FINAL EXAMINATION
NAME SCORE /100

| Problem | 0 | 2 | 3 | 4 | 5 | 6 | 7 | Sum |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Points |  |  |  |  |  |  |  |  |

INSTRUCTIONS: This exam is closed book. You are permitted four sheets of notes (three of which are your sheets for the 3 midterms plus a new sheet for the final exam). There should be no xerox reductions on your note sheets. The exam consists of 7, 20point problems of which you are to work only 5 for a total of 100 points. Problem 1 must be worked and you may choose any four of the last six problems for a total of five problems. Please circle the number in the table above of the five problems you wish graded. If you do not indicate the problems to be graded, then problems 1 through 5 will be graded regardless of whether they are worked or not. Be sure to turn in only the 5 problems you wish graded in proper numerical order. Please show your work leading to your answers so that maximum partial credit may be given where appropriate.
TRANSISTOR INFORMATION:
The following is for an NMOS transistor only (with appropriate sign changes they can be used for PMOS transistors):

$$
V_{T}=V_{T 0}+\gamma\left(\sqrt{\left|2 \phi_{F}\right|+\mathrm{VSB}}-\sqrt{\left|2 \phi_{F}\right|}\right.
$$

For long-channel devices, the current equations are:
If $V_{G S} \geq V_{T}$ and $V_{D S} \geq V_{G S}-V_{T}$ (the saturation region) $\quad I_{D S}=\frac{\mathrm{k}}{2}\left(V_{G S}-V_{T}\right)^{2}\left(1+\lambda V_{D S}\right)$ or $V_{D S} \leq V_{G S}-V_{T}$ (the linear region) $\quad I_{D S}=\frac{\mathrm{k}}{2}\left[2\left(V_{G S}-V_{T}\right) V_{D S}-V_{D S}{ }^{2}\right]$

If $V_{G S}<V_{T}$ (subthreshold region)
$I_{D S} \approx 0$
For velocity saturated short-channel devices, the current equations are:
If $V_{G S} \geq V_{T}$
and $V_{D S} \geq \frac{\left(V_{G S}-V_{T}\right) E_{c} L}{\left(V_{G S}-V_{T}\right)+E_{c} L}$ (saturation region) $I_{D S}=W v_{s a t} C_{o x} \frac{\left(V_{G S}-V_{T}\right)^{2}}{\left(V_{G S}-V_{T}\right)+E_{c} L}$ or $V_{D S}<\frac{\left(V_{G S}-V_{T}\right) E_{c} L}{\left(V_{G S}-V_{T}\right)+E_{c} L}$ (linear region) $I_{D S}=\frac{W}{L} \frac{\mu_{e} C_{o x}}{\left(1+\frac{v_{D S}}{E_{c} L}\right)}\left(V_{G S}-V_{T}-\frac{V_{D S}}{2}\right) V_{D S}$
If $V_{G S}<V_{T}$ (subthreshold region) $I_{D S}=I_{S} \exp \left(\frac{q\left(V_{G S}-V_{T}-V_{\text {offset }}\right)}{n k T}\right)\left[1-\exp \left(\frac{-q V_{D S}}{k T}\right)\right]$

|  |  | $0.18 \mu \mathrm{~m}$ |  | $0.13 \mu \mathrm{~m}$ |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Name | Symbol | NMOS | PMOS | NMOS | PMOS | Units |
| Oxide Thickness | $t_{o x}$ | 35 | 35 | 22 | 22 | $\AA$ |
| Oxide Capacitance | $C_{o x}$ | 1.0 | 1.0 | 1.6 | 1.6 | $\mu \mathrm{~F} / \mathrm{cm}^{2}$ |
| Threshold Voltage | $V_{T O}$ | 0.5 | -0.5 | 0.4 | -0.4 | V |
| Body-Effect Term | $\gamma$ | 0.3 | 0.3 | 0.2 | 0.2 | $\mathrm{~V}^{0.5}$ |
| Fermi Potential | $2 \mid \phi_{F} \backslash$ | 0.84 | 0.84 | 0.88 | 0.88 | V |


