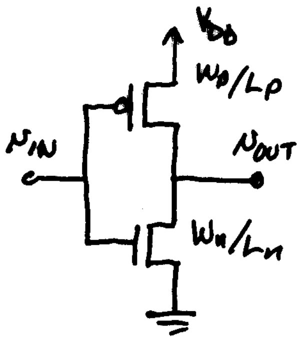
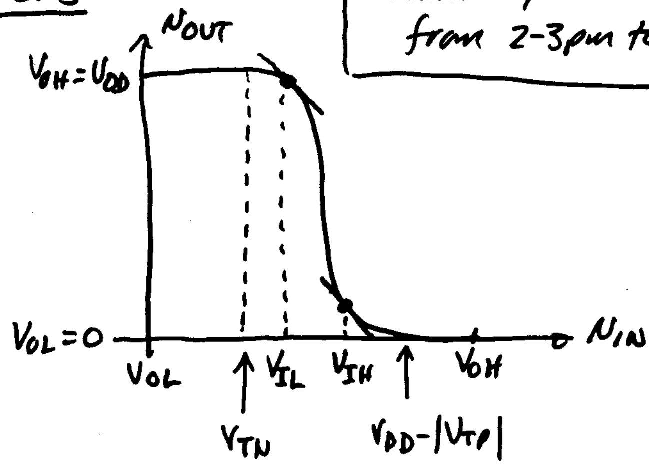
Exam No. 1 – Monday, Feb. 9, 2004, 12:05–12:55pm, C341
Problem Session – Sunday, Feb. 8, 2004, 4–5pm, C341?

Wednesday OH are moved from 2-3pm to 3-4pm

CMOS Inverters





$V_{IL}=?$ (NMOS sat., PMOS lin.)

$$\frac{W_n v_{sat} C_{ox} (v_{IN}-V_{TN})^2}{(v_{IN}-V_{TN}) + E_{CN} L_n} = \frac{W_p}{L_p}\left(\frac{\mu_p C_{ox}}{1+\frac{V_{DD}-v_{OUT}}{E_{CP}L_p}}\right)\left[(V_{DD}-v_{IN}-|V_{TP}|)(V_{DD}-v_{OUT}) - \frac{(V_{DD}-v_{OUT})^2}{2}\right]$$

Assumptions: $v_{IN}-V_{TN}\approx 0$ and $v_{OUT}\approx V_{DD} \rightarrow \frac{V_{DD}-v_{OUT}}{E_{CP}L_p}\approx 0$

Eq. (1) $$\frac{W_n}{L_n}\frac{\mu_n}{2}(v_{IN}-V_{TN})^2 = \frac{W_p}{L_p}\mu_p\left[(V_{DD}-v_{IN}-|V_{TP}|)(V_{DD}-v_{OUT}) - \frac{(V_{DD}-v_{OUT})^2}{2}\right]$$

$$\frac{\partial}{\partial v_{IN}} \rightarrow \frac{W_n}{L_n}\mu_n(v_{IN}-V_{TN}) = \frac{W_p}{L_p}\mu_p\Big[(V_{DD}-v_{IN}-|V_{TP}|)\Big(\overset{=-1}{-\frac{\partial v_{OUT}}{\partial v_{IN}}}\Big)$$

$$+ (V_{DD}-v_{OUT})(-1) - (V_{DD}-v_{OUT})\left(-\frac{\partial v_{OUT}}{\partial v_{IN}}\right)\Big]$$

If $k_N \equiv \mu_n \frac{W_n}{L_n}$ and $k_p \equiv \mu_p \frac{W_p}{L_p}$ $(v_{OUT} = \tilde{V}_{OUT})$

and $v_{IN} = V_{IL}$, then

Eq. (2) $$V_{IL} = \frac{2V_{OUT} - V_{DD} - |V_{TP}| + (k_N/k_p)(V_{TN})}{1+\frac{k_N}{k_p}}$$

Use Eqs. (1) and (2) to solve for V_{IL} or

If $V_{OUT}\approx V_{DD}$, $$V_{IL}\approx \frac{V_{DD} - |V_{TP}| + \left(\frac{k_N}{k_p}\right)V_{TN}}{1+(k_N/k_p)}$$

$V_{IH} = ?$

$$i_{DN}(\text{lin}) = i_{DP}(\text{sat.})$$

$$\frac{W_n}{L_n}\left(\frac{\mu_n C_{ox}}{1+\underbrace{\frac{v_{OUT}}{E_{cn}L_n}}_{\to 0}}\right)\left[(v_{IN}-V_{TN})v_{OUT}-\frac{v_{OUT}^2}{2}\right] = \frac{W_p\, v_{sat.}\, C_{ox}\left(V_{DD}-v_{IN}-|V_{TP}|\right)^2}{\underbrace{\left(V_{DD}-v_{IN}-|V_{TP}|\right)}_{0}+E_{cp}L_p}$$

Assumptions: $\frac{v_{OUT}}{E_{cn}L_n} < 1$ and $\left(V_{DD}-v_{IN}-|V_{TP}|\right) < E_c L_p$

