Homework No. 5 – Solutions

Problem 1 – P4.3

For this problem, you are required to use the formulae:

$$V_{IL} = \frac{2V_{out} - V_{DD} - |V_{TP}| + (k_N / k_P)(V_{TN})}{1 + (k_N / k_P)} \qquad V_{IH} = \frac{2V_{out} + V_{TN} + (k_P / k_N)(V_{DD} - |V_{TP}|)}{1 + (k_P / k_N)}$$

We already know that V_{OH}=1.2 V and V_{OL}=0 V. For V_S use:

$$W_N = 4\lambda, \quad W_P = 16\lambda:$$

$$X = \sqrt{\frac{\frac{W_N}{E_{CN} L_N}}{\frac{W_P}{E_{CP} L_P}}} = \sqrt{\frac{W_N E_{CP}}{W_P E_{CN}}} = \sqrt{\frac{(4)(24)}{(16)(6)}} = 1$$

$$V_S = \frac{0.8 + (0.4)1}{1+1} = 0.6V$$

Next V_{IL} and V_{IH} are estimated as follows:

$$V_{IL} = \frac{2V_{out} - V_{DD} - |V_{TP}| + (k_N / k_P)(V_{TN})}{1 + (k_N / k_P)} = \frac{2V_{out} - 1.2 - |-0.4| + (1)(0.4)}{1 + (1)} = \frac{2V_{out} - 1.2}{2} = 0.55V$$

$$V_{IH} = \frac{2V_{out} + V_{TN} + (k_P / k_N)(V_{DD} - |V_{TP}|)}{1 + (k_P / k_N)} = \frac{2V_{out} + 0.4 + (1)(1.2 - 0.4)}{1 + (1)} = \frac{2V_{out} + 1.2}{2} = 0.65V$$

Therefore

$$NM_L = 0.55 - 0 = 0.55V$$
$$NM_H = 1.2 - 0.65V = 0.55V$$

When we cut the size of the PMOS device in half, the VTC shifts to the left. So V_{IL}, V_S, and V_{IH} will all shift to the left. The recalculation of the switching threshold produces V_S=0.566V.

We can compute V_{IL} to be roughly 0.533V and V_{IH} to be roughly 0.667V.

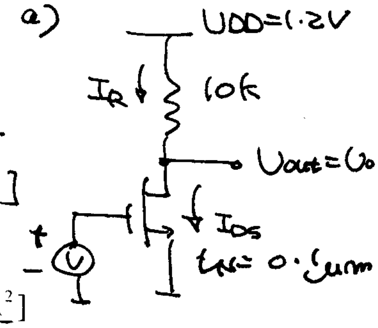
Therefore

$$NM_L = 0.533 - 0 = 0.533V$$
$$NM_H = 1.2 - 0.667V = 0.533V$$

⟨Hw#4 Solution⟩

Design the circuit for $V_{OL}=0.1V$, $L=0.1\mu m$ in $0.13\mu m$ technology.

Problem 2 – P4.9



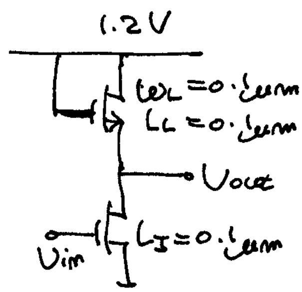
* Resistive Load inverter:

$E_CL=0.6V$
$V_{OH}=1.2V$
$V_{OL}=0.1V$
$C_{ox}=1.6\mu F/cm^2$
$\mu_n=270\ \frac{cm^2}{V\cdot s}$
$V_{sat}=8\times10^6$

$$\frac{V_{DD}-V_{OL}}{R_L}=\frac{W_N}{L_N}\frac{\mu_n C_{ox}}{\left(1+\frac{V_{OL}}{E_c L}\right)}\cancel{[2(V_{OH}-V_T)V_{OL}-V_{OL}^2]}\left[(V_{OH}-V_T)V_{OL}-\frac{V_{OL}^2}{2}\right]$$

$$\frac{1.2-0.1}{10k}=\frac{W_N}{0.1}\frac{(270)(1.6\times10^{-6})}{\left(1+\frac{0.1}{0.6}\right)}[\cancel{2}(1.2-0.4)0.1-\frac{0.1^2}{2}]$$

