Homework Assignment No. 2 – Solutions

P2.4 – For each transistor, first determine if the transistor is in cutoff by checking to see if V_{GS} is less than or greater than V_T. V_T may have to be recalculated if the source of the transistor isn't grounded. If V_{GS} is less than V_T, then it is in cutoff, otherwise, it is in either triode or saturation.

To determine if it is in the triode saturation region, check to see if V_{DS} is less than or greater than V_{DSAT}. If V_{DS} is less than V_{DSAT}, then it is in triode, otherwise, it is in saturation.

a. Cutoff

$$V_{GS} = V_G - V_S = 0.2 - 0 = 0.2\text{V}$$
$$V_T = V_{T0} = 0.4\text{V}$$
$$\therefore V_{GS} < V_T$$

b. Cutoff

$$V_{GS} = V_G - V_S = 1.2 - 1.2 = 0\text{V}$$
$$V_T = V_{T0} = 0.4\text{V}$$
$$\therefore V_{GS} < V_T$$

c. Linear

$$V_{GS} = V_G - V_S = 1.2 - 0 = 1.2\text{V}$$
$$V_T = V_{T0} = 0.4\text{V}$$
$$\therefore V_{GS} > V_T$$

The transistor is not in the cutoff region.

$$V_{DSAT} = \frac{(V_{GS} - V_T)E_C L}{V_{GS} - V_T + E_C L} = \frac{(1.2 - 0.4)(6)(0.2)}{1.2 - 0.4 + (6)(0.2)} = 0.48\text{V}$$
$$V_{DS} = 0.2\text{V}$$
$$\therefore V_{DS} < V_{DSAT}$$

Saturation: In this case, because $V_D > V_G$ the transistor is in the saturation region. To see this, recognize that in a long-channel transistor if $V_D > V_G$, the transistor is in saturation. Since the saturation drain voltage V_{Dsat} is smaller in a velocity-saturated transistor than in a long-channel transistor, if the long-channel saturation region equation produces a saturated transistor, than the velocity-saturated saturation region equation will also.

P2.5 - In both cases, the first step it to calculate the maximum value of V_X given V_G. If the voltage at the drain is higher than this maximum value, then $V_X = V_{X,\max}$, otherwise, $V_X = V_D$. The maximum value of V_X is $V_G - V_T$ but $V_T \neq V_{T0}$ because of body effect and we consider its effect.

$$\begin{aligned} V_{X,\max} &= V_G - V_T = V_G - \left(V_{T0} + \gamma\left(\sqrt{V_{SB} + 2|\phi_F|} - \sqrt{2|\phi_F|}\right)\right) \\ &= V_G - V_{T0} - \gamma\left(\sqrt{V_{X,\max} + 2|\phi_F|} - \sqrt{2|\phi_F|}\right) \\ &= V_G - V_{T0} - \gamma\sqrt{V_{X,\max} + 2|\phi_F|} + \gamma\sqrt{2|\phi_F|} \\ &= 1.2 - 0.4 - 0.2\sqrt{V_{X,\max} + 0.88} + 0.2\sqrt{0.88} \\ &= 0.988 - 0.2\sqrt{V_{X,\max} + 0.88} \end{aligned}$$

There are two ways to calculate this, either through iteration or through substitution.

Iteration:
For the iteration method, we need a starting value for $V_{X,\max}$. A good starting value would be $V_G - V_{T0} = 1.2 - 0.4 = 0.8\text{V}$. We plug this value on the RHS of the equation, calculate a new $V_{X,\max}$ and repeat until we reach a satisfactory converged value.

Old Vx,max	New Vx,max
0.800	0.728
0.728	0.734
0.734	0.734

In this, only three iterations are needed to reach 0.734V.

Substitution:

The $\sqrt{V_{X,\max} + 0.88}$ term makes things a bit tricky, we get around this by making the following substitution:

$$x^2 = V_{X,\max} + 0.88$$
$$\therefore$$
$$V_{X,\max} = x^2 - 0.88$$

Therefore:

$$\begin{aligned} V_{X,\max} &= 0.988 - 0.2\sqrt{V_{X,\max} + 0.88} \\ x^2 - 0.88 &= 0.988 - 0.2\sqrt{x^2} \\ 0 &= x^2 + 0.2x - 1.87 \end{aligned}$$

P2.5 - Continued

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-0.2 \pm \sqrt{0.2^2 - 4(1)(-1.87)}}{2(1)} = 1.27, -1.47$$

$$V_{X,\max} = x^2 - 0.88 = 0.733, \cancel{1.28}$$

We use the first value since second value is above V_{DD}.

d. Since $V_D > V_{X,\max}$, $V_X = V_{X,\max} = 0.733\text{V}$.

e. Since $V_D < V_{X,\max}$, $V_X = V_{X,\max} = 0.6\text{V}$.