| Junction Cap. Coeff. | $C_{j 0}$ | 1.6 | 1.6 | 1.6 | 1.6 | $\mathrm{fF} / \mu \mathrm{m}^{2}$ |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Built-In Junct. Potential | $\phi_{B}$ | 0.9 | 0.9 | 1.0 | 1.0 | V |  |  |
| Grading Coefficient | $m$ | 0.5 | 0.5 | 0.5 | 0.5 | - |  |  |
| Nominal Mobility | $\mu_{o}$ | 540 | 180 | 540 | 180 | $\mathrm{~cm}^{2} / \mathrm{V} \cdot \mathrm{s}$ |  |  |
| Effective Mobility | $\mu_{e}$ | 270 | 70 | 270 | 70 | $\mathrm{~cm}^{2} / \mathrm{V} \cdot \mathrm{s}$ |  |  |
| Critical Field | $E_{c}$ | $6 \times 10^{4}$ | $24 \times 10^{4}$ | $6 \times 10^{4}$ | $24 \times 10^{4}$ | $\mathrm{~V} / \mathrm{cm}$ |  |  |
| Critical Field x $L$ | $E_{c} L$ | 1.2 | 4.8 | 0.6 | 2.4 | V |  |  |
| Effective Resistance | $R_{e f f}$ | 12.5 | 30 | 12.5 | 30 | $\mathrm{k} \Omega / \mathrm{sq}$. |  |  |
| Saturation Velocity | $v_{\text {sat }}$ | $8 \times 10^{6}$ |  |  |  |  |  | $\mathrm{~cm} / \mathrm{s}$ |

CAPACITOR INFORMATION:

|  | Cutoff | Linear | Saturation |
| :--- | :--- | :--- | :--- |
| $C_{g s}$ | $C_{o l}$ | $C_{o l}+0.5 C_{g}$ | $C_{o l}+0.67 C_{g}$ |
| $C_{g d}$ | $C_{o l}$ | $C_{o l}+0.5 C_{g}$ | $C_{o l}$ |
| $C_{g b}$ | $1 /\left(1 / C_{g}+1 / C_{j c}\right)<C_{g b}<C_{g}$ | 0 | 0 |
| $C_{s b}$ | $C_{j S B}$ | $C_{j S B}+\alpha_{1} C_{j c}$ | $C_{j S B}+\beta_{1} C_{j c} \quad(\alpha, \beta$ small $)$ |
| $C_{d b}$ | $C_{j D B}$ | $C_{j D B}+\alpha_{2} C_{j c}$ | $C_{j D B}+\beta_{2} C_{j c} \quad(\alpha, \beta$ small $)$ |

where,$\quad C_{o l}=$ overlap capacitance $C_{j c}=$ channel to substrate depeletion capacitance

$$
C_{g}=C_{o x} L \quad C_{j}=\frac{C_{j b}\left(A_{b}+A_{s w}\right)}{\left(1-\frac{V_{j}}{\phi_{B}}\right)^{m j}}
$$

where $A_{b}=$ area of source/drain and $A_{s w}=$ area of side of source/drain facing the channel
Gate capacitance coefficient: $C_{g}=C_{o x} L+2 C_{o l} \approx 2 f F / \mu \mathrm{m}$
Self capacitance coefficient: $C_{e f f}=C_{j}+2 C_{o l} \approx 1 \mathrm{fF} / \mu \mathrm{m}$
Wire capacitance coefficient: $C_{w}=C_{\text {int }}=0.1 \mathrm{fF} / \mu \mathrm{m}$
OTHER INFORMATION:

$$
\varepsilon_{o x}=4 \varepsilon_{o}=4 \cdot 8.85 \times 10^{-14} \mathrm{~F} / \mathrm{cm}=3.54 \times 10^{-13} \mathrm{~F} / \mathrm{cm}
$$

Logical Effort:

| Type of Gate | 1 input | 2 inputs | 3 inputs | 4 inputs |
| :---: | :---: | :---: | :---: | :---: |
| Inverter | 1 | - | - | - |
| NAND | - | $4 / 3$ | $5 / 3$ | $6 / 3$ |
| NOR | - | $5 / 3$ | $7 / 3$ | $9 / 3$ |

## Parasitic Terms:

| Type of Gate | 1 input | 2 inputs | 3 inputs | 4 inputs |
| :---: | :---: | :---: | :---: | :---: |
| Inverter | $1 / 2$ | - | - | - |
| NAND | - | 1 | $3 / 2$ | 2 |
| NOR | - | $3 / 2$ | $9 / 4$ | 3 |