$\therefore W_N=0.2\mu m$ $\boxed{W=0.396\mu m}$ ✓



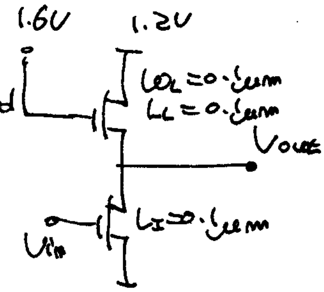
* Saturated Enhancement Load inverter (ignoring body-effect):

$I_I(lin) = I_L(sat)$

$$\frac{W_I}{L_I}\frac{\mu_N C_{ox}}{\left(1+\frac{V_{out}}{E_{CN}L_I}\right)}\left[(V_{in}-V_{TI})V_{out}-\frac{V_{out}^2}{2}\right]=\frac{W_L v_{sat}C_{ox}(V_{DD}-V_{out}-V_{TL})^2}{(V_{DD}-V_{out}-V_{TL})+E_{CN}L_L}$$

$$\frac{W_I}{0.1}\frac{(270)(1.6\times10^{-6})}{\left(1+\frac{0.1}{0.6}\right)}[\cancel{2}(1.2-0.4)0.1-\frac{0.1^2}{2}]=\frac{0.1(10^{-4})(8)(1.6)(1.2-0.1-0.4)^2}{(1.2-0.1-0.4)+0.6}$$ ✓

$\therefore W_N=0.1\mu m$ $\boxed{W=0.174\mu m}$



* Linear Enhancement Load inverter (ignoring body-effect): Load is saturated

$$\frac{W_I}{L_I}\frac{\mu_N C_{ox}}{\left(1+\frac{V_{out}}{E_{CN}L_I}\right)}\left[(V_{in}-V_{TI})V_{out}-\frac{V_{out}^2}{2}\right]=\frac{W_L v_{sat}C_{ox}(V_{DD}-V_{out}-V_{TL})^2}{(V_{DD}-V_{out}-V_{TL})+E_{CN}L_L}$$

$$\frac{W_I}{0.1}\frac{(270)(1.6\times10^{-6})}{\left(1+\frac{0.1}{0.6}\right)}[\cancel{2}(1.2-0.4)0.1-\frac{0.1^2}{2}]=\frac{0.1(10^{-4})(8)(1.6)(1.6-0.1-0.4)^2}{(1.6-0.1-0.4)+0.6}$$ ✓

$\therefore W_N=0.6\mu m$ $\boxed{W=0.328\mu m}$

The linear enhancement load inverter requires the largest pull-down device since it has the strongest pull up device. The resistive load inverter is next and the saturated enhancement load requires the smallest pull-down device.

Problem 3 – P4.10

We will illustrate the process and estimate the solutions for this problem.

We already know that V_{OH}=1.2 V and V_{OL}=0 V. For V_S use:

$$W_N = 0.4um, \quad W_P = 0.8um:$$

$$X = \sqrt{\frac{\dfrac{W_N}{E_{CN}L_N}}{\dfrac{W_P}{E_{CP}L_P}}} = \sqrt{\frac{W_N E_{CP}}{W_P E_{CN}}} = \sqrt{\frac{(0.4)(24)}{(0.8)(6)}} = 1.41$$

$$V_S = \frac{0.8 + (0.4)1.41}{1 + 1.41} = 0.566$$

Next V_{IL} and V_{IH} are estimated as follows:

$$V_{IL} = \frac{2V_{out} - V_{DD} - |V_{TP}| + (k_N / k_P)(V_{TN})}{1 + (k_N / k_P)}$$

$$V_{IL} = \frac{2V_{out} - 1.2 - |-0.4| + (2)(0.4)}{1 + (2)} = \frac{2V_{out} - 0.6}{3}$$

We can compute V_{IL} to be roughly 0.533V.

$$V_{IH} = \frac{2V_{out} + V_{TN} + (k_P / k_N)(V_{DD} - |V_{TP}|)}{1 + (k_P / k_N)}$$

$$V_{IH} = \frac{2V_{out} + 0.4 + (2)(1.2 - 0.4)}{1 + (2)} = \frac{2V_{out} + 2}{3}$$

We can compute V_{IH} to be roughly 0.667V.

