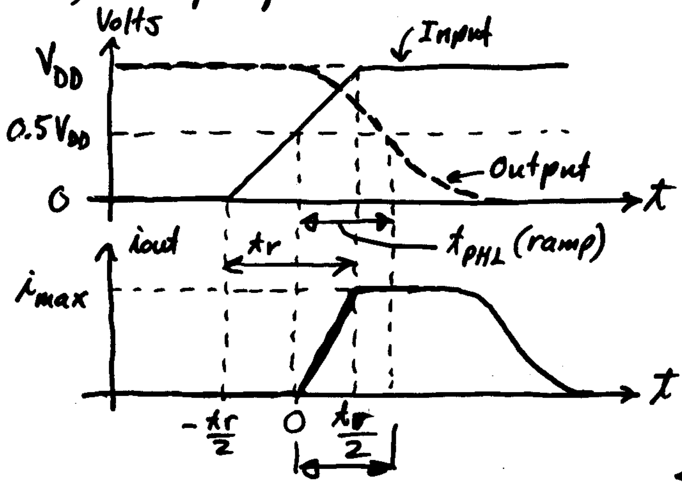
Improving the Delay Calculations including the Input Stage

1.) Step input

$$t_{PHL}(\text{step}) = C_L \frac{V_{DD}}{2\, i_{max}}$$

Problem Session on 3/4 at 7pm

2.) Ramp input



$$i_{out} = C_L \frac{dV_{out}}{dt}$$

$$\int_0^{t_{PHL}} i_{OUT}\, dt = C_L \int_0^{V_{DD}/2} dV_{out}$$

$$\int_0^{t_r/2} i_{max}\left(\frac{t}{t_r/2}\right) dt + \int_{\frac{t_r}{2}}^{t_{PHL}(\text{ramp})} i_{max}\, dt = C_L \frac{V_{DD}}{2}$$

$$\frac{i_{max}}{0.5 t_r} \frac{t^2}{2}\Bigg|_0^{t_r/2} + i_{max} t\Bigg|_{\frac{t_r}{2}}^{t_{PHL}(\text{ramp})} = C_L \frac{V_{DD}}{2}$$

$$\cancel{i_{max}}\left(\frac{t_r}{4}\right) + \cancel{i_{max}}\left[t_{PHL}(\text{ramp}) - \frac{t_r}{2}\right] = \frac{C_L \frac{V_{DD}}{2}}{i_{max}} = \underbrace{\frac{C_L V_{DD}}{2\, i_{max}}}_{t_{PHL}(\text{step})}$$

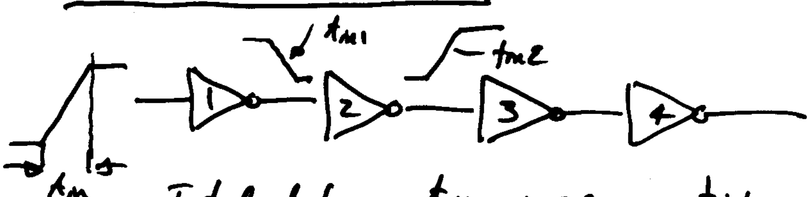
$$\frac{t_r}{4} + t_{PHL}(\text{ramp}) - \frac{t_r}{2} = t_{PHL}(\text{step})$$

$$\therefore \boxed{t_{PHL}(\text{ramp}) = \frac{t_r}{4} + t_{PHL}(\text{step})}$$

If t_{in} is the input rise or fall time, then

$$t_{ramp} \cong \frac{t_{in}}{2} + t_{step} = \frac{t_{in}}{2} + 0.7RC$$

Inverter Chain Delay



$$\text{Total delay} = \frac{t_{in}}{2} + 0.7R_1C_1 + \frac{t_{in1}}{2} + 0.7R_2C_2 + \frac{t_{in2}}{2} + 0.7R_3C_3 + \frac{t_{in3}}{2} + 0.7R_4C_4$$