$v_{sat} = \frac{\mu E_c}{2}$

$$\therefore \quad \mu_n C_{ox}\frac{W_n}{L_n}\left[(v_{IN}-V_{TN})\,v_{OUT}-\frac{v_{OUT}^2}{2}\right] = \frac{\mu_p C_{ox}}{2}\frac{W_p}{L_p}\left(V_{DD}-v_{IN}-|V_{TP}|\right)^2 \qquad \text{Eq. (1)}$$

$$k_N = \mu_n C_{ox}\frac{W_n}{L_n} \quad \text{and} \quad k_P = \mu_p C_{ox}\frac{W_p}{L_p}$$

Eq. (1) -

$$k_N\left[(v_{IN}-V_{TN})v_{OUT}-\frac{v_{OUT}^2}{2}\right] = \frac{k_P}{2}\left(V_{DD}-v_{IN}-|V_{TP}|\right)^2$$

$$\frac{\partial}{\partial v_{IN}}\left[\qquad \text{''} \qquad\right]$$

$v_{IN} = V_{IH}$ and $v_{OUT} = V_{OUT}$

Eq. (2)

$$V_{IH} = \frac{2V_{OUT} + V_{TN} + \left(\frac{k_P}{k_N}\right)\left(V_{DD}-|V_{TP}|\right)}{1+\left(\frac{k_P}{k_N}\right)}$$

Solve Eqs. (1) and (2) for V_{IH}.

An approx. $V_{OUT} \simeq 0 \rightarrow V_{IH} \simeq \dfrac{V_{TN} + \left(\frac{k_P}{k_N}\right)\left(V_{DD}-|V_{TP}|\right)}{1+\frac{k_P}{k_N}}$

Example

Find NM_H and NM_L for a CMOS inverter if

$W_n = 400nm$, $W_L = 800nm$, $\mu_n = 270 \frac{cm^2}{V \cdot s}$, $\mu_p = 70 \frac{cm^2}{V \cdot s}$,

$C_{ox} = 1\mu F/cm^2$, $V_{TN} = 0.5V$, $V_{TP} = -0.5V$, $E_{cn}L_n = 1.2V$,

$E_{cp}L_p = 4.8V$, $v_{sat} = 8 \times 10^6 \frac{cm}{s}$, $V_{DD} = 1.8V$ and $L_p = L_n = 200nm$.

Solution:

$V_{OH} = V_{DD} = 1.8V$ and $V_{OL} = 0V$

$k_N = \mu_n C_{ox} \frac{W_n}{L_n} = 540 \frac{\mu A}{V^2}$ and $k_p = \mu_p C_{ox} \frac{W_p}{L_p} = 280 \mu A/V^2$

$\underline{V_{IL} = ?}$

Eq.(2) $\longrightarrow$ $$V_{IL} = \frac{2V_{OUT} - 1.8 - 0.5 + \left(\frac{540}{280}\right)0.5}{1 + \frac{540}{280}}$$

$$V_{OUT} = 0.7593 V_{IL} + 0.5875$$

Eq.(1) $$\frac{540}{2}(V_{IL} - 0.5)^2 = 280\left[(1.8 - V_{IL} - 0.5)(1.8 - V_{OUT}) - \frac{(1.8 - V_{OUT})^2}{2}\right]$$

Too simplify, assume that $V_{OUT} \approx 1.8 \rightarrow (1.8 - V_{OUT})^2 \approx 0$

$$(V_{IL} - 0.5)^2 = 1.037\left[(1.3 - V_{IL})(1.8 - V_{OUT})\right]$$

Quadratic $\rightarrow$ $V_{IL}^2 - 4.141V_{IL} + 2.46 = 0 \rightarrow \underline{\underline{V_{IL} = 0.719V}}$

(SPICE: 0.626V)

$\underline{V_{IH} = ?}$

Eq.(2) $$V_{IH} = \frac{2V_{OUT} + 0.5 + \frac{280}{540}(1.8 - 0.5)}{1 + \frac{280}{540}}$$

$$V_{OUT} = 0.7593 V_{IH} - 0.5875$$

Eq.(1) $$540\left[(V_{IH} - 0.5)V_{OUT} - \frac{V_{OUT}^2}{2}\right] = \frac{280}{2}\left[1.8 - V_{IN} - 0.5\right]^2$$

Assume that $V_{OUT} \approx 0$ so that $V_{OUT}^2 \approx 0$

$$\therefore (V_{IH} - 0.5) V_{OUT} \approx \frac{280}{2 \cdot 540} [1.3 - V_{IH}]^2$$

↑ Eq. (2)

(SPICE: 0.909)

$$V_{IH}^2 - 0.6299 V_{IH} - 0.3098 = 0 \longrightarrow \underline{\underline{V_{IH} = 0.954}}$$

Finally,

$$NM_H = V_{OH} - V_{IH} = 1.8 - 0.954 = 0.846\,V \quad (0.891)$$

$$NM_L = V_{IL} - V_{OL} = 0.719 - 0 = 0.719\,V \quad (0.676)$$