

**Homework No. 4 – Solutions**Problem 1 – P4.2

Problem should refer to Figure P4.1

- a. Resistive load

$$V_{OH} = V_{DD}$$

$$\begin{aligned} V_{OL} &= \frac{V_{DD}}{1 + kR_L(V_{DD} - V_T)} = \frac{V_{DD}}{1 + \mu_N C_{OX} \frac{W}{L} R_L (V_{DD} - V_T)} \\ &= \frac{1.2}{1 + (260)(10^{-6})\left(\frac{1}{0.1}\right)(10 \times 10^3)(1.2 - 0.4)} = 0.055\text{V} \end{aligned}$$

- b. Saturated-enhancement load

$$\begin{aligned} V_{OH} &= V_{DD} - V_T = V_{DD} - \left( V_{T0} + \gamma \left( \sqrt{V_{SB} + 2|\phi_f|} - \sqrt{2|\phi_f|} \right) \right) \\ &= V_{DD} - V_{T0} - \gamma \sqrt{V_{OH} + 2|\phi_f|} + \gamma \sqrt{2|\phi_f|} \\ &= 1.2 - 0.4 - 0.2\sqrt{V_{OH} + 0.88} + 0.2\sqrt{0.88} \\ &= 0.988 - 0.2\sqrt{V_{OH} + 0.88} \end{aligned}$$

Iterate to produce:

$$V_{OH} = 0.733\text{V}$$

To compute  $V_{OL}$  we can ignore body effect and equate currents:

$$\begin{aligned} \therefore \frac{W_I}{L_I} \frac{\mu_N}{\left(1 + \frac{V_{OL}}{E_{CN} L_I}\right)} \left[ (V_{DD} - V_{TI}) V_{OL} - \frac{V_{OL}^2}{2} \right] &= \frac{W_L V_{sat} (V_{DD} - V_{OL} - V_{TL})^2}{(V_{DD} - V_{OL} - V_{TL}) + E_{CN} L_L} \\ \therefore \frac{1.2}{0.1} \frac{(270 \text{ cm}^2 / \text{V} \cdot \text{s})}{\left(1 + \frac{V_{OL}}{0.6}\right)} \left[ (1.2 - 0.4) V_{OL} - \frac{V_{OL}^2}{2} \right] &= \frac{(0.2 \mu\text{m})(8 \times 10^6 \text{ cm} / \text{s})(1.2 - V_{OL} - 0.4)^2}{(1.2 - V_{OL} - 0.4) + 0.6} \end{aligned}$$

Solve for  $V_{OL} \approx 0.03\text{V}$ 

- c. Linear-enhancement load

$$\begin{aligned}
V_{OH} &= V_{GG} - V_T = V_{DD} - \left( V_{T0} + \gamma \left( \sqrt{V_{SB} + 2|\phi_f|} - \sqrt{2|\phi_f|} \right) \right) \\
&= V_{GG} - V_{T0} - \gamma \sqrt{V_{OH} + 2|\phi_f|} + \gamma \sqrt{2|\phi_f|} \\
&= 1.6 - 0.4 - 0.2 \sqrt{V_{OH} + 0.88} + 0.2 \sqrt{0.88} \\
&= 1.388 - 0.2 \sqrt{V_{OH} + 0.88}
\end{aligned}$$

Iterate to produce:

$$V_{OH} = 1.11\text{V}$$

This tells us that  $V_{GG}$  should have been above 1.6V (closer to 1.7 V).

