional stages can be implemented as inverters attached at the input.