When we double the size of the PMOS device, the VTC shifts to the right. So V_{IL}, V_S, and V_{IH} will all shift to the right. The recalculation of the switching threshold produces V_S=0.6V.

We can compute V_{IL} to be roughly 0.55V and V_{IH} to be roughly 0.65V.

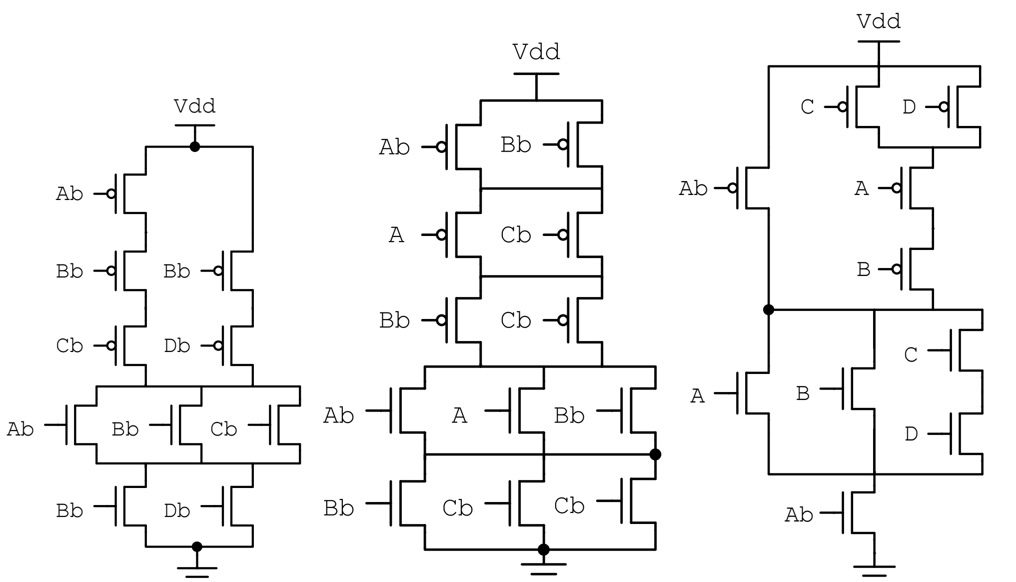
Problem 2 – P5.1

For each problem, restate each Boolean equation into a form such that it can be translated into the p and n-complex of a CMOS gate.

a. $Out = ABC + BD = \overline{\overline{ABC + BD}} = \overline{(\overline{A} + \overline{B} + \overline{C})(\overline{B} + \overline{D})}$

b. $Out = AB + \overline{A}C + BC = \overline{\overline{AB + \overline{A}C + BC}} = \overline{(\overline{A} + \overline{B})(A + \overline{C})(\overline{B} + \overline{C})}$

c. $Out = \overline{A + B + CD} + A = \overline{A}\,\overline{B}(\overline{C} + \overline{D}) + A = \overline{\overline{\overline{A + B + CD} + A}} = \overline{(A + B + CD)\overline{A}}$



Problem 3 – P5.8

The solution is shown below. Notice that there is no relevance with the lengths and widths of the transistors when it comes to V_{OH}, although they the do matter when calculating V_{OL}.

$$V_{out} = V_{GG} - V_T$$

$$V_{GG} = V_{out} + V_{T0} + \gamma\left(\sqrt{V_{out} + 2|\phi_F|} - \sqrt{2|\phi_F|}\right)$$

$$= 1.8 + 0.5 + 0.3\left(\sqrt{1.8 + 0.88} - \sqrt{0.88}\right) = 2.51\text{V}$$

Problem 4 – P5.9

For t_{PLH}, we need to size the pull-up PMOS appropriately.

$$t_{PLH} = 0.7RC = 0.7R_{eqp}\frac{L}{W}C_{LOAD}$$

$$W_p = 0.7R_{SQ}\frac{L}{t_{PLH}}C_{LOAD} = 0.7\left(30k\Omega\right)\frac{\left(2\lambda\right)}{\left(50\times10^{-12}\right)}\left(100\times10^{-15}\right) = 84\lambda$$

For V_{OL}:

