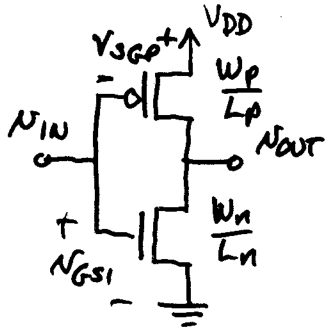
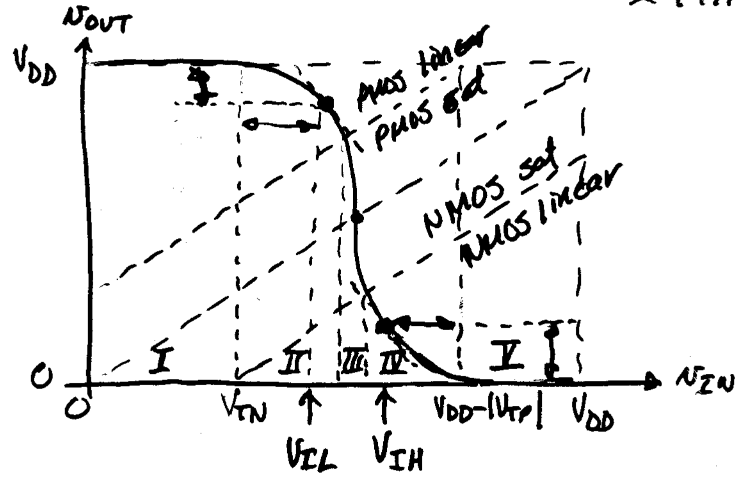
DC Analysis of the CMOS Inverter – Cont'd

Exam #1 Aver. ≈ 79/100





V_{IL} (Region II)

Procedure:

1.) Equate currents for the correct operating region of FETs
2.) Perform $\frac{\partial}{\partial v_{IN}}$ and set $\frac{\partial v_{OUT}}{\partial v_{IN}} = -1$
3.) Solve the above two eqs. for V_{IL}

$$I_{DN}(sat) = I_{DP}(lin)$$

$$\frac{W_N v_{sat} C_{ox} (v_{IN} - V_{TN})^2}{(v_{IN} - V_{TN}) + E_{CN} L_N} = \frac{W_p}{L_p} \frac{\mu_p C_{ox}}{1 + \frac{V_{DD} - v_{OUT}}{E_{CP} L_p}} \left[(V_{DD} - v_{IN} - |V_{TP}|)(V_{DD} - v_{OUT}) - \frac{(V_{DD} - v_{OUT})^2}{2} \right]$$

Assumptions: $v_{IN} - V_{TN} \approx 0$ and $v_{OUT} \approx V_{DD} \Rightarrow v_{OUT} - v_{DD} = \text{small}$

$$\frac{W_N v_{sat} C_{ox} (v_{IN} - V_{TN})^2}{E_{CN} L_N} \approx \frac{W_p}{L_p} \mu_p C_{ox} \left[(V_{DD} - v_{IN} - |V_{TP}|)(V_{DD} - v_{OUT}) - \frac{(V_{DD} - v_{OUT})^2}{2} \right]$$

Since $v_{sat} = \frac{\mu_n E_{CN}}{2}$

$$(1) \quad \frac{W_N}{L_N} \frac{\mu_n}{2} (v_{IN} - V_{TN})^2 = \frac{W_p}{L_p} \mu_p \left[(V_{DD} - v_{IN} - |V_{TP}|)(V_{DD} - v_{OUT}) - \frac{(V_{DD} - v_{OUT})^2}{2} \right]$$

Differentiating w.r.t. v_{IN},

$$\frac{W_N}{L_N} \mu_n (v_{IN} - V_{TN}) = \frac{W_p}{L_p} \mu_p \left[(V_{DD} - v_{IN} - |V_{TP}|)\left(-\frac{\partial v_{OUT}}{\partial v_{IN}}\right) + (V_{DD} - v_{OUT})(-1) - (V_{DD} - v_{OUT})\left(-\frac{\partial v_{OUT}}{\partial v_{IN}}\right) \right]$$

If $v_{IN} = V_{IL}$ and $v_{OUT} = V_{OUT}$

$$\frac{W_N}{L_N}\mu_n\left(V_{IL}-V_{TN}\right) = \frac{W_P}{L_P}\mu_P\left[\left(V_{DD}-V_{IL}-|V_{TP}|\right)+\left(V_{DD}-V_{OUT}\right) + \left(V_{DD}-V_{OUT}\right)\right]$$

(2) $$V_{IL} = \frac{2V_{OUT} - V_{DD} - |V_{TP}| + (k_N/k_P)(V_{TN})}{1 + k_N/k_P}$$

$$k_N = \mu_n C_{ox}\frac{W_N}{L_N}, \quad k_P = \mu_P C_{ox}\frac{W_P}{L_P}$$

Solve eqs. (1) and (2) to find V_{IL}.