P2.7 – First, let's convert the units into terms of fF and μm.

$$L = 100\text{nm} \times \frac{10^6 \mu\text{m}}{10^9 \text{nm}} = 0.1\mu\text{m}$$

$$W = 400\text{nm} \times \frac{10^6 \mu\text{m}}{10^9 \text{nm}} = 0.4\mu\text{m}$$

$$Y = 300\text{nm} \times \frac{10^6 \mu\text{m}}{10^9 \text{nm}} = 0.3\mu\text{m}$$

$$\text{x}_\text{j} = 65\text{nm} \times \frac{10^6 \mu\text{m}}{10^9 \text{nm}} = 0.065\mu\text{m}$$

$$C_{ox} = 1.6 \times 10^{-6} \frac{\text{F}}{\text{cm}^2} \times \frac{10^{15}\text{fF}}{1\text{F}} \times \left(\frac{100\text{cm}}{10^6 \mu\text{m}}\right)^2 = 16 \frac{\text{fF}}{\mu\text{m}^2}$$

Now we can calculate the capacitances.

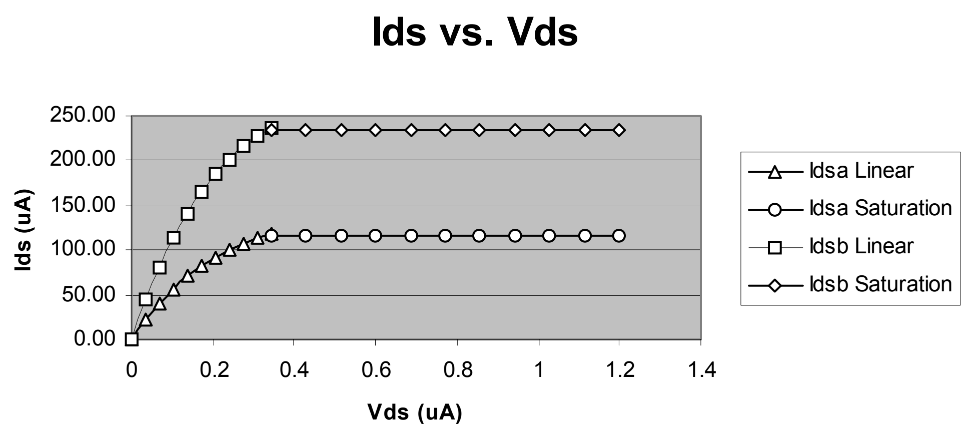
$$C_G = C_{ox} WL = (16)(0.4)(0.1) = 0.64\text{fF}$$

$$C_J = K_{eq} C_{jb} (Y + x_j) W = (0.8)(1.6)(0.3 + 0.065)(0.4) = 0.19\text{fF}$$

P2.9 – Since the lengths are the same, the saturation voltage V_{Dsat} will be the same.

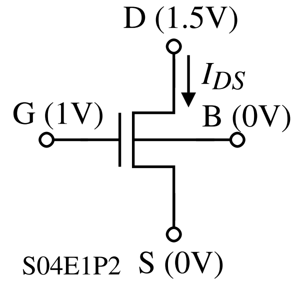
$$V_{Dsat} = \frac{(V_{GS} - V_T) E_C L}{V_{GS} - V_T + E_C L} = \frac{(1.2 - 0.4)(6)(0.1)}{1.2 - 0.4 + (6)(0.1)} = 0.34\text{V}$$

The graphs of the two transistors are shown in Figure 0. Notice that the main difference between the two curves is that when we double the width, we double the current.


Ids vs. Vds
250.00
200.00
150.00
100.00
50.00
0.00
Ids (uA)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
Vds (uA)
Idsa Linear
Idsa Saturation
Idsb Linear
Idsb Saturation

Problem 4

Solve for the dc value of the drain current, I_{DS}, for the NMOS transistor shown assuming 0.18µm CMOS technology. The W and L for this transistor are given in Problem 3.