## Problem 1-(20 points - This problem is required)

a.) a.) An interconnect line is 10 mm long, $0.5 \mu \mathrm{~m}$ wide, and has a resistance of $27 \mathrm{~m} \Omega / \mathrm{sq}$. and a capacitance of $0.1 \mathrm{fF} / \mu \mathrm{m}$ and is driven by a 50 X inverter (an inverter with an $200 \lambda$ PMOS and a $100 \lambda$ NMOS where $\lambda=0.1 \mu \mathrm{~m}$ and $L=2 \lambda$ ). What is the total delay of the circuit as calculated from the Elmore delay assuming the pi model for the wire from the input of the inverter to the end of the interconnect line?

b.) Divide the 10 mm wire above into 5 equal segments and use a buffer of x 50 to drive each segment (use the existing buffer to drive the first segment). Find the new propagation delay calculated from the Elmore delay.

## Problem 2-(20 points - This problem is optional)

A bipolar-resistor inverter is shown along with the generic voltage transfer function. Use the large signal model of the BJT and the circuit to find values for $V_{O H}, V_{O L}, V_{I L}$, and $V_{I H}$. The large signal model for the BJT is
$I_{c}=I_{s} \exp \left(\frac{V_{\text {in }}}{V_{t}}\right)$

where $I_{s}=1 \mathrm{fA}$ and $V_{t}=25 \mathrm{mV}$. The collector-emitter saturation voltage of the BJT is $V_{C E}(\mathrm{sat})=0.2 \mathrm{~V}$. Use this information to find the values of $N M_{H}$ and $N M_{L}$.

Problem 3-(20 points - This problem is optional)
Design a pseudo-NMOS inverter in $0.13 \mu \mathrm{~m}$ technology to give $V_{O H}=$ $V_{D D}=1.2 \mathrm{~V}$ and $V_{O L}=0.1 \mathrm{~V}$. Assume $L=200 \mathrm{~nm}$ and find the value of the $W^{\prime}$ 's if $W_{p}=200 \mathrm{~nm}$. Assume $v_{s a t}=8 \times 10^{6} \mathrm{~cm} / \mathrm{s}$.


## Problem 4-(25 points)

Calculate the optimum delay (in ps ) and the size of the transistors (in $\mu \mathrm{m}$ ) for the logic circuit shown. All devices are standard CMOS and all transistors have a minimum length of $L=0.1 \mu \mathrm{~m} . C_{i n v}$ is the input capacitance of a minimum size inverter $(W=0.2 \mu \mathrm{~m})$.


## Problem 5-(20 points - This problem is optional)

For the logic circuit shown below, assume that the transmission gates are all $4 \lambda: 2 \lambda$ and that the inverters driving the transmission gates have PMOS transistors that are $8 \lambda: 2 \lambda$, and NMOS transistors that are $4 \lambda: 2 \lambda$, where $\lambda=0.1 \mu \mathrm{~m}$. The output inverter is to drive a 50 fF load. The output inverter is 4 times larger than the input inverters.
(a.) Write the logic expression for the output function in terms of $A, B$, sel, and selB.
(b.) Draw an equivalent $R C$ circuit model for the path from $A$ to $C$ assuming that the sel
 signal is high. Write down the individual contributions for each resistance and capacitance and place the total values at the appropriate nodes.
(c.) Find the Elmore delay from $A$ to $C$.

## Problem 6 - ( 20 points - This problem is optional)

A dynamic logic gate is shown. The pre-charge device has a W/L of 5, the n-channel devices have a $\mathrm{W} / \mathrm{L}$ of 3 , and inverter has a pull-up of 4 and a pull-down of 1. (a.) What is the logic function of the gate at the output of the inverter? (b.) Why is the output inverter skewed? (c.) Using the device sizes above, what is the logical effort of input $B$ of the first stage of the domino logic (when its immediate output is falling), and the inverter (when its output is rising). Compute these two values separately.


## Problem 7-(20 points - This problem is optional)

Design the values of the access and pull-down transistor in the memory cell shown. Assume you are designing a $256 \times 256$ memory array and that the bit lines must change by 150 mV in 300 ps at which point the sense amplifier is activated. The total capacitance of the bit lies are 450 fF each. Assume that $W_{D} / W_{A}=1.5$. Be sure to state any other assumptions you use in working this problem. $W_{A} . V_{D D}=1.2 \mathrm{~V}$.


Extra Sheet