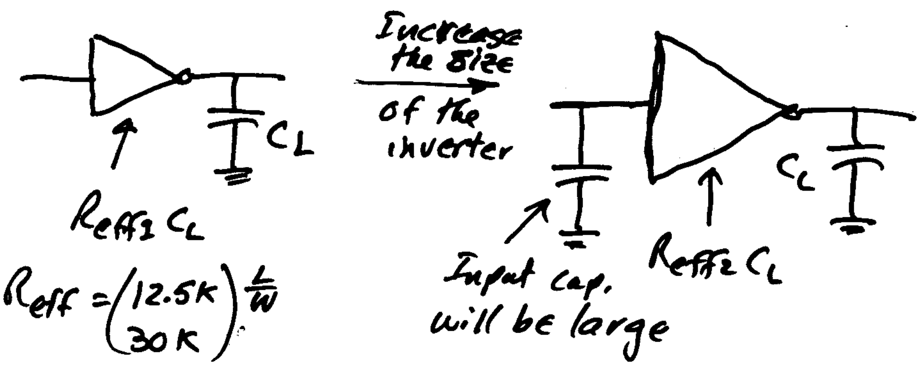
$$\text{Total delay} = \frac{t_{in}}{2} + 0.7R_1C_1 + \overbrace{\frac{0.7R_1C_1}{2}}^{0.35R_1C_1 \approx 0.3R_1C_1} + 0.7R_2C_2 + \frac{0.7R_2C_2}{2}$$

$$+ 0.7R_3C_3 + \frac{0.7R_3C_3}{2} + 0.7R_4C_4$$

$$\text{Total delay} \approx \underbrace{\frac{t_{in}}{2} + R_1C_1 + R_2C_2 + R_3C_3}_{\approx R_4C_4} + 0.7R_4C_4 = \sum_{i=1}^{n} R_iC_i$$

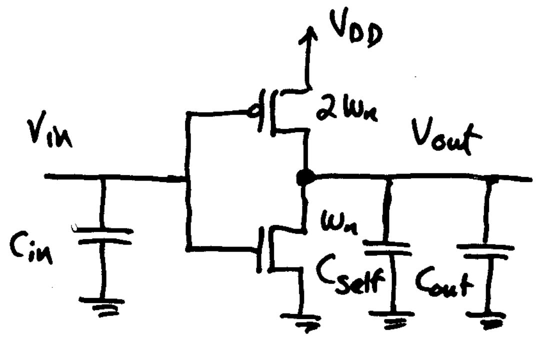
Gate Sizing for Optimal Delay

The problem: Minimize the global delay.



$$R_{eff} = \begin{pmatrix} 12.5K \\ 30K \end{pmatrix} \frac{L}{W}$$

Inverter Delay



Define the gate time constant of an inverter as,

$$\tau_{inv} = R_{eff}C_{in}$$

$$\tau_{inv} = R_{eqn}\left(\frac{L_n}{W_n}\right)C_g(3W_n) = 3R_{eqn}C_gL_n$$

$$t_{delay} = R_{eff}\left[C_{self} + C_{out}\right] \times \frac{C_{in}}{C_{in}} = R_{eff}C_{in}\left[\frac{C_{self}}{C_{in}} + \frac{C_{out}}{C_{in}}\right]$$

$$t_{delay} = \tau_{inv}\left[\frac{C_{out}}{C_{in}} + \gamma_{inv}\right] \quad \text{where} \quad \gamma_{inv} = \frac{C_{self}}{C_{in}}$$

Note: $\frac{C_{out}}{C_{in}}$ = fanout = f and C_{self} is strongly dependent on layout

Exercise 6.3

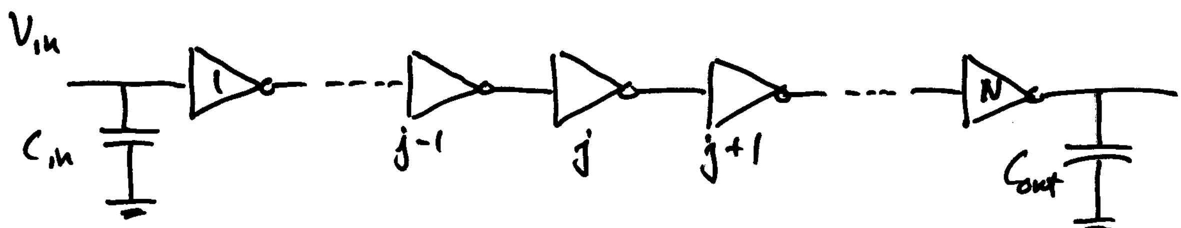
Find τ_{inv} and γ_{inv} for an inverter in $0.12\mu m$ technology