To compute  $V_{OL}$  we can ignore body effect and equate currents. Note that the load is saturated even though we call it a linear-enhancement load. The driver is also saturated due to the device sizes used.

$$\begin{aligned}
\therefore \frac{W_I V_{sat} (V_{DD} - V_{TI})^2}{(V_{DD} - V_{TI}) + E_{CN} L_I} &= \frac{W_L V_{sat} (V_{GG} - V_{OL} - V_{TL})^2}{(V_{GG} - V_{OL} - V_{TL}) + E_{CN} L_L} \\
\therefore \frac{(0.1\mu\text{m})(8 \times 10^6 \text{ cm/s})(1.2 - 0.4)^2}{(1.2 - 0.4) + 0.6} &= \frac{(0.1\mu\text{m})(8 \times 10^6 \text{ cm/s})(1.6 - V_{OL} - 0.4)^2}{(1.6 - V_{OL} - 0.4) + 0.6}
\end{aligned}$$

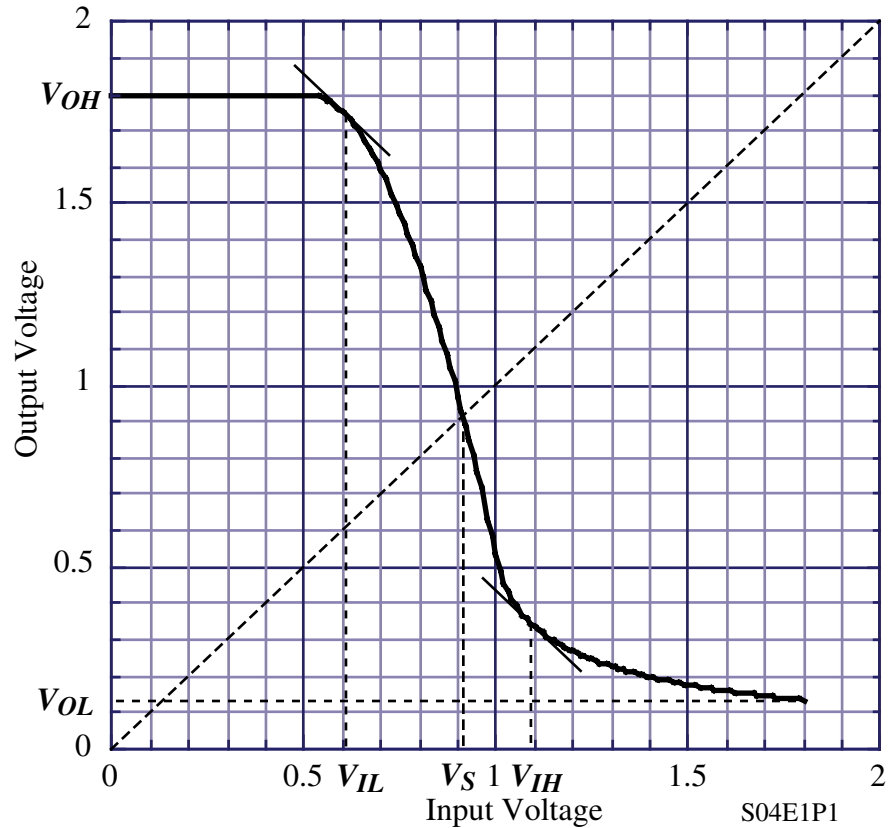
Solve for  $V_{OL} \approx 0.69\text{V}$

d. CMOS

$$V_{OH} = V_{DD} \quad V_{OL} = 0\text{V}$$

#### Problem 4

From the voltage transfer function curve shown, numerically identify,  $V_{OH}$ ,  $V_{OL}$ ,  $V_{IL}$ ,  $V_{IH}$ , and  $V_S$ . From these values, find the value of  $NM_H$  and  $NM_L$ .



From the curve we get the following numerical values.

$$V_{OH} = \underline{1.8V}, V_{OL} = \underline{0.13V}, V_{IL} = \underline{0.6V}, V_{IH} = \underline{1.08V}, \text{ and } V_S = \underline{0.92V}$$

The noise margins are,

$$NM_H = V_{OH} - V_{IH} = 1.80 - 1.08 = \underline{0.72V}$$

and

$$NM_L = V_{IL} - V_{OL} = 0.60 - 0.13 = \underline{0.47V}$$