V_{IH} (Regim IV)

$$i_{DN}(lin) = i_{DP}(sat)$$

$$\frac{W_N}{L_N}\frac{\mu_N C_{ox}}{\left(1+\frac{v_{OUT}}{E_{CN}L_N}\right)}\left[\left(v_{IN}-V_{TN}\right)v_{OUT} - \frac{v_{OUT}^2}{2}\right] = \frac{W_P v_{sat} C_{ox}\left(V_{DD}-v_{IN}-|V_{TP}|\right)^2}{\left(V_{DD}-v_{IN}-|V_{TP}|\right)+E_{CP}L_P}$$

Assumptions:

v_{OUT} is small $\rightarrow \frac{v_{OUT}}{E_{CN}L_N} < 1$ and $V_{DD}-v_{IN}-|V_{TP}| < E_{CP}L_P$

(1) $\therefore$ $$\mu_n C_{ox}\frac{W_N}{L_N}\left[\left(v_{IN}-V_{TN}\right)v_{OUT} - \frac{v_{OUT}^2}{2}\right] \approx \frac{W_P}{L_P}\frac{C_{ox}\mu_P}{2}\left[V_{DD}-v_{IN}-|V_{TP}|\right]^2$$

Differentiating w.r.t. v_{IN}, gives

$$\mu_N C_{ox}\frac{W_N}{L_N}\left[\left(V_{IN}-V_{TN}\right)\left(\frac{\partial v_{OUT}}{\partial v_{IN}}\right) + v_{OUT} - v_{OUT}\frac{\partial v_{OUT}}{\partial v_{IN}}\right]$$

$$= \mu_P C_{ox}\frac{W_P}{L_P}\left[\left(V_{DD}-V_{IN}-|V_{TP}|\right]\right](-1)$$

If $\mu_P C_{ox}\frac{W_P}{L_P} = k_P$ and $\mu_n C_{ox}\frac{W_N}{L_N} = k_N$ and $\frac{\partial v_{OUT}}{\partial v_{IN}} = -1$

(2) $\therefore$ $$V_{IH} = \frac{2V_{OUT} + V_{TN} + (k_P/k_N)(V_{DD}-|V_{TP}|)}{1 + k_P/k_N}$$

Solve Eqs. (1) and (2) for V_{IH}.

Example

Find NM_H and NM_L for a CMOS inverter if
$W_N = 400nm$, $W_P = 800nm$, $\mu_n = 270\ cm^2/V\cdot s$,
$\mu_p = 70 cm^2/V\cdot s$, $C_{ox} = 1\mu F/cm^2$, $V_{TN} = 0.5V$,
$V_{TP} = -0.5V$, $E_{CN}L_N = 1.2V$, $E_{CP}L_P = 4.8V$, $N_{sat} = 8\times10^{-6}\ cm/s$,
$V_{DD} = 1.8V$ and $L_P = L_N = L = 200nm$.

Solution:

$V_{OH} = V_{DD} = \underline{1.8V}$, $V_{OL} = \underline{0}$, $k_N = (270)(10^{-6})(2) = 540\ \mu A/V^2$

$k_P = (70)(10^{-6})(4) = 280\ \mu A/V^2$

V_{IL}:
$$V_{IL} = \frac{2V_{OUT} - 1.8 - 0.5 + \left(\frac{540}{280}\right)0.5}{1 + \frac{540}{280}} \rightarrow V_{OUT} = 1.4645 V_{IL} + 0.668 \quad ①$$

2nd eq.
$$\frac{540}{2}(V_{IL} - 0.5)^2 = 280\left[(1.8 - V_{IL} - 0.5)(1.8 - V_{OUT}) - \frac{(1.8 - V_{OUT})^2}{2}\right]$$

Assume $V_{OUT} \approx 1.8 \rightarrow 1.8 - V_{out} \approx 0$ (so the last term $\rightarrow 0$)

$$(V_{IL} - 0.5)^2 \simeq 1.037\,(1.3 - V_{IL})(1.8 - V_{OUT}) \quad ②$$

Substituting ① into ② gives

$$V_{IL}^2 - 4.141 V_{IL} + 2.460 = 0 \rightarrow V_{IL} = \sout{0.7093}\ 0.719V$$
$(0.626V)$

V_{IH}:
$$V_{IH} = \frac{2V_{OUT} + 0.5 + \frac{280}{540}(1.8 - 0.5)}{1 + \frac{280}{540}} \rightarrow V_{OUT} = 0.7593 V_{IH} - 0.5875 \quad ①$$

$$540\left[(V_{IH} - 0.5)V_{OUT} - \frac{V_{OUT}^2}{2}\right] = \frac{280}{2}\left[1.8 - V_{IH} - 0.5\right]$$

Assume $V_{OUT} \approx 0$ (so $\frac{V_{OUT}^2}{2} \rightarrow 0$)

$$\therefore (V_{IH} - 0.5)V_{OUT} \simeq \frac{280}{2\cdot 540}(1.3 - V_{IH})^2 \quad ②$$

$$① \rightarrow ② \Rightarrow V_{IH}^2 - 0.6299 V_{IH} - 0.3098 = 0$$

$$V_{IH} = 0.954V \quad (0.909V)$$

$NM_H = V_{OH} - V_{IH} = 1.8 - 0.954 = \underline{\underline{0.846\,V}}$

$NM_L = V_{IL} - V_{OL} = 0.719 - 0 = \underline{\underline{0.719\,V}}$

Layout

(See layout of CMOS inverter)