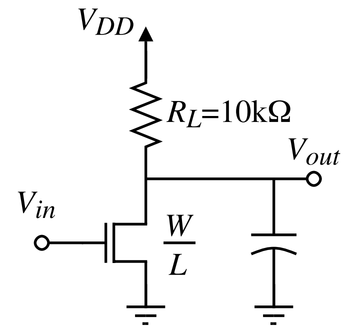
$$I_P\left(sat\right) = \frac{W_P v_{sat} C_{OX}\left(V_{GS}-V_T\right)^2}{V_{GS}-V_T+E_{CP}L} = \frac{\left(4.2\times10^{-4}\right)\left(8\times10^{6}\right)\left(1.6\times10^{-6}\right)\left(1.2-0.4\right)^2}{1.2-0.4+\left(24\right)\left(0.1\right)} = 1.08\text{mA}$$

$$I_P\left(sat\right) = \frac{W_N\mu_N C_{OX}\left(V_{OL}-V_{TN}-\frac{V_{OL}}{2}\right)V_{OL}}{L_N\left(1+\frac{V_{OL}}{E_{CN}L}\right)} = \frac{W_N(270)(1.6\times10^{-6})\left(1.2-0.4-\frac{0.1}{2}\right)0.1}{L_N\left(1+\frac{0.1}{0.6}\right)}$$

$$\frac{W_N}{L_N} = 38.5 \quad W_N = 77\lambda \qquad W_3 = 3\times77\lambda = 232\lambda \quad (3\ stack) \quad W_2 = 155\lambda \quad (2\ stack)$$

Problem 4

An NMOS transistor with a 10kΩ resistor as a load is used to implement a simple inverter as shown. The alpha-power model of Section 2.6 is used to fit the measured data for the NMOS transistor to produce the following two equations:



$$i_{DS} = (W/L)K_L(v_{GS} - V_{TN})v_{DS} \qquad v_{DS} \le V_{DS(\text{sat})}$$

$$i_{DS} = (W/L)K_S(v_{GS} - V_{TN})^{1.5} \qquad v_{DS} \ge V_{DS(\text{sat})}$$

where K_L = 100µA/V^2 and K_S = 100µA/V$^{1.5}$ and V_{TN} = 0.6V.

a.) Derive the expression for $V_{DS(\text{sat})}$ assuming the model above.

b.) Design V_{DD} and W/L of the resistively loaded inverter above to achieve V_{OH} = 3.3V and V_{OL} = 0.3V.

c.) For the inverter of part b.) derive an expression for V_{IL} using the given alpha-power model. Using the previous values, evaluate V_{IL}.

Solution

a.) Equate the two equations for the linear and saturation regions to get,

$$\frac{W}{L}K_L(V_{GS}\text{-}V_{TN})V_{DS(\text{sat})} = \frac{W}{L}K_S(v_{GS} - V_{TN})^{1.5} \rightarrow \boxed{V_{DS(\text{sat})} = \frac{K_S}{K_L}\sqrt{v_{GS} - V_{TN}}}$$

b.) Since $V_{OH} = V_{DD}$, let V_{DD} = $\underline{\underline{3.3\text{V}}}$. Solve for V_{OL} by assuming the MOSFET is in the linear region.

$$\frac{V_{DD} - V_{OL}}{R_L} = \frac{3.3\text{-}0.3}{10\text{k}\Omega} = 300\mu\text{A} = \frac{W}{L}K_L(V_{DD} - V_{TN})V_{OL} = \frac{W}{L}\,100\mu\text{A/V}^2(3.3\text{-}0.6)0.3$$

$$\frac{W}{L} = \frac{300\mu\text{A}}{81\mu\text{A}} = \underline{\underline{3.7}}$$

c.) For V_{IL} assume the MOSFET is saturated. Therefore,

$$\frac{V_{DD} - V_{out}}{R_L} = \frac{W}{L}K_S(V_{in} - V_{TN})^{1.5}$$

Differentiating with respect to V_{in} gives,

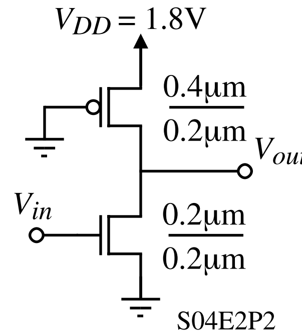
$$-\frac{1}{R_L}\frac{dV_{out}}{dV_{in}} = 1.5\frac{W}{L}K_S(V_{in} - V_{TN})^{0.5} \rightarrow \qquad \frac{1}{R_L} = 1.5\,(3.7)\,K_S(V_{IL} - V_{TN})^{0.5}$$