Solution

Check for saturation.

$$V_{DS}(\text{sat}) = \frac{(V_{GS} - V_T)E_cL}{(V_{GS} - V_T) + E_cL} = \frac{(1\text{-}0.5)(1.2)}{(1\text{-}0.5) + 1.2} = 0.353\text{V}$$

$V_{DS} = 1.5\text{V} \Rightarrow$ NMOS in saturation

$$\therefore I_{DS} = Wv_{sat}C_{ox}\frac{(V_{GS} - V_T)^2}{(V_{GS} - V_T) + E_cL})$$

$W = 0.6\text{x}10^{-4}$ cm

$$v_{sat} = \frac{\mu_e E_c}{2} = \frac{270\ (\text{cm}^2/\text{V}\cdot\text{s})\ 6\text{x}10^4(\text{V/cm})}{2} = 8.1\text{x}10^6\ \text{cm/sec}$$

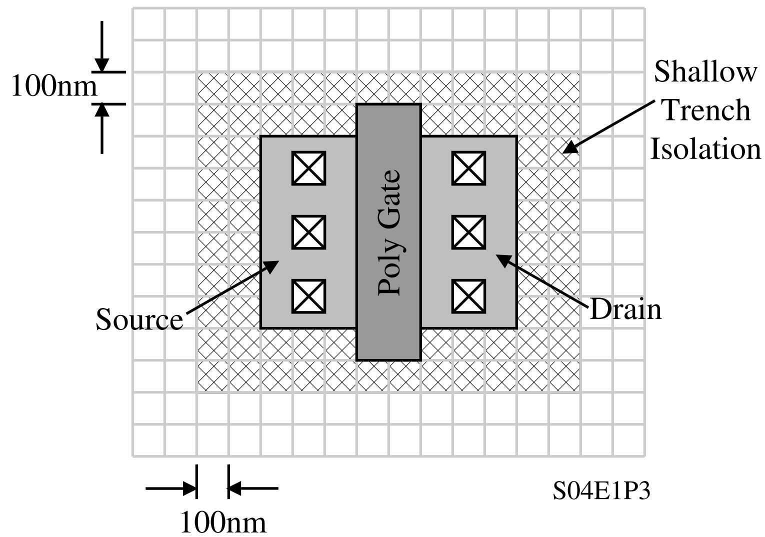
$$C_{ox} = \frac{\varepsilon_r(\varepsilon_o)}{t_{ox}} = \frac{4\cdot 8.85\text{x}10^{-14}\ (\text{F/cm})}{35\text{x}10^{-8}\ \text{cm}} = 1.01\text{x}10^{-6}\ \text{F/cm}^2$$

$$\therefore \quad I_{DS} = (0.6\text{x}10^{-4}\text{cm})(\ 8.1\text{x}10^6\ \text{cm/sec})(\ 1.01\text{x}10^{-6}\ \text{F/cm}^2)\left(\frac{0.5^2}{0.5 + 1.2}\right)(\text{V})$$

$$= \underline{\underline{72.18\ \mu\text{A}}}$$

Problem 5

Given the layout for the NMOS transistor of Problem 2, find the value of C_{gs}, C_{gd}, C_{gb}, C_{db}, and C_{sb} assuming that the junction depth of the source-drain diffusions is x_j = 50 nm, m = 0.5 and the lateral diffusion is 10nm.



Solution

From Problem 2, we know that the NMOS transistor is in saturation. To make the calculations, we will need C_g and C_{ov}. They are calculated as follows,

$$C_g = C_{ox} \cdot W \cdot L = 1.01\text{x}10^{-6}\ (\text{F/cm2}) \cdot 0.6\text{x}10^{-4}(\text{cm}) \cdot 0.2\text{x}10^{-4}(\text{cm}) = 1.212\text{x}10^{-15}\ \text{F}$$

(C_{ox} was calculated in Problem 2)

$$C_{ol} = C_{ox} \cdot LD = 1.01\text{x}10^{-6}\ (\text{F/cm2}) \cdot 10\text{x}10^{-7}(\text{cm}) = 1.01\text{x}10^{-12}\ \text{F/cm}$$

$$\therefore\ C_{gs} = C_{ol} \cdot W + 0.667C_g = (1.01\text{x}10^{-12})(0.6\text{x}10^{-4}) + (0.667)(1.212) = (0.061+0.808)\text{fF}$$

$$= \underline{\underline{0.868\ \text{fF}}}$$

$$C_{gd} = 0.061\text{fF} \approx \underline{\underline{0}}$$

$$C_{gb} = \underline{\underline{0}}$$

$$C_J = \frac{C_{jb}\,(A_b + A_{sw})}{\left(1 - \frac{V_j}{\phi_B}\right)} = \frac{C_{j0}\,(A_b + A_{sw})}{\left(1 - \frac{V_j}{\phi_B}\right)}$$

$$C_{bd} = \frac{1.6\text{fF}/\mu\text{m}^2[(0.3)(0.6)+(0.05)(0.6)]}{\sqrt{1 + \frac{1.5}{0.9}}} = \underline{\underline{0.206\ \text{fF}}}$$

$$C_{bs} = \frac{1.6\text{fF}/\mu\text{m}^2[(0.3)(0.6)+(0.05)(0.6)]}{\sqrt{1+0}} = \underline{\underline{0.336\ \text{fF}}}$$