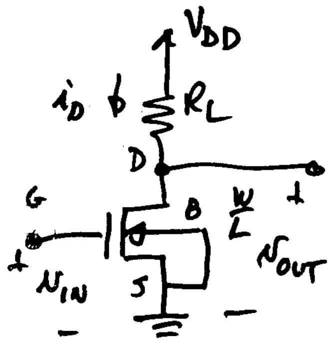
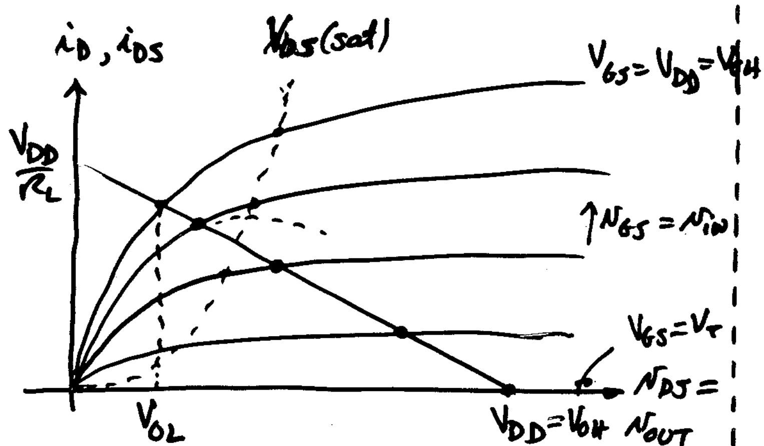
Exam 1 - Friday, Feb. 11 (see website for Exam 1 Preview)

Problem Session - 7pm - 8pm, Thurs. Homework: P4.3 & P4.9 → #5

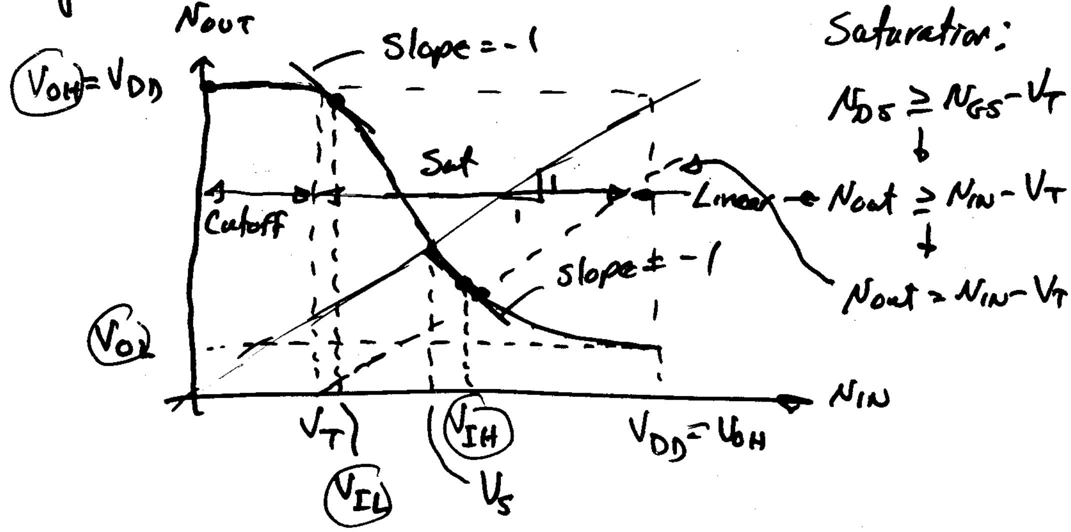
RESISTOR LOAD INVERTER DESIGN

Graphical Perspective





Voltage Transfer Curve (VTC):



$$NM_H = V_{OH} - V_{IH} \qquad NM_L = V_{IL} - V_{OL}$$

Mathematical Perspective

1.) $V_{OH} = V_{DD}$ If $v_{in} \le V_T$

2.) $V_{OL} = ?$ Assume the MOSFET is in the linear region & min. channel length (velocity saturation)

$I_R = I_{DS}(\text{linear})$ and $V_{GS} = V_{DD} = V_{OH}$ & $V_{DS} = V_{OL}$

↓

$$\frac{V_{DD} - V_{OL}}{R_L} = \frac{1}{2} \frac{W_n}{L_n} \left(\frac{\mu_n C_{ox}}{1 + \frac{V_{OL}}{E_c L_n}} \right) \left[2(V_{OH} - V_T) V_{OL} - V_{OL}^2 \right]$$

Assume also that $E_c L_n > V_{OL}$,

$$\frac{V_{DD} - V_{OL}}{R_L} \approx \underbrace{\frac{\mu_n C_{ox}}{2} \frac{W_n}{L_n}}_{\frac{k}{2}} \left[2(V_{OH} - V_T) V_{OL} - V_{OL}^2 \right]$$

where $k = \mu_n C_{ox} \frac{W_n}{L_n}$

$$\frac{2(V_{DD} - V_{OL})}{k R_L} = 2(V_{OH} - V_T) V_{OL} - V_{OL}^2$$

Writing as a quadratic,

$$V_{OL}^2 - 2\left(\frac{1}{k R_L} + V_{DD} - V_T \right) V_{OL} + \frac{2 V_{DD}}{k R_L} = 0$$