$$\tau_{inv} = 3 R_{equ} C_g L_n = 3(12.5k\Omega)(2fF/\mu m)(0.2\mu m) = 15ps$$

$$\gamma_{inv.} = \frac{C_{self}}{C_{in}} = \frac{C_{eff}(3W)}{C_g(3W)} = \frac{1fF/\mu m}{2fF/\mu m} = \frac{1}{2}$$

$$0 < \gamma_{inv} < 2$$

Optimal Sizing on An Inverter Chain



$$\text{Total Delay} = \sum_{j=1}^{N} \tau_{inv}\left(\frac{C_{j+1}}{C_j} + \gamma_{inv}\right) = \sum_{j=1}^{N} \tau_{inv}\left(\left(\frac{\cancel{C_g} W_{j+1}}{\cancel{C_g} W_j}\right) + \gamma_{inv}\right)$$

Define the consecutive delay as

$$D_j = \tau_{inv}\left(\frac{W_j}{W_{j-1}} + \gamma_{inv}\right) + \tau_{inv}\left(\frac{W_{j+1}}{W_j} + \gamma_{inv}\right)$$

$$\frac{\partial D_j}{\partial W_j} = 0 \longrightarrow \frac{\partial D_j}{\partial W_j} = \tau_{inv}\frac{1}{W_{j-1}} - \tau_{inv}\left(\frac{W_{j+1}}{W_j^2}\right) = 0$$

$$\frac{W_j}{W_{j-1}} = \frac{W_{j+1}}{W_j} \rightarrow \underline{\underline{W_j = \sqrt{W_{j+1} W_{j-1}}}}$$

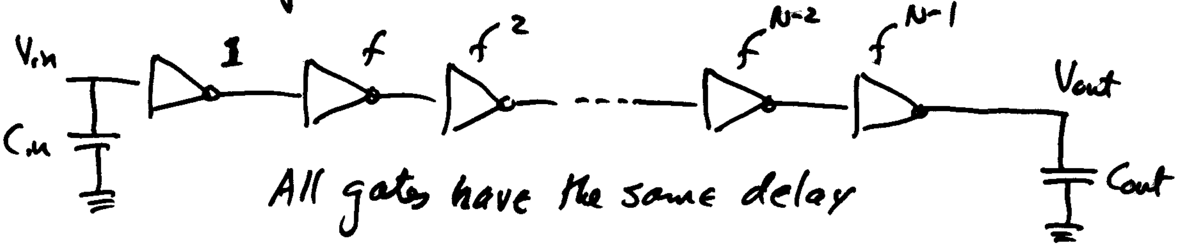
In general,

$$W_j = \left(W_1 W_2 \cdots W_{j-1} W_{j+1} \cdots W_{N-1} W_N\right)^{1/N}$$

Q. How do we achieve geometrical sizing?

A. We will increase size of each inverter by the fanout $= \frac{W_{j+1}}{W_j}$

$$f_j \equiv \frac{W_{j+1}}{W_j}$$



All gates have the same delay

Note that: $C_{out} = f^N C_{in} \rightarrow N = \frac{\ln(C_{out}/C_{in})}{\ln f}$

What optimizes the problem is knowing f and N given C_{in} and C_{out}.

$$\text{Total Delay} = N \times \text{Gate Delay} = N \times \tau_{inv}\left(\frac{C_{j+1}}{C_j} + \gamma_{inv}\right)$$

$$\boxed{\text{Total Delay} = \frac{\ln(C_{out}/C_{in})}{\ln f} \times \tau_{inv}(f + \gamma_{inv})}$$

For $\gamma_{inv} = 0 \rightarrow f = e = 2.72$

$0.5 < \gamma_{inv} < 2 \rightarrow 2.5 < f < 4$

Ex. 6.9

a.) Find the optimal fanout for $N=3$ and total delay if $C_{out} = 200fF$ and $C_{in} = 1fF$. Assume $\tau_{inv} = 7.5ps$ & $\gamma_{inv} = 0.5$

$$\ln f = \frac{\ln(C_{out}/C_{in})}{N} = \frac{\ln 200}{3} = 1.766 \rightarrow f = \underline{\underline{5.85}}$$

$$\text{Total delay} = 3(7.5ps)(5.85 + 0.5) = \underline{\underline{143ps}}$$

b.)