$$V_{IL} = V_{TN} + \frac{1}{[(1.5\cdot(W/L))K_S\,R_L]^2} = 0.6 + \frac{1}{(1.5\cdot 3.7\cdot 100\cdot 0.01)^2}$$

$$= 0.6 + 0.0325 = \underline{\underline{0.6325\text{V}}}$$

Problem 5

For the pseudo-NMOS load inverter shown using 0.18μm CMOS technology, determine V_{OH} and estimate V_{OL} using the velocity saturated model with effective mobility (high vertical field). Be sure to clearly state any assumptions used in estimating V_{OL}.



S04E2P2

Solution

For this inverter, we know that $V_{OH} \approx V_{DD} = \underline{\underline{1.8\text{V}}}$

For V_{OL}, we need to assume the state of the transistors. To help in this calculate $V_{DS(\text{sat})}$ for the NMOS and PMOS transistors.

$$\text{PMOS: } V_{SD(\text{sat})} = \frac{(V_{SG} - |V_{TP}|)E_{CP}L_P}{(V_{SG} - |V_{TP}|) + E_{CP}L_P} = \frac{(1.8\text{-}0.5)4.8}{(1.8\text{-}0.5)+4.8} = 1.023\text{V}$$

$$\text{NMOS: } V_{DS(\text{sat})} = \frac{(V_{GS} - V_{TN})E_{CN}L_N}{(V_{GS} - V_{TN}) + E_{CN}L_N} = \frac{(1.8\text{-}0.5)1.2}{(1.8\text{-}0.5)+1.2} = 0.624\text{V}$$

So if $V_{OL} < 0.624$V, the PMOS is saturated and the NMOS is linear. Assuming this to be the case, we get:

$$\frac{W_n}{L_n}\frac{\mu_e C_{ox}}{1+\frac{V_{DS}}{E_{CN}L_N}}\left(V_{GS} - V_{TN} - \frac{V_{DS}}{2}\right)V_{DS} = W_p v_{sat} C_{ox}\frac{(V_{SG} - |V_{TP}|)^2}{(V_{SG} - |V_{TP}|) + E_{CP}L_P}$$

Assuming V_{DS} which is V_{OL} is small, we can simplify the above to,

$$\frac{W_n}{L_n}\mu_e(V_{GS} - V_{TN})V_{OL} \approx W_p v_{sat}\frac{(V_{SG} - |V_{TP}|)^2}{(V_{SG} - |V_{TP}|) + E_{CP}L_P}$$

Substituting the values gives,

$$270(\text{cm}^2/\text{v}\cdot\text{s})(1.8\text{-}0.5)V_{OL} = 0.4\text{x}10^{-4}(\text{cm})(8\text{x}10^6)(\text{cm/s})\frac{(1.8\text{-}0.5)^2}{1.8\text{-}0.5+4.8}$$

$$351V_{OL} = 88.656 \quad \rightarrow \quad V_{OL} = \underline{\underline{0.253\text{V}}}$$

This problem can also be solved exactly for V_{OL} as follows.

$$\frac{270(1.8\text{-}0.5\text{-}0.5V_{OL})V_{OL}}{1+V_{OL}/1.2} = (0.4\text{x}10^{-4})(8\text{x}10^6)\frac{(1.8\text{-}0.5)^2}{1.8\text{-}0.5+4.8} = 88.656$$

$$270(1.3\text{-}0.5V_{OL})V_{OL} = 88.656(1+0.833V_{OL})$$

$$351V_{OL} - 135V_{OL}^2 = 88.656 + 73.88V_{OL}$$

$$\therefore \quad 135V_{OL}^2 + (73.88\text{-}351)V_{OL} + 88.656 = 0 \;\rightarrow\; V_{OL}^2 - 2.053V_{OL} + 0.6567 = 0$$

$$V_{OL} = 1.0265 \pm 0.5\sqrt{2.053^2\text{-}4\cdot 0.6567} = 1.0265 \pm 0.5\sqrt{1.588} = 1.0265 \pm 0.6300$$

$$\therefore \quad V_{OL} = \underline{\underline{0.3964\text{V}}}$$

Either answer will be accepted provided the work is correct and the assumptions consistent.