Assume further that V_{OL}^2 is small, i.e. $V_{OL}^2 \approx 0$

$$V_{OL} \approx \frac{V_{DD}}{1 + k R_L (V_{DD} - V_T)} \quad (V_{DD} = V_{OH})$$

Tradeoffs:

$k \uparrow \Rightarrow V_{OL} \downarrow$, Area ↑, Fall time ↓, Power ↑

$R_L \uparrow \Rightarrow V_{OL} \downarrow$, Area ↑, Rise time ↑, Power ↓

3.) $V_{IL} = ?$ Assume the MOSFET is saturated and that $V_{IN} - V_T$ is small.

$$I_R = I_{DS} \rightarrow \frac{V_{DD} - V_{OUT}}{R_L} = \frac{W v_{sat} C_{ox} (V_{IN} - V_T)^2}{(V_{IN} - V_T) + E_c L} \approx \frac{W v_{sat} C_{ox} (V_{IN} - V_T)^2}{E_c L}$$

However, $v_{sat} = \frac{\mu E_c}{2}$,

$$\frac{V_{DD} - V_{out}}{R_L} = \overbrace{\frac{\mu_n C_{ox}}{2}}^{k'} \frac{W_n}{L_n} (V_{IN} - V_T)^2 = \frac{k}{2} (V_{IN} - V_T)^2$$

Differentiate w.r.t. V_{IN},

$$-\frac{1}{R_L} \frac{\partial V_{out}}{\partial V_{IN}} = k (V_{IN} - V_T) 1 \rightarrow \frac{1}{R_L} = k (V_{IL} - V_T)$$

$$V_{IL} = V_T + \frac{1}{k R_L}$$

$$NM_L = V_{IL} - V_{OL} = V_T + \frac{1}{k R_L} - \frac{V_{DD}}{1 + k R_L (V_{DD} - V_T)}$$

4.) V_{IH} Assume that the MOSFET is in linear operation and $V_{OUT} < E_c L$

$$I_{DS} = I_R \rightarrow \frac{W}{L} \frac{\mu_n C_{ox}}{1 + \frac{V_{OUT}}{E_c L}} \left[(V_{IN} - V_T) V_{OUT} - \frac{V_{OUT}^2}{2} \right] = \frac{V_{DD} - V_{OUT}}{2}$$

$$k \left[(V_{IN} - V_T) V_{OUT} - \frac{V_{OUT}^2}{2} \right] \approx \frac{V_{DD} - V_{OUT}}{R_L}$$

Take the partial derivative w.r.t. V_{IN},

$$k \left[(V_{IN} - V_T) \frac{\partial V_{OUT}}{\partial V_{IN}} + V_{OUT} - V_{OUT} \frac{\partial V_{OUT}}{\partial V_{IN}} \right] = -\frac{1}{R_L} \frac{\partial V_{OUT}}{\partial V_{IN}}$$

$$k \left[-(V_{IN} - V_T) + 2 V_{OUT} \right] = \frac{1}{R_L} \rightarrow V_{IN} = V_{IH} = 2 V_{OUT} + V_T - \frac{1}{k R_L}$$

Solve for V_{IH} from the following eqs.

1.) $V_{IH} = 2V_{OUT} + V_T - \frac{1}{kR_L}$

2.) $\frac{k}{2}\left[2(V_{IH}-V_T)V_{OUT} - V_{OUT}^2\right] = \frac{V_{DD}-V_{OUT}}{R_L}$

$$V_{IH} = V_T + \sqrt{\frac{8V_{DD}}{3kR_L}} - \frac{1}{kR_L}$$

$$\therefore\ NM_H = V_{OH} - V_{IH} = V_{DD} - V_T - \sqrt{\frac{8V_{DD}}{3kR_L}} + \frac{1}{kR_L}$$

Example

Find NM_L and NM_H if $k' = 430\frac{\mu A}{V^2}$, $V_T = 0.4V$, $\frac{W}{L} = 2$, $V_{DD} = 1.2V$ and $R_L = 20k\Omega$.

Solution;

$V_{OH} = V_{DD} = 1.2V \qquad k = k'\frac{W}{L} = 860\frac{\mu A}{V^2}$

$$V_{OL} \approx \frac{1.2}{1+(0.86)(20)(1.2-0.4)} = 0.081V$$

$$V_{IL} = 0.4 + \frac{1}{(0.86)(20)} = 0.46V$$

$$V_{IH} = 0.4 + \sqrt{\frac{8(1.2)}{3(0.86)(20)}} - \frac{1}{(0.86)(20)} = 0.773$$

$$NM_L = V_{IL} - V_{OL} = 0.46 - 0.081 = 0.38V$$

$$NM_H = V_{OH} - V_{IH} = 1.2 - 0.773 = 0